1) a)

I) $A^2 = \begin{pmatrix}0&1&0\\0&0&1\\1&0&0\end{pmatrix}\begin{pmatrix}0&1&0\\0&0&1\\1&0&0\end{pmatrix} = \begin{pmatrix}0&0&1\\1&0&0\\0&1&0\end{pmatrix}$

$|A| = 0+0+1-0-0-0 = 1$

$Adj(A) = \begin{pmatrix}0&1&0\\0&0&1\\1&0&0\end{pmatrix}$ $\quad (Adj(A))^t = \begin{pmatrix}0&0&1\\1&0&0\\0&1&0\end{pmatrix}$

$A^{-1} = \begin{pmatrix}0&0&1\\1&0&0\\0&1&0\end{pmatrix} = A^2$

II) si $A^2 = A^{-1} \Rightarrow A \cdot A^2 = A \cdot A^{-1} \Rightarrow A^3 = I_3$

$A^{100} = (A^3)^{333} \cdot A = (I_3)^{333} \cdot A = I_3 \cdot A = \underline{\underline{A}}$

$A^{100} = \begin{pmatrix}0&1&0\\0&0&1\\1&0&0\end{pmatrix}$

b)

$I \rightarrow x$

$O.A \rightarrow y$

$J \rightarrow z$

$$\left.\begin{array}{l} 0'6x+0'5y = 0'3(x+y+z) \\ 0'2x+0'6y+0'6z = \frac{x+y+z}{2} \\ y = x+100 \end{array}\right\}$$

$0'6x+0'5y = 0'3x+0'3y+0'3z$

$0'4x + 1'2y + 1'2z = x+y+z$

$y = x+100$

$$\left\{\begin{array}{l} 0'3x+0'2y-0'3z = 0 \\ -0'6x+0'2y+0'2z = 0 \\ -x+y \qquad = 100 \end{array}\right\}$$

$\left(\begin{array}{ccc|c} 0'3 & 0'2 & -0'3 & 0 \\ -0'6 & 0'2 & 0'2 & 0 \\ -1 & 1 & 0 & 100 \end{array}\right) F1 \rightleftarrows F3 \quad \left(\begin{array}{ccc|c} -1 & 1 & 0 & 100 \\ -0'6 & 0'2 & 0'2 & 0 \\ 0'3 & 0'2 & -0'3 & 0 \end{array}\right)$

$-0'6F1+F2 \rightarrow F2$

$0'3F1+F3 \rightarrow F3$

$$\left(\begin{array}{ccc|c} -1 & 1 & 0 & 100 \\ 0 & -0'4 & 0'2 & -60 \\ 0 & 0'5 & -0'3 & 30 \end{array}\right) \xrightarrow{0'5F2+0'4F3} F3 \left(\begin{array}{ccc|c} -1 & 1 & 0 & 100 \\ 0 & -0'4 & 0'2 & -60 \\ 0 & 0 & -0'02 & -18 \end{array}\right)$$

$$\left.\begin{array}{l} -x+y = 100 \\ -0'4y + 0'2z = -60 \\ -0'02z = -18 \end{array}\right\} \quad z = \frac{-18}{-0'02} = 900//$$

$-0'4y + 180 = -60$; $-0'4y = -240$

$y = \dfrac{-240}{-0'4} = 600//$

$(500, 600, 900)$

$-x + 600 = 100$; $x = 500//$

2) a) I) $A \times B = B^2$; $\underbrace{A^{-1}A}_{In} \times \underbrace{BB^{-1}}_{In} = A^{-1}\underbrace{B^2B^{-1}}_{B}$

$X = A^{-1}B$

II) $|A| = 0 + 0 + 0 + 1 + 0 + 0 = 1$

$$Adj(A) = \begin{pmatrix} -1 & -1 & -1 \\ 1 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix} ; \quad (Adj(A))^t = \begin{pmatrix} -1 & 1 & 1 \\ -1 & 1 & 0 \\ -1 & 0 & 0 \end{pmatrix}$$

$$A^{-1} = \frac{(Adj(A))^t}{|A|} = \begin{pmatrix} -1 & 1 & 1 \\ -1 & 1 & 0 \\ -1 & 0 & 0 \end{pmatrix}$$

$$X = \begin{pmatrix} -1 & 1 & 1 \\ -1 & 1 & 0 \\ -1 & 0 & 0 \end{pmatrix} \begin{pmatrix} 1 & 0 & 2 \\ 0 & 1 & 3 \\ 1 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 0 & 1 & 2 \\ -1 & 1 & 1 \\ -1 & 0 & -2 \end{pmatrix}$$

b) $\begin{vmatrix} x & x-1 & 1 \\ 0 & 1 & 2 \\ -1 & 2 & 0 \end{vmatrix} = -2(x-1)+1-4x =$

$= -2x+2+1-4x = -6x+3$

$$\begin{vmatrix} 1 & -1 & x & 1 \\ -1 & -1 & 0 & 0 \\ 1 & 0 & x & 1 \\ x & 0 & -2 & x \end{vmatrix} = 1 \cdot \begin{vmatrix} -1 & x & 1 \\ 0 & x & 1 \\ 0 & -2 & x \end{vmatrix} - 1 \underbrace{\begin{vmatrix} 1 & x & 1 \\ 1 & x & 1 \\ x & -2 & x \end{vmatrix}}_{\overset{\shortparallel}{0}} =$$

$F1 = F2$

$= -x^2 - 2$; $(-6x+3) - (-x^2 - 2) = 0$

$-6x+3+x^2+2=0$; $\underline{\underline{x^2 - 6x + 5 = 0}}$

$$x = \frac{6 \pm \sqrt{36-20}}{2} = \frac{6 \pm 4}{2} = \begin{cases} 5 \\ 1 \end{cases}$$

3) $|A| = \begin{vmatrix} k-2 & -1 & 1 \\ 1 & 2k-1 & -k \\ 1 & k & -1 \end{vmatrix} = -(k-2)(2k-1)+k+k$

$-(2k-1)+k^2(k-2)-1 =$

$= -2k^2+k+4k-\underline{2}+k+k-2k+\underline{1}+k^3-2k^2\underline{-1} =$

$= k^3-4k^2+5k-2 = (k-1)^2(k-2)$

$$\begin{array}{c|cccc} & 1 & -4 & 5 & -2 \\ ① & & 1 & -3 & 2 \\ \hline & 1 & -3 & 2 & \lfloor 0 \end{array}$$

$k^2-3k+2=0$; $k = \dfrac{3 \pm \sqrt{9-8}}{2} = \dfrac{3 \pm 1}{2} = \begin{cases} ② \\ ① \end{cases}$

Si $k = 1$

$$\left(\begin{array}{ccc|c} -1 & -1 & 1 & -2 \\ 1 & 1 & -1 & 2 \\ 1 & 1 & -1 & 2 \end{array}\right) \begin{array}{l} F1+F2 \\ \longrightarrow F2 \\ F1+F3 \\ \longrightarrow F3 \end{array}$$

$$\left(\begin{array}{ccc|c} -1 & -1 & 1 & -2 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right) \quad \left.\begin{array}{l} \text{rang } A = 1 \\ \text{rang } A/B = 1 \end{array}\right\} \begin{array}{l} \underline{\underline{S.C.I}} \\ G.I = 3-1 = 2 \,// \end{array}$$

$$-x - y + z = -2 \,; \qquad z = x + y - 2$$

$$\underline{\underline{(\lambda, \mu, \lambda + \mu - 2)}} \quad \lambda, \mu \in \mathbb{R}$$

Si $k = 2$

$$\left(\begin{array}{ccc|c} 0 & -1 & 1 & -2 \\ 1 & 3 & -2 & 2 \\ 1 & 2 & -1 & 2 \end{array}\right) F1 \rightleftarrows F2 \left(\begin{array}{ccc|c} 1 & 3 & -2 & 2 \\ 0 & -1 & 1 & -2 \\ 1 & 2 & -1 & 2 \end{array}\right)$$

$$\begin{array}{l} F1-F3 \\ \rightarrow F3 \end{array} \left(\begin{array}{ccc|c} 1 & 3 & -2 & 2 \\ 0 & -1 & 1 & -2 \\ 0 & 1 & -1 & 0 \end{array}\right) \begin{array}{l} F2+F3 \\ \rightarrow F3 \end{array} \left(\begin{array}{ccc|c} 1 & 3 & -2 & 2 \\ 0 & -1 & 1 & -2 \\ 0 & 0 & 0 & -2 \end{array}\right)$$

$$\text{rang } A = 2 \;; \quad \text{rang } A/B = 3 \quad \Longrightarrow \underline{\underline{S.I}}$$

Si $k \neq 1, 2 \longrightarrow \underline{\underline{S.C.D}}$ rang A = rang A/B = 3

$$X = \frac{\begin{vmatrix} -2 & -1 & 1 \\ 2 & 2k-1 & -k \\ 2 & k & -1 \end{vmatrix}}{(k-1)^2 (k-2)} = \frac{-2\cancel{(k-1)^2}}{\cancel{(k-1)^2}(k-2)} = \frac{-2}{k-2} \;//$$

$$4k - 2 + 2k + 2k - 4k + 2 - 2k^2 - 2 =$$

$$= -2k^2 + 4k - 2 = -2(k^2 - 2k + 1)$$

$$y = \frac{\begin{vmatrix} k-2 & -2 & 1 \\ 1 & 2 & -k \\ 1 & 2 & -1 \end{vmatrix}}{(k-1)^2(k-2)} = \frac{-\underline{2k}+\underline{4}+\underline{2}+\underline{2k}-\underline{2}+\underline{2k^2}-\underline{4k}-\underline{2}}{} =$$

$$= 2k^2-4k-2 = \frac{2(k^2-2k+2)}{(k-1)^2(k-2)} = \frac{2(k-\cancel{1})^2}{(\cancel{k-1})^2(k-2)} = \frac{2}{k-2}$$

$$z = \frac{\begin{vmatrix} k-2 & -1 & -2 \\ 1 & 2k-1 & 2 \\ 1 & k & 2 \end{vmatrix}}{(k-1)^2(k-2)} = \frac{2(k-2)(2k-1)-2k-2+2(2k-1)-2k(k-2)}{}$$

$$+2 = \overbrace{4k^2}-\overbrace{2k}-\overbrace{8k}+\underline{4}-2k-\underline{2}+\overbrace{4k}-\underline{2}-\overbrace{2k^2}+\overbrace{4k}$$

$$+2 = \frac{2k^2-4k+2}{(k-1)^2(k-2)} = \frac{2(k^2-2k+1)}{(k-1)^2(k-2)} =$$

$$= \frac{2\cancel{(k-1)^2}}{\cancel{(k-1)^2}(k-2)} = \frac{2}{k-2}$$

1)

①

a) A → x sgect
B − y ″
C − z ″

$$\left.\begin{array}{l} 3x+4y+5z=720 \\ x+y+z=200 \\ 4x+3y+5z=740 \end{array}\right\}$$

$$\left(\begin{array}{ccc|c} 3 & 4 & 5 & 720 \\ 1 & 1 & 1 & 200 \\ 4 & 3 & 5 & 740 \end{array}\right) \xrightarrow{F1 \rightleftarrows F2} \left(\begin{array}{ccc|c} 1 & 1 & 1 & 200 \\ 3 & 4 & 5 & 720 \\ 4 & 3 & 5 & 740 \end{array}\right) \begin{array}{l} -3F1+F2 \to F2 \\ -4F1+F3 \to F3 \end{array}$$

$$\left(\begin{array}{ccc|c} 1 & 1 & 1 & 200 \\ 0 & 1 & 2 & 120 \\ 0 & -1 & 1 & -60 \end{array}\right) \begin{array}{l} F2+F3 \\ \to F3 \end{array} \left(\begin{array}{ccc|c} 1 & 1 & 1 & 200 \\ 0 & 1 & 2 & 120 \\ 0 & 0 & 3 & 60 \end{array}\right)$$

$$\left.\begin{array}{r} x+y+z=200 \\ y+2z=120 \\ 3z=60 \end{array}\right\} \begin{array}{l} \to x+80+20=200 \Rightarrow x=100 \\ \to y+40=120 \to y=80 \\ \to z=\frac{60}{3}=20 \end{array}$$

$$\underline{\underline{(100,\ 80,\ 20)}}$$

b)
$$\begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix}\begin{pmatrix} x & y \\ y & z \end{pmatrix} + \begin{pmatrix} x & y \\ y & z \end{pmatrix}\begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix} = \begin{pmatrix} 5 & 2 \\ 3 & -1 \end{pmatrix}$$

$$\begin{pmatrix} x & y \\ x+y & y+z \end{pmatrix} + \begin{pmatrix} x+y & y \\ y+z & z \end{pmatrix} = \begin{pmatrix} 5 & 2 \\ 3 & -1 \end{pmatrix}$$

$$\begin{pmatrix} x+x+y & y+y \\ x+y+y+z & y+z+z \end{pmatrix} = \begin{pmatrix} 5 & 2 \\ 3 & -1 \end{pmatrix}$$

$$\begin{pmatrix} 2x+y & 2y \\ x+2y+z & y+2z \end{pmatrix} = \begin{pmatrix} 5 & 2 \\ 3 & -1 \end{pmatrix}$$

$$\left.\begin{array}{r} 2x+y=5 \\ 2y=2 \\ x+2y+z=3 \\ y+2z=-1 \end{array}\right\} \begin{array}{l} \to x=2 \\ \to y=1 \\ \to z=-1 \end{array}$$

$$\underline{\underline{X=\begin{pmatrix} 2 & 1 \\ 1 & -1 \end{pmatrix}}}$$

$|X| = -2-1 = -3$ $\quad \operatorname{Adj}(A) = \begin{pmatrix} -1 & -1 \\ -1 & 2 \end{pmatrix}$

$(\operatorname{Adj}(A))^t = \begin{pmatrix} -1 & -1 \\ -1 & 2 \end{pmatrix}$ $\quad \underline{\underline{A^{-1} = \begin{pmatrix} 1/3 & 1/3 \\ 1/3 & -2/3 \end{pmatrix}}}$

2) a) $3B = \begin{pmatrix} 3 & 6 & 0 \\ 3\lambda & 0 & 3 \\ 0 & 3 & -6 \end{pmatrix}$

$B^2 = \begin{pmatrix} 1 & 2 & 0 \\ \lambda & 0 & 1 \\ 0 & 1 & -2 \end{pmatrix} \begin{pmatrix} 1 & 2 & 0 \\ \lambda & 0 & 1 \\ 0 & 1 & -2 \end{pmatrix} = \begin{pmatrix} 1+2\lambda & 2 & 2 \\ \lambda & 2\lambda+1 & -2 \\ \lambda & -2 & 5 \end{pmatrix}$

$3B + B^2 = \begin{pmatrix} 4+2\lambda & 8 & 2 \\ 4\lambda & 2\lambda+1 & 1 \\ \lambda & 1 & -1 \end{pmatrix}$

$\begin{array}{r} 34 \\ 34 \\ \hline 136 \\ 102 \\ \hline 1156 \end{array} \qquad \begin{array}{r} 32 \\ 8 \\ \hline 256 \end{array}$

$|3B+B^2| = -(4+2\lambda)(2\lambda+1) + 8\lambda + 8\lambda - 2\lambda(2\lambda+1)$

$-(4+2\lambda) + 32\lambda = -8\lambda - \underline{4} - 4\lambda^2 - 2\lambda + 8\lambda + 8\lambda$

$-4\lambda^2 - 2\lambda - \underline{4} - 2\lambda + 32\lambda = -8\lambda^2 + 34\lambda - 8$

$\lambda = \frac{-34 \pm \sqrt{1156-256}}{-16} = \frac{-34 \pm 30}{-16} = \begin{cases} \frac{-64}{-16} = 4 \text{//} \\ \frac{-4}{-16} = 1/4 \text{//} \end{cases}$

b) $\underbrace{\begin{vmatrix} a & b & c \\ -a & -b & -c \\ a+c & b-a & c+b \end{vmatrix}}_{0} + \begin{vmatrix} a & b & c \\ c & -a & b \\ a+c & b-a & c+b \end{vmatrix} =$

$= \underbrace{\begin{vmatrix} a & b & c \\ c & -a & b \\ a & b & c \end{vmatrix}}_{\overset{\shortparallel}{0}} + \underbrace{\begin{vmatrix} a & b & c \\ c & -a & b \\ c & -a & b \end{vmatrix}}_{\overset{\shortparallel}{0}} = 0$

c) $$\begin{vmatrix} x & 1/4 & 4 \\ y & 0 & 4 \\ z & 1/2 & 12 \end{vmatrix} = \begin{vmatrix} x & y & z \\ 1/4 & 0 & 1/2 \\ 4 & 4 & 12 \end{vmatrix} =$$

$$= \frac{1}{4} \begin{vmatrix} x & y & z \\ 1 & 0 & 2 \\ 4 & 4 & 12 \end{vmatrix} = \underbrace{\frac{1}{4} \cdot 4}_{1} \underbrace{\begin{vmatrix} x & y & z \\ 1 & 0 & 2 \\ 1 & 1 & 3 \end{vmatrix}}_{1} = \underline{\underline{1}}$$

d) $$\left| \frac{1}{2} A \right| = \left(\frac{1}{2}\right)^3 |A| = \frac{1}{8} \cdot (-8) = \frac{-8}{8} = \underline{\underline{-1}}$$

3) $$\begin{vmatrix} 1 & \lambda & 0 \\ \lambda & 1 & \lambda-1 \\ \lambda & 1 & 0 \end{vmatrix} = \lambda^2(\lambda-1) - (\lambda-1) = \lambda^3 - \lambda^2 - \lambda + 1$$
$$= \underline{\underline{(\lambda-1)^2(\lambda+1)}}$$

$$\begin{array}{c|cccc} & 1 & -1 & -1 & 1 \\ 1 & & 1 & 0 & -1 \\ \hline & 1 & 0 & -1 & 0 \end{array}$$

$\lambda^2 - 1 = 0 \Rightarrow \lambda = \pm 1$

$\underline{\underline{\lambda = +1}}$ $$\left(\begin{array}{ccc|c} 1 & 1 & 0 & 1 \\ 1 & 1 & 0 & 1 \\ 1 & 1 & 0 & 3 \end{array}\right) \sim \left(\begin{array}{ccc|c} 1 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 2 \end{array}\right)$$

$$\left(\begin{array}{ccc|c} 1 & 1 & 0 & 1 \\ 0 & 0 & 0 & 2 \\ 0 & 0 & 0 & 0 \end{array}\right)$$ rang $A = 1$; rang $A/B = 2$

$\underline{\underline{\text{S.I}}}$

$\underline{\underline{\lambda = -1}}$ $$\left(\begin{array}{ccc|c} 1 & -1 & 0 & -1 \\ -1 & 1 & -2 & 1 \\ -1 & 1 & 0 & 1 \end{array}\right) \sim \left(\begin{array}{ccc|c} 1 & -1 & 0 & -1 \\ 0 & 0 & -2 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right)$$

rang $A = 2$
rang $A/B = 2$ } $\underline{\underline{\text{S.C.I}}}$

$x - y = -1 \rightarrow x = -1 + y$
$-2z = 0 \rightarrow z = 0$

$$\underline{\underline{(-1+\mu,\ \mu,\ 0)}} \quad \mu \in \mathbb{R}$$

Si $\lambda \neq -1, 1 \Rightarrow$ S.C.D

$$x = \frac{\begin{vmatrix} \lambda & \lambda & 0 \\ 1 & 1 & \lambda-1 \\ 2+\lambda & 1 & 0 \end{vmatrix}}{(\lambda-1)^2(\lambda+1)} = \frac{\lambda(\lambda-\cancel{1})\cancel{(\lambda+1)}}{(\lambda-1)^{\cancel{2}}\cancel{(\lambda+1)}} = \boxed{\frac{\lambda}{\lambda-1}}$$

$$\lambda(\lambda-1)(2+\lambda) - \lambda(\lambda-1) = \\ = \lambda(\lambda-1)\underbrace{[(2+\lambda)-1]}_{\lambda+1}$$

$$y = \frac{\begin{vmatrix} 1 & \lambda & 0 \\ \lambda & 1 & \lambda-1 \\ \lambda & 2+\lambda & 0 \end{vmatrix}}{(\lambda-1)^2(\lambda+1)} = \frac{\cancel{(\lambda-1)}(\lambda-2)\cancel{(\lambda+1)}}{(\lambda-1)^{\cancel{2}}\cancel{(\lambda+1)}} = \boxed{\frac{\lambda-2}{\lambda-1}}$$

$$\lambda^2(\lambda-1) - (\lambda-1)(2+\lambda) = \\ = (\lambda-1)\underbrace{(\lambda^2-\lambda-2)}_{(\lambda-2)(\lambda+1)}$$

$$z = \frac{\begin{vmatrix} 1 & \lambda & \lambda \\ \lambda & 1 & 1 \\ \lambda & 1 & 2+\lambda \end{vmatrix}}{(\lambda-1)^2(\lambda+1)}$$

$$2+\lambda+\lambda^2+\cancel{\lambda^2}-\cancel{\lambda^2}-1-\lambda^2(2+\lambda) = \\ = 2+\lambda+\lambda^2-1-2\lambda^2-\lambda^3 = \\ = -\lambda^3-\lambda^2+\lambda+1 \\ = \frac{-\cancel{(\lambda-1)}(\lambda+1)^{\cancel{2}}}{(\lambda-1)^{\cancel{2}}\cancel{(\lambda+1)}} = \boxed{\frac{-(\lambda+1)}{\lambda-1}}$$

$$\begin{array}{c|cccc} & -1 & -1 & 1 & 1 \\ 1 & & -1 & -2 & -1 \\ \hline & -1 & -2 & -1 & 0 \end{array}$$

$$-\lambda^2-2\lambda-1=0 \qquad \lambda = \frac{2 \pm \sqrt{4-4}}{-2} = \begin{cases} -1 \\ -1 \end{cases}$$

①

1)

a) $\begin{vmatrix} 1 & -2 & 2 \\ 1 & \alpha & -1 \\ 2 & -2 & \beta \end{vmatrix} = 0$; $2 - 2\alpha - \beta = 1$

$\alpha\beta \cancel{-4} \cancel{+4} - 4\alpha - 2 + 2\beta = 0$; $2\alpha + \beta = 1$

$$\left.\begin{array}{l} \alpha\beta - 4\alpha + 2\beta - 2 = 0 \\ 2\alpha + \beta = 1 \end{array}\right\} \quad \beta = 1 - 2\alpha$$

$\alpha(1 - 2\alpha) - 4\alpha + 2(1 - 2\alpha) - 2 = 0$

$\alpha - 2\alpha^2 - 4\alpha \cancel{+2} - 4\alpha \cancel{-2} = 0$

$-2\alpha^2 - 7\alpha = 0$; $\alpha(-2\alpha - 7) = 0 \begin{array}{l} \nearrow \alpha = 0 \; // \\ \searrow \alpha = -7/2 \; // \end{array}$

$\boxed{\alpha_1 = 0 \Rightarrow \beta_1 = 1}$ $\boxed{\alpha_2 = -\frac{7}{2} \Rightarrow \beta_2 = 8}$

b) $2 - 2\alpha - \beta = 0$

$\begin{vmatrix} 1 & -2 & 2 \\ 1 & \alpha & -1 \\ 2 & -2 & \beta \end{vmatrix} = \alpha\beta - 4\alpha + 2\beta - 2 = \pm 6$

$$\left.\begin{array}{l} \alpha\beta - 4\alpha + 2\beta - 2 = \pm 6 \\ 2\alpha + \beta = 2 \end{array}\right\} \quad \beta = 2 - 2\alpha$$

$\alpha(2 - 2\alpha) - 4\alpha + 2(2 - 2\alpha) - 2 = \pm 6$

$2\alpha - 2\alpha^2 - 4\alpha + 4 - 4\alpha - 2 = \pm 6$

$-2\alpha^2 - 6\alpha - 4 = 0$; $\alpha^2 + 3\alpha + 2 = 0$

$\alpha = \dfrac{-3 \pm \sqrt{9 - 8}}{2} = \dfrac{-3 \pm 1}{2} = \begin{cases} -1 \\ -2 \end{cases}$

$-2\alpha^2 - 6\alpha + 8 = 0$

$\alpha^2 + 3\alpha - 4 = 0 \begin{array}{l} \nearrow \alpha = -4 \\ \searrow \alpha = 1 \end{array}$

$\alpha_1 = -1 \Rightarrow \beta_1 = 4$

$\alpha_2 = -2 \Rightarrow \beta_2 = 6$

~~[illegible]~~

$\alpha_1 = -4 \Rightarrow \beta_1 = 10$; $\alpha_2 = 1$; $\beta_2 = 0$

c) $\vec{u} \perp \vec{v} \Rightarrow 1-2\alpha-2=0$; $-2\alpha=1$; $\alpha=-1/2$ //

$\vec{u} \perp \vec{w} \Rightarrow 2+4+2\beta=0 \Rightarrow 2\beta=-6$; $\beta=-3$ //

$\vec{v}\,(1,-1/2,-1)$; $\vec{w}\,(2,-2,-3)$

$\vec{v}\cdot\vec{w} = 2+1+3=7\neq 0$ No puede ser base ortogonal

2) a) $\vec{u_r}=(2,0,a)$

$$\vec{u_s} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & -1 & 2 \\ 0 & 1 & 1 \end{vmatrix} = (-3,-1,1)$$

$P_r(-1,-3,-2)$; $P_s(7,-1,0)$

$\overrightarrow{P_rP_s}\,(8,2,2)$

$$\begin{vmatrix} 2 & 0 & a \\ -3 & -1 & 1 \\ 8 & 2 & 2 \end{vmatrix} = 0$$

$-4-6a+8a-4=0$; $2a-8=0$, $2a=8$, $\underline{\underline{a=4}}$

Si $a\neq 4 \Rightarrow$ Se cruzan

Si $a=4 \Rightarrow \begin{array}{l}\vec{u_r}=(2,0,4)\\ \vec{u_s}=(-3,-1,1)\end{array} \Rightarrow$ Coplanarias secantes

¿ $-\frac{3}{2}=\frac{-1}{0}=\frac{1}{4}$? No

$\boxed{a=4}$

$$r: \begin{cases} x=-1+2t_1 \\ y=-3 \\ z=-2+4t_1 \end{cases} \quad ; \quad s: \begin{cases} x=7-3t_2 \\ y=-1-t_2 \\ z=t_2 \end{cases}$$

$$\left.\begin{array}{l} -1+2t_1 = 7-3t_2 \\ -3 = -1-t_2 \\ -2+4t_1 = t_2 \end{array}\right\} \begin{array}{l} \to t_2 = 2 // \\ \to -2+4t_1 = 2 \; ; \; \Rightarrow t_1 = 1 // \end{array}$$

$$\underline{\underline{I = (1, -3, 2)}}$$

$$\vec{n} = \vec{u_r} \wedge \vec{u_s} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 2 & 0 & 4 \\ -3 & -1 & 1 \end{vmatrix} = (4, -14, -2)$$

$4x - 14y - 2z + D = 0 \to$ Pasa por $(1, -3, 2)$

$4 + 42 - 4 + D = 0$; $D = -42$

$4x - 14y - 2z - 42 = 0$; $\underline{\underline{\pi: 2x - 7y - z - 21 = 0}}$

b) $\vec{u_r} \perp \vec{u_s} \Rightarrow -6 + a = 0 \Rightarrow \underline{a = 6}$

$$d(r,s) = \frac{\left| \begin{vmatrix} 2 & 0 & 6 \\ -3 & -1 & 1 \\ 8 & 2 & 2 \end{vmatrix} \right|}{|(6, -20, -2)|} = \frac{|-4-36+48-4|}{\sqrt{6^2+(-20)^2+(-2)^2}} =$$

$$\overline{u_r} \wedge \overline{u_s} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 2 & 0 & 6 \\ -3 & -1 & 1 \end{vmatrix} = (6, -20, -2)$$

$$= \frac{|4|}{\sqrt{440}} = \frac{4}{2\sqrt{110}} = \frac{2}{\sqrt{110}} = \frac{2\sqrt{110}}{110} =$$

$$\underline{\underline{= \frac{\sqrt{110}}{55} \; u}}$$

3) a) $\overrightarrow{u_r} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & 1 & 0 \\ 1 & 0 & -1 \end{vmatrix} = (-1, +1, -1)$

$P_r(0, 1, 2)$ $r: \begin{cases} x = -t \\ y = 1 + t \\ z = 2 - t \end{cases}$

$M_{AB} = (-1, 1, 3)$ $\overrightarrow{AB}(-4, -4, 2)$

$\pi_{med}: -4x - 4y + 2z + D = 0$; $4 - 4 + 6 + D = 0$. $D = -6$

$-4x - 4y + 2z - 6 = 0$

$r \cap \pi_{med}$; $-4(-t) - 4(1+t) + 2(2-t) - 6 = 0$

$\cancel{4t} \cancel{-4} \cancel{-4t} \cancel{+4} - 2t - 6 = 0$; $-2t = 6$, $t = -3$

$\boxed{P(3, -2, 5)}$

b) $\overrightarrow{u_r} = (-1, 1, -1)$ $\overrightarrow{AB}(-4, -4, 2)$

$P_r(0, 1, 2)$

$\vec{n} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ -1 & 1 & -1 \\ -4 & -4 & 2 \end{vmatrix} = (-2, 6, 8)$

$-2x + 6y + 8z + D = 0$; $6 \cdot 1 + 8 \cdot 2 + D = 0$

$6 + 16 + D = 0$. $D = -22$. $-2x + 6y + 8z - 22 = 0$

$\underline{\underline{\pi: x - 3y - 4z + 11 = 0}}$

c) $-x+y-z+D=0$

$-1+3-2+D=0$; $D=0$

$-x+y-z=0$; $\pi_A: x-y+z=0$

$P_{med}=r\cap\pi_A$: $(-t)-(1+t)+(2-t)=0$

$-t-1-t+2-t=0$; $-3t+1=0$; $t=1/3$

$P_{med}=\left(-\frac{1}{3},\frac{4}{3},\frac{5}{3}\right)$, $P_{sim}=(x,y,z)$

$\frac{x+1}{2}=-\frac{1}{3}\Rightarrow 3x+3=-2\Rightarrow 3x=-5\Rightarrow x=-5/3$

$\frac{y+3}{2}=\frac{4}{3}\Rightarrow 3y+9=8\Rightarrow 3y=-1\Rightarrow y=-1/3$

$\frac{z+2}{2}=\frac{5}{3}\Rightarrow 3z+6=10\underset{5}{\Rightarrow} 3z=4\Rightarrow z=\frac{4}{3}$

$$P_{sim}=\left(-\frac{5}{3},-\frac{1}{3},\frac{4}{3}\right)$$

4) a) $\overline{u_r}=\overline{n_2}=\begin{vmatrix}\vec{i}&\vec{j}&\vec{k}\\1&2&0\\-1&-3&-1\end{vmatrix}=(-2,1,-1)$

$$r:\begin{cases}x=1-2t\\y=1+t\\z=-2-t\end{cases}$$

b) $\overline{n_2}=(-2,1,-1)$ $\overline{n_1}(1,3,1)$

$\overline{u_s}=\begin{vmatrix}\vec{i}&\vec{j}&\vec{k}\\1&3&1\\-2&1&-1\end{vmatrix}=(-4,-1,7)$

$$s:\begin{cases}x=1-4t\\y=1-t\\z=-2+7t\end{cases}$$

c) $P_{\pi_2} = (1, 0, 2)$ $\quad \vec{n_2} = (-2, 1, -1)$

$\pi_2 : -2x + y - z + D = 0$

$-2 + 0 - 2 + D = 0 \Rightarrow D = 4$

$\pi_2 : -2x + y - z + 4 = 0$; $\quad \pi_2 : 2x - y + z - 4 = 0$

$$r : \pi_1 \wedge \pi_2 : \begin{cases} x + 3y + z - 8 = 0 \\ 2x - y + z - 4 = 0 \end{cases}$$

Plano $\underline{\underline{OXY}}$: $\underline{\underline{z = 0}}$ $\quad \left.\begin{matrix} x + 3y = 8 \\ 2x - y = 4 \end{matrix}\right\} \underline{\underline{\left(\frac{20}{7}, \frac{12}{7}, 0\right)}}$

plano $\underline{\underline{OXZ}}$: $\underline{\underline{y = 0}}$ $\quad \left.\begin{matrix} x + z = 8 \\ 2x + z = 4 \end{matrix}\right\} \underline{\underline{(-4, 0, 12)}}$

plano $\underline{\underline{OYZ}}$: $\underline{\underline{x = 0}}$ $\quad \left.\begin{matrix} 3y + z = 8 \\ -y + z = 4 \end{matrix}\right\} \underline{\underline{(0, 1, 5)}}$

1) $\overrightarrow{AB}(x-1, 0, -1)$ $\quad$ $\overrightarrow{AC}(2, 1, y-2)$ $\quad$ $\overrightarrow{AD}(x-1, 2, y-2)$

$$\overrightarrow{AB}\cdot\overrightarrow{AC} = 2(x-1)+0-(y-2) = 2x-2-y+2 =$$

$$= 2x-y=0$$

$$\frac{\left|[\overrightarrow{AB},\overrightarrow{AC},\overrightarrow{AD}]\right|}{6} = \frac{\left|\begin{vmatrix} x-1 & 0 & -1 \\ 2 & 1 & y-2 \\ x-1 & 2 & y-2 \end{vmatrix}\right|}{6} = \frac{5}{6}$$

$$\frac{\left|(x-1)(y-2)-4+(x-1)-2(x-1)(y-2)\right|}{\cancel{6}} = \frac{5}{\cancel{6}}$$

$$\left|xy-2x-y+2-4+x-1-2xy+4x+2y-4\right| = 5$$

$$\left|-xy+3x+y-7\right| = 5 \Rightarrow \begin{matrix} -xy+3x+y-7=5 \\ -xy+3x+y-7=-5 \end{matrix}$$

$$\begin{matrix} 2x-y=0 \\ -xy+3x+y-7=5 \end{matrix} \quad \begin{cases} y=2x \\ -x(2x)+3x+2x-7=5 \end{cases}$$

$$-2x^2+3x+2x-12=0; \quad -2x^2+5x-12=0$$

$$x = \frac{-5\pm\sqrt{25-96}}{-4} \Rightarrow \nexists \text{ sol}$$

$$\begin{matrix} 2x-y=0 \\ -xy+3x+y-7=-5 \end{matrix} \quad \begin{cases} y=2x \\ -x(2x)+3x+2x-7=-5 \end{cases}$$

$$-2x^2+3x+2x-2=0; \quad -2x^2+5x-2=0$$

$$x = \frac{-5\pm\sqrt{25-16}}{-4} = \frac{-5\pm3}{-4} = \left\langle \begin{matrix} \frac{-2}{-4} = \frac{1}{2} \\ \frac{-8}{-4} = 2 \end{matrix} \right.$$

$$\boxed{x = 1/2 \Rightarrow y = 1} \qquad \boxed{x = 2 \Rightarrow y = 4}$$

2) a) $\overline{u_r}(2,4,5)$; $\overline{u_s}(2,1,3)$

$P_r(1,2,0)$, $P_s(3,0,k)$; $\overline{P_rP_s}(2,-2,k)$

$$[\overline{u_r}, \overline{u_s}, \overline{P_rP_s}] = \begin{vmatrix} 2 & 4 & 5 \\ 2 & 1 & 3 \\ 2 & -2 & k \end{vmatrix} = 2k - 20 + 24 - 10 + 12 - 8k =$$

$$= -6k + 6 = 0 \Longrightarrow \underline{\underline{k = 1}}$$

Si $k=1 \Rightarrow$ Coplanarias como $\overline{u_r} \not\parallel \overline{u_s} \Rightarrow$

Coplanarias secantes $\quad \Downarrow \quad \frac{2}{2} \neq \frac{1}{4} \neq \frac{3}{5}$

Si $k \neq 1 \Rightarrow$ Se cruzan

b) $\underline{\underline{k=1}}$

$$r: \begin{cases} x = 1 + 2t_1 \\ y = 2 + 4t_1 \\ z = 5t_1 \end{cases} \qquad s: \begin{cases} x = 3 + 2t_2 \\ y = t_2 \\ z = 1 + 3t_2 \end{cases}$$

$$\left.\begin{array}{l} 1 + 2t_1 = 3 + 2t_2 \\ 2 + 4t_1 = t_2 \\ 5t_1 = 1 + 3t_2 \end{array}\right\} \begin{array}{l} t_1 = -1 \\ t_2 = -2 \end{array} \qquad \underline{\underline{I(-1,-2,-5)}}$$

c) $$d(r,s) = \frac{|[\overline{u_r}, \overline{u_s}, \overline{P_rP_s}]|}{|\overline{u_r} \wedge \overline{u_s}|}$$

$$|[\overline{u_r}, \overline{u_s}, \overline{P_rP_s}]| = |-6k + 6|$$

$$\overline{u_r} \wedge \overline{u_s} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 2 & 4 & 5 \\ 2 & 1 & 3 \end{vmatrix} = (7, 4, -6)$$

$$|\overline{u_r} \wedge \overline{u_s}| = \sqrt{7^2 + 4^2 + (-6)^2} = \sqrt{49 + 16 + 36} = \sqrt{101}$$

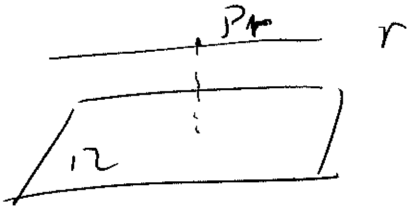
$$\frac{|-6k+6|}{\sqrt{101}} = 6 \Rightarrow |-6k+6| = 6\sqrt{101}$$

$$-6k+6 = 6\sqrt{101} \Rightarrow k = \frac{6\sqrt{101}-6}{-6} = \underline{-\sqrt{101}+1}$$

$$-6k+6 = -6\sqrt{101} \Rightarrow k = \frac{-6\sqrt{101}-6}{-6} = \underline{\underline{\sqrt{101}+1}}$$

3) a) $r \parallel \pi \Rightarrow \overline{u_r} \perp \vec{n}$, $\overline{u_r}(k,-1,k)$; $\vec{n}(2,-3,1)$

$$\overline{u_r} \cdot \overline{n} = 2k+3+k = 3k+3 = 0 \Rightarrow \boxed{k=-1}$$

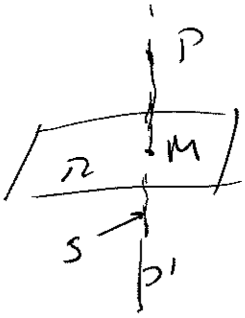


b) $\boxed{k=-1}$

$P_r(2,0,-1)$

$$d(r,\pi) = d(P_r,\pi) = \frac{|2\cdot 2-3\cdot 0+(-1)|}{\sqrt{2^2+(-3)^2+1^2}} = \frac{3}{\sqrt{14}} = \underline{\underline{\frac{3\sqrt{14}}{14}u}}$$

c)



$\overline{u_s} = \overline{n} = (2,-3,1)$

$$s: \begin{cases} x = 1+2t \\ y = -3t \\ z = -3+t \end{cases}$$

$M = s \cap \pi$; $2(1+2t)-3(-3t)+(-3+t) = 0$; $2+4t+9t-3+t=0$; $14t = 1$; $t = \frac{1}{14}$

$$M = \left(1+\frac{2}{14}, -3\cdot\frac{1}{14}, -3+\frac{1}{14}\right) = \left(\frac{8}{7}, \frac{-3}{14}, \frac{-41}{14}\right)$$

$P'(x,y,z)$

$$\frac{x+1}{2} = \frac{8}{7} \Rightarrow x+1 = \frac{16}{7} \Rightarrow x = \frac{16}{7}-1 = \frac{9}{7}$$

$$\frac{y+0}{2} = \frac{-3}{14} \Rightarrow y = \frac{-6}{14} = \frac{-3}{7}$$

$$\frac{z+3}{2} = \frac{-41}{14} \Rightarrow z-3 = \frac{-82}{14} = \frac{-41}{7}$$

$$z = \frac{-41}{7}+3 = \frac{-41+21}{7} = \frac{-20}{7}$$

$$\underline{\underline{P'\left(\frac{9}{7}, \frac{-3}{7}, \frac{-20}{7}\right)}}$$

4) a) $r \subset \pi \Rightarrow \begin{cases} r \parallel \pi \Rightarrow \overline{u_r} \perp \overrightarrow{n} \\ \quad y \\ P_r \in \pi \end{cases}$

$\overline{u_r}\ (4, -4, 1) \qquad \overline{n}\ (a, 2, -4)$

$\overline{u_r} \cdot \overline{n} = 4a - 8 - 4 = 4a - 12 = 0 \Rightarrow \underline{\underline{a = 3}}$

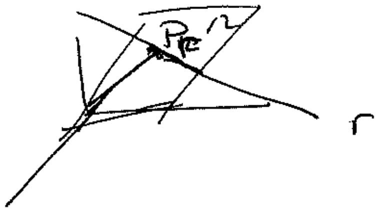
$P_r\ (3, 1, -3) \qquad \pi: 3x + 2y - 4z + b$

$P_r \in \pi \Rightarrow 3 \cdot 3 + 2 \cdot 1 - 4 \cdot (-3) + b = 0 ; \ 9 + 2 + 12 + b = 0$

$\Rightarrow \underline{\underline{b = -23}}$

b) $r \perp \pi \Rightarrow \overline{u_r} \parallel \overrightarrow{n}$

$\frac{a}{4} = \frac{2}{-4} = \frac{-4}{1} \Rightarrow$ <u>No puede ser</u>



c) $\overline{n} = \overline{u_r} = (4, -4, 1)$

$\pi: 4x - 4y + z + D = 0$

Como pasa por $O(0,0,0) \Rightarrow D = 0$

$\pi: 4x - 4y + z = 0 \qquad r: \begin{cases} x = 3 + 4t \\ y = 1 - 4t \\ z = -3 + t \end{cases}$

$P_r = r \cap \pi ; \quad 4(3 + 4t) - 4(1 - 4t) + (-3 + t) = 0$

$12 + 16t - 4 + 16t - 3 + t = 0 ; \quad 33t = -5 \Rightarrow t = \frac{-5}{33}$

$P_r \left(3 - \frac{20}{33}, 1 + \frac{20}{33}, -3 - \frac{5}{33}\right)$

$P_r \left(\frac{79}{33}, \frac{53}{33}, \frac{-104}{33}\right)$

1) a) $$\lim_{x\to+\infty}\left(\frac{x^2(x-\sqrt{x})-x^2(x+\sqrt{x})}{(x+\sqrt{x})\;(x-\sqrt{x})}\right)=$$

$$=\lim_{x\to+\infty}\frac{\cancel{x^3}-x^2\sqrt{x}-\cancel{x^3}-x^2\sqrt{x}}{x^2-x}=$$

$$=\lim_{x\to+\infty}\frac{-2x^2\sqrt{x}}{x^2-x}=\lim_{x\to+\infty}\frac{-2x^{5/2}}{x^2-x}=\underline{\underline{-\infty}}$$

b) $$\lim_{x\to+\infty}\left(\frac{4x^2-1+12x^2+4}{16x^2-4}\right)^{\frac{x^2+x}{x-1}}=$$

$$=\lim_{x\to+\infty}\left(\frac{16x^2+3}{16x^2-4}\right)^{\frac{x^2+x}{x-1}}=$$

$$=\lim_{x\to+\infty}\left(\frac{16^2\cancel{+4}+7}{16x^2-4}\right)^{\frac{x^2+x}{x-1}}=\lim_{x\to+\infty}\left(1+\frac{7}{16x^2-4}\right)^{\frac{x^2+x}{x-1}}=$$

$$=\left(\lim_{x\to+\infty}\left(1+\frac{1}{\frac{16x^2-4}{7}}\right)^{\frac{16x^2-4}{7}}\right)^{\frac{x^2+x}{x-1}\cdot\frac{7}{16x^2-4}}=$$

$$\lim_{x\to+\infty}\frac{7x^2+7x}{16x^3-16x^2-4x+4}$$

$$=\lim_{x\to+\infty}e^{\frac{x^2+x}{x-1}\cdot\frac{7}{16x^2-4}}=e\qquad=\underline{\underline{e^0=1}}$$

c) $$\lim_{x\to-1}\left(\frac{2x(x^2+3x+2)-(x+2)(x^2-1)}{(x^2-1)\;(x^2+3x+2)}\right)=$$

$$=\lim_{x\to-1}\left(\frac{2x^3+6x^2+4x-x^3+x-2x^2+2}{(x^2-1)\;(x^3+3x+2)}\right)=$$

$$=\lim_{x\to-1}\left(\frac{x^3+4x^2+5x+2}{(x^2-1)\;(x^3+3x+2)}\right)=$$

$$= \lim_{x \to -1} \frac{\cancel{(x+1)}\cancel{(x+1)}\cancel{(x+2)}}{\cancel{(x+1)}(x-1)\cancel{(x+1)}\cancel{(x+2)}} =$$

$$\begin{array}{r|rrrr} & 1 & 4 & 5 & 2 \\ -1 & & -1 & -3 & -2 \\ \hline & 1 & 3 & 2 & 0 \end{array}$$

$$x^2+3x+2=0 \begin{array}{l} \nearrow -1 \\ \searrow -2 \end{array}$$

$$= \lim_{x \to -1} \frac{1}{x-1} = \frac{1}{-1-1} = -\frac{1}{2} \,//$$

2) a) $x-1=0 \Rightarrow x=1$

$$\begin{array}{ccc} - & | & + \\ & 1 & \end{array}$$

$$f(x) = \begin{cases} (1-x)\cdot e^{1-x} & \text{si } x<1 \\ (x-1)\cdot e^{x-1} & \text{si } x \geq 1 \end{cases}$$

CONTINUIDAD

FUNCIONES

Continua

CONEXIONES

$x=1$

$$\left.\begin{array}{l} \lim_{x \to 1^-} f(x) = 0\cdot e^0 = 0 \\ \lim_{x \to 1^+} f(x) = 0\cdot e^0 = 0 \\ f(1)=0 \end{array}\right\} \text{Continua.}$$

$$f'(x) = \begin{cases} -e^{1-x} \text{[illegible]} (1-x)\cdot e^{1-x} = e^{1-x}\cdot(x-2) & \text{si } x<1 \\ e^{x-1}+(x-1)e^{x-1} = e^{x-1}\cdot x & \text{si } x>1 \end{cases}$$

FUNCIONES

Continua

CONEXIONES

$x=1$

$\lim_{x \to 1^-} f'(x) = e^0 \cdot (1-2) = -1$ //

$\lim_{x \to 1^+} f'(x) = e^0 \cdot 1 = 1$ //

En $\underline{\underline{x=1}}$ continua, no derivable

b) $g(0)=0 \Rightarrow \dfrac{e^{a0}+b}{0^2+1} = \dfrac{1+b}{1} = 0 \Rightarrow \underline{\underline{b=-1}}$

$$g(x) = \frac{e^{ax}-1}{x^2+1} \; ; \qquad g'(x) = \frac{a \cdot e^{ax}(x^2+1) - 2x(e^{ax}-1)}{(x^2+1)^2} =$$

$$= \frac{e^{ax}(ax^2+a-2x)+2x}{(x^2+1)^2}$$

$$g'(0) = \frac{e^0(a \cdot 0^2 + a - 2 \cdot 0) + 2 \cdot 0}{(0^2+1)^2} = \frac{1 \cdot a}{1} = a$$

$g'(0) = 1 \Rightarrow \underline{\underline{a=1}}$

3) I) Continua en $x=2$

$$\left.\begin{aligned} &\lim_{x \to 2^+} f(x) = \frac{2c}{2^2-2} = \frac{2c}{2} = c \;// \\ &\lim_{x \to 2^-} f(x) = a \cdot 2^2 + b = 4a+b \;// \\ &f(2) = 4a+b \;// \end{aligned}\right\}$$

$$\boxed{c = 4a+b}$$

Continue en $x = -1$

$$\left.\begin{aligned} &\lim_{x\to -1^+} f(x) = a\cdot(-1)^2 + b = a+b \\ &\lim_{x\to -1^-} f(x) = \frac{-4}{(-1)^2} = -4 \\ &f(-1) = a+b \end{aligned}\right\}$$

$$\boxed{a+b=-4}$$

$$f'(x) = \begin{cases} \dfrac{8}{x^3} & \text{și } x < -1 \\ 2ax & \text{și } -1 < x < 2 \\ \dfrac{-cx^2-2c}{(x^2-2)^2} & \text{și } \cancel{x} > 2 \end{cases}$$

$$\frac{c(x^2-2) - 2x\cdot cx}{(x^2-2)^2} = \frac{cx^2 - 2c - 2cx^2}{(x^2-2)^2}$$

Derivable en $x = -1$

$$\left.\begin{aligned} &\lim_{x\to -1^+} f'(x) = 2\cdot a\cdot(-1) = -2a \\ &\lim_{x\to -1^-} f'(x) = \frac{8}{(-1)^3} = -8 \end{aligned}\right\} \boxed{-2a=-8}$$

$$\left.\begin{aligned} &c = 4a+b \\ &a+b = -4 \\ &-2a = -8 \end{aligned}\right\} \rightarrow \boxed{\begin{aligned} &c = 8 \\ &b = -8 \\ &a = 4 \end{aligned}}$$

II)

$$f(x) = \begin{cases} -\dfrac{4}{x^2} & \text{si } x < -1 \\ 4x^2-8 & \text{si } -1 \leq x \leq 2 \\ \dfrac{8x}{x^2-2} & \text{si } x > 2 \end{cases}$$

CONTINUIDAD

FUNCIONES

$x^2=0 \Rightarrow x=0 \rightarrow$ No es del dominio

$4x^2-8 \Rightarrow$ Es polinómica (continua)

$x^2-2=0 \Rightarrow x = \begin{cases} \sqrt{2} \\ -\sqrt{2} \end{cases}$ (No son del dominio)

SON CONTINUAS

CONEXIONES

$x=-1 \; ; \; x=2$

$$\lim_{x\to -1^-} f(x) = \frac{-4}{(-1)^2} = -4$$

$$\underbrace{\begin{aligned} &\lim_{x\to -1^+} f(x) = 4(-1)^2-8 = -4 \\ &f(-1) = -4 \end{aligned}}_{\text{Continua}}$$

$$\lim_{x\to 2^-} f(x) = 4\cdot 2^2-8 = 8$$

$$\lim_{x\to 2^+} f(x) = \frac{8\cdot 2}{2^2-2} = \frac{16}{2} = 8$$

$$\underbrace{f(2) = 8}_{\text{CONTINUA}}$$

$$f'(x) = \begin{cases} \dfrac{8}{x^3} & \text{si } x < -1 \\ 8x & \text{si } -1 < x < 2 \\ \dfrac{-8x^2-16}{(x^2-2)^2} & \text{si } x > 2 \end{cases}$$

$$\frac{8(x^2-2) - 2x \cdot 8x}{(x^2-2)^2} = \frac{8x^2-16-16x^2}{(x^2-2)^2} = \frac{-8x^2-16}{(x^2-2)^2}$$

DERIVABILIDAD

FUNCIONES CONEXIONES

Son derivables $x = -1$; $x = 2$

$$\left.\begin{array}{l} \lim_{x \to -1^-} f'(x) = \dfrac{8}{(-1)^3} = -8 \,// \\ \lim_{x \to -1^+} f'(x) = 8 \cdot (-1) = -8 \,// \end{array}\right\} \text{Derivable}$$

$$\lim_{x \to 2^-} f'(x) = 8 \cdot 2 = 16 \,//$$

$$\lim_{x \to 2^+} f'(x) = \frac{-8 \cdot 2^2 - 16}{(2^2-2)^2} = \frac{-48}{4} = -12 \,//$$

Continua, no derivable

4) a) $y = \ln \sqrt{\dfrac{1}{\operatorname{sen} x \cdot \cos x}} = \ln \left(\dfrac{1}{\operatorname{sen} x \cdot \cos x}\right)^{1/2} = \dfrac{1}{2} \ln \left(\dfrac{1}{\operatorname{sen} x \cdot \cos x}\right) =$

$= \dfrac{1}{2} \left[\underbrace{\ln 1}_{0} - \ln(\operatorname{sen} x \cdot \cos x) \right] = -\dfrac{1}{2} \ln(\operatorname{sen} x \cdot \cos x) =$

$= -\dfrac{1}{2} \left[\ln \operatorname{sen} x + \ln \cos x \right]$

$y' = -\dfrac{1}{2} \left[\dfrac{\cos x}{\operatorname{sen} x} - \dfrac{\operatorname{sen} x}{\cos x} \right] =$

$= -\dfrac{1}{2} \left[\dfrac{\cos^2 x - \operatorname{sen}^2 x}{\operatorname{sen} x \cos x} \right] = -\dfrac{\cos^2 x \operatorname{sen}^2 x}{2 \operatorname{sen} x \cos x} = -\dfrac{\cos 2x}{\operatorname{sen} 2x} =$

$= -\operatorname{cotg} 2x \,//$

2) $y = \operatorname{arc\,tg} \ln x - \operatorname{arc\,tg}(\underbrace{\ln 1}_{} - \ln x) =$

$= \operatorname{arc\,tg} \ln x - \operatorname{arc\,tg}(-\ln x)$

$$y' = \frac{1}{1+(\ln x)^2} \cdot \frac{1}{x} - \frac{1}{1+(-\ln x)^2} \cdot \left(-\frac{1}{x}\right) =$$

$$= \frac{1}{x(1+(\ln x)^2)} + \frac{1}{x(1+(\ln x)^2)} = \frac{2}{x(1+(\ln x)^2)} \quad //$$

b) $$f'(x) = \frac{2x}{x^2+1} - \frac{2x}{x^2-1} =$$

$$= \frac{2x(x^2-1) - 2x(x^2+1)}{(x^2+1)\cdot(x^2-1)} = \frac{\cancel{2x^3} - 2x - \cancel{2x^3} - 2x}{x^4-1} =$$

$$= \frac{-4x}{x^4-1}$$

$f'(x) = 0 \Rightarrow \frac{-4x}{x^4-1} = 0 \Rightarrow -4x = 0 \Rightarrow \underline{\underline{x = 0}}$

$f(0) = \ln(0^2+1) - \ln(0^2-1) = \ln 1 - \ln(-1)$

Pero $\ln(-1) \nexists \Rightarrow$ no puede haber

①

1) a) I) $\lim_{x\to0} \frac{e^x - e^{\operatorname{sen}x}}{x^2} \underset{\substack{\uparrow\\ \text{l'Hôp}}}{=} \lim_{x\to0} \frac{e^x - e^{\operatorname{sen}x}\cdot\cos x}{2x} =$

$\underset{\substack{\uparrow\\ \text{l'Hôp}}}{=} \lim_{x\to0} \frac{e^x - e^{\operatorname{sen}x}\cdot\cos^2 x + e^{\operatorname{sen}x}\cdot\operatorname{sen}x}{2} =$

$= \frac{1-1+0}{2} = 0$ //

II) $L = \lim_{x\to\frac{\pi}{4}^-} (\operatorname{tg}x)^{\operatorname{tg}2x}$; $\ln L = \ln\left(\lim_{x\to\frac{\pi}{4}^-} (\operatorname{tg}x)^{\operatorname{tg}2x}\right) =$

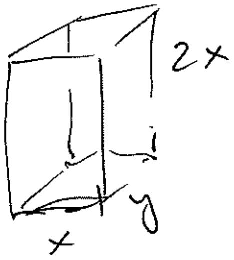
$= \lim_{x\to\frac{\pi}{4}^-} \ln(\operatorname{tg}x)^{\operatorname{tg}2x} = \lim_{x\to\frac{\pi}{4}^-} \operatorname{tg}2x\cdot\ln\operatorname{tg}x =$

$= \lim_{x\to\frac{\pi}{4}^-} \frac{\ln\operatorname{tg}x}{\frac{1}{\operatorname{tg}2x}} \overset{\substack{\text{l'Hôp}\\ \downarrow}}{=} \lim_{x\to\frac{\pi}{4}^-} \frac{\frac{1}{\operatorname{tg}x}\cdot\frac{1}{\cos^2x}}{\frac{\frac{-2}{\cos^2 2x}}{(\operatorname{tg}2x)^2}} =$

$= \lim_{x\to\frac{\pi}{4}^-} \frac{\frac{1}{\operatorname{sen}x\cos x}}{\frac{-2}{\operatorname{sen}^2 2x}} = \frac{\frac{1}{\frac{\sqrt2}{2}\frac{\sqrt2}{2}}}{\frac{-2}{1}} = \frac{\frac{1}{\frac{1}{2}}}{-2} = \frac{2}{-2} = -1$

$\ln L = -1 \Rightarrow L = e^{-1} = \frac{1}{e}$ //

b)



$V = 2x^2y = 784 \rightarrow y = \frac{784}{2x^2} = \frac{392}{x^2}$

$C = 30xy + 10(4xy + 4x^2) =$

$= 30xy + 40xy + 40x^2 =$

$= 70xy + 40x^2$

$C_1(x) = 70\cdot x\cdot\frac{392}{x^2} + 40x^2 = \frac{27440}{x} + 40x^2 =$

$= \frac{27440 + 40x^3}{x}$

$C_1'(x) = \frac{120x^2\cdot x - (27440 + 40x^3)}{x^2} = \frac{80x^3 - 2744}{x^2} = 0$

$x^3 = \frac{27440}{80} = 343$; $x = \sqrt[3]{343} = \underline{7}$; $y = \frac{392}{49} = \underline{\underline{8}}$

$x \in [0,+\infty)$ $A(0)=+\infty$; $A(\infty)=+\infty$

$\underline{\underline{x=7\,m}}$; $\underline{\underline{y=8\,m}}$. $2x=\underline{\underline{14\,m}}$

2) a) $y(2)=3 \Rightarrow \dfrac{4a+b}{a-2}=3$

$4a+b=3a-6$; $\boxed{a+b=-6}$

$$\lim_{x\to\infty}\frac{\frac{ax^2+b}{a-x}}{x}=\lim_{x\to\infty}\frac{ax^2+b}{ax-x^2}=-a=-4$$

$\Rightarrow \underline{\underline{a=4}}$; $\underline{\underline{b=-10}}$

b) $y=x\cdot e^{1-x}$; $y'=e^{1-x}+x\cdot e^{1-x}\cdot(-1)$

$y'=e^{1-x}(1-x)=0 \Rightarrow \underline{\underline{x=1}}$

$y''=-e^{1-x}(1-x)-1\cdot e^{1-x}=-e^{1-x}(2-x)=0$

$\Rightarrow x=2$

$E_1(1,1)$ Màx relat

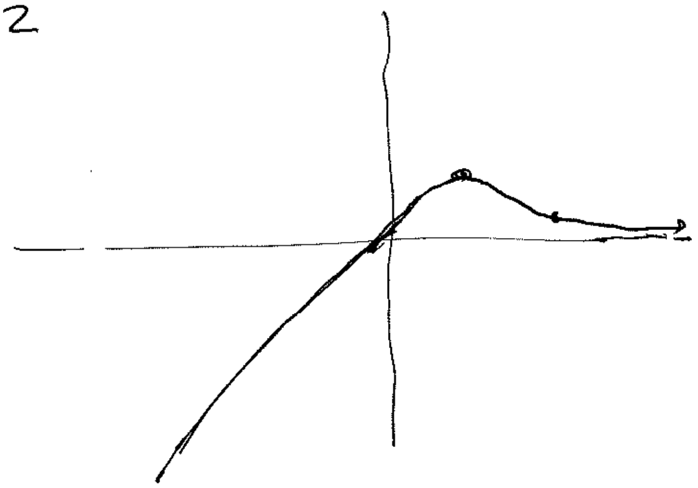
$E_2\left(2,\frac{2}{e}\right)$ Inflex

$y''(1)=-e^0(2-1)=-1<0$

$f'>0$ Crec	1	$f'<0$ Decrec	2	$f'<0$ Decrec

Creciente: $(-\infty,1)$

Decreciente: $(1,+\infty)$



3) a) $\sqrt{x} = t$

$\frac{dx}{2\sqrt{x}} = dt$

$I = \int \text{arc sen}\, t \,.\, 2\,dt$

$= 2 \int \text{arc sen}\, t \, dt$;

$u = \text{arc sen}\, t$ $\quad$ $v = t$

$du = \frac{dt}{\sqrt{1-t^2}}$ $\quad$ $dv = dt$

$$I = 2\left[t\, \text{arc sen}\, t - \underbrace{\int \frac{t\,dt}{\sqrt{1-t^2}}}_{I_1} \right]$$

$1 - t^2 = z$

$-2t\,dt = dz$

$$I_1 = \int \frac{dz/-2}{\sqrt{z}} = \int \frac{dz}{-2\sqrt{z}} = -\sqrt{z} = -\sqrt{1-t^2}$$

$$I = 2\left[t\, \text{arc sen}\, t + \sqrt{1-t^2} \right] = 2\left[\sqrt{x}\, \text{arc sen} \sqrt{x} + \sqrt{1-x}\right] + \text{cte}$$

$x^2 + 4x + 4 = 0 \begin{cases} x = -2 \\ x = -2 \end{cases}$

b)

$$\begin{array}{c|cccc} & 1 & 2 & -4 & -8 \\ 2 & & 2 & 8 & 8 \\ \hline & 1 & 4 & 4 & 0 \end{array}$$

$$\begin{array}{l|l} 2x^3 + 0x^2 + 0x + 1 & x^3 + 2x^2 - 4x - 8 \\ \hline -2x^3 - 4x^2 + 8x + 16 & 2 \\ -4x^2 + 8x + 17 & \end{array}$$

$$2x^3 + 1 = 2(x^3 + 2x^2 - 4x - 8) + (-4x^2 + 8x + 17)$$

$$\int \frac{2x^3+1}{x^3+2x^2-4x-8}\,dx = \int 2\,dx + \int \frac{-4x^2+8x+17}{x^3+2x^2-4x-8}\,dx$$

$$\frac{-4x^2+8x+17}{x^3+2x^2-4x-8} = \frac{A}{(x-2)} + \frac{B}{(x+2)} + \frac{C'}{(x+2)^2} =$$

$$= \frac{A(x+2)^2 + B(x-2)(x+2) + C'(x-2)}{(x-2)(x+2)^2}$$

$x=2 \Rightarrow 17 = 16A \Rightarrow A = \frac{17}{16}$

$x=-2 \Rightarrow -15 = -4C' \Rightarrow C' = \frac{15}{4}$

$x=0 \Rightarrow 17 = 4A - 4B - 2C'$

$17 = \frac{17}{4} - 4B - \frac{15}{2} = -4B - \frac{13}{4}$

$4B = -17 - \frac{13}{4} = \frac{-81}{4}$, $B = \frac{-81}{16}$

$$\int \frac{2x^3+1}{x^3+2x^2-4x-8}\,dx = \int 2\,dx + \int \frac{17/16}{x-2}\,dx + \int \frac{-81/16}{x+2}\,dx + \int \frac{15/4}{(x+2)^2}\,dx = \boxed{2x + \frac{17}{16}\ln|x-2| - \frac{81}{16}\ln|x+2| - \frac{15}{4(x+2)} + cte}$$

4) a) $I = \int \frac{e^{tg x} \cdot tg x}{\cos^2 x}\,dx$

$tg x = t$

$\frac{dx}{\cos^2 x} = dt$

$u = t$ $v = e^t$

$du = dt$ $dv = e^t dt$

$$I = \int t\, e^t\, dt =$$

$$= t \cdot e^t - \int e^t\, dt =$$

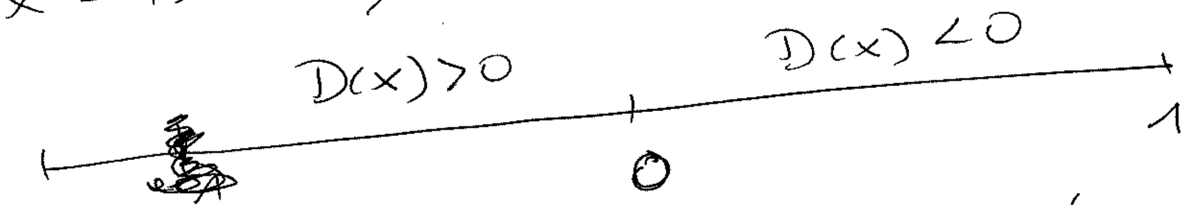
$$= t \cdot e^t - e^t = e^t(t-1) = e^{tg x}(tg x - 1)$$

$$I_D = \left[e^{tg x} (tg x - 1) \right]_0^{\frac{\pi}{4}} =$$

$$= \left[\underbrace{e^1 (1-1)}_{0} - \underbrace{e^0 (0-1)}_{-1} \right] = 1 \,//$$

b) $D(x) = x - \frac{5x}{1+x^2} = \frac{x + x^3 - 5x}{1+x^2} = \frac{x^3 - 4x}{1+x^2}$

$x^3 - 4x = 0 \; ; \; x(x^2-4) = 0 \; < \begin{matrix} x=0 \\ x=-2 \\ x=2 \end{matrix}$



-1

$\int x \, dx = \frac{x^2}{2}$

$1 + x^2 = t$
$2x \, dx = dt$

$$\int \frac{5x}{1+x^2} dx =$$

$$= 5 \int \frac{dt/2}{t} = \frac{5}{2} \ln t = \frac{5}{2} \ln(1+x^2)$$

$$A = \left[\frac{x^2}{2} - \frac{5}{2} \ln(1+x^2) \right]_{-2}^{0} - \left[\frac{x^2}{2} - \frac{5}{2} \ln(1+x^2) \right]_0^2 =$$

$$= \left[(0-0) - \left(2 - \frac{5}{2} \ln 5\right) \right] - \left[\left(2 - \frac{5}{2} \ln 5\right) - (0-0) \right] =$$

[illegible]

$$= \underline{\underline{(5 \ln 5 - 4) \, u^2}}$$

1)

a) FUNCIONES — CONEXIONES

$x = 0$

Continua

i) $\lim_{x\to 0^-} f(x) = 1 + a\sqrt{1}\cdot \ln 1 = 1 + a\cdot 0 = 1$ //

$\lim_{x\to 0^+} f(x) = (0-1)^2 = (-1)^2 = 1$ //

$f(0) = (0-1)^2 = (-1)^2 = 1$ //

Continua para todo "a"

$$f'(x) = \begin{cases} \dfrac{a}{2\sqrt{x+1}}\cdot \ln(x+1) + \dfrac{a\sqrt{x+1}}{x+1} & \text{si } x > 0 \\ 2(x-1) & \text{si } x < 0 \end{cases}$$

$$\left.\begin{array}{l} \lim_{x\to 0^-} f'(x) = \dfrac{a}{2\sqrt{1}}\cdot \ln 1 + \dfrac{a\sqrt{1}}{1} = a \\ \lim_{x\to 0^+} f'(x) = 2(0-1) = -2 \end{array}\right\} \boxed{a=-2}$$

b) $f(x) = \text{arc sen}\sqrt{1-\text{sen}^2 x}$

$$f'(x) = \frac{1}{\sqrt{1-\left(\sqrt{1-\text{sen}^2 x}\right)^2}}\cdot \frac{1}{2\sqrt{1-\text{sen}^2 x}}\cdot -2\,\text{sen}\,x\cos x =$$

$$= \frac{-\cancel{2}\,\cancel{\text{sen}\,x}\cos x}{\cancel{\text{sen}\,x}\cdot \cancel{2}\sqrt{1-\text{sen}^2 x}} = \frac{-\cos x}{\sqrt{1-\text{sen}^2 x}} = \frac{-\cos x}{\cos x} = -1$$

$f'(0) = -1$; $P\left(0, \frac{\pi}{2}\right)$, $y - \frac{\pi}{2} = -1(x-0)$

$$\boxed{y = -x + \frac{\pi}{2}}$$

2) a)

i) $\lim\limits_{x\to 0} \dfrac{\ln(1+x) - \text{sen}\, x}{x\, \text{sen}\, x} \overset{\text{L'Hôp}}{\underset{\uparrow}{=}}$

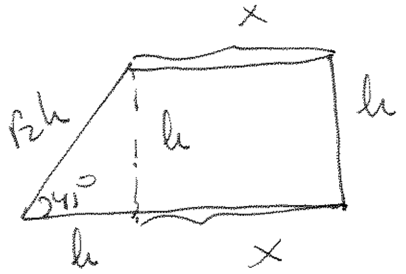
$= \lim\limits_{x\to 0} \dfrac{\dfrac{1}{1+x} - \cos x}{\text{sen}\, x + x\cos x} \overset{\text{L'Hôp}}{\underset{\uparrow}{=}} \lim\limits_{x\to 0} \dfrac{\dfrac{-1}{(1+x)^2} + \text{sen}\, x}{\cos x + \cos x - x\,\text{sen}\, x} =$

$= \lim\limits_{x\to 0} \dfrac{\dfrac{-1}{(1+x)^2} + \text{sen}\, x}{2\cos x - x\,\text{sen}\, x} = \dfrac{-1+0}{2-0} = \dfrac{-1}{2}$ //

ii) $\lim\limits_{x\to +\infty} x\cdot\left(e^{1/x} - 1\right) = \lim\limits_{x\to +\infty} \dfrac{e^{1/x} - 1}{\dfrac{1}{x}} =$

$\overset{\text{L'Hôp}}{\underset{\uparrow}{=}} \lim\limits_{x\to +\infty} \dfrac{e^{1/x}\cdot\left(-\dfrac{1}{x^2}\right)}{\left(-\dfrac{1}{x^2}\right)} = \lim\limits_{x\to +\infty} e^{1/x} = e^0 = \underline{\underline{1}}$

b)



$2x + 2h + \sqrt{2}h = 10$

$2x + (2+\sqrt{2})h = 10$

$x = \dfrac{10 - (2+\sqrt{2})h}{2}$

$A = \dfrac{((x+h)+x)\cdot h}{2} = \dfrac{(2x+h)\,h}{2} =$

$= \dfrac{(10 - (2+\sqrt{2})h + h)\,h}{2} = \dfrac{(10 - (1+\sqrt{2})h)\,h}{2} =$

$= \dfrac{10h - (1+\sqrt{2})h^2}{2} \to$ màx

$10 - 2(1+\sqrt{2})h = 0$

$A' = \dfrac{10 - 2(1+\sqrt{2})h}{2} = 0$

$h = \dfrac{10}{2(1+\sqrt{2})} = \dfrac{5}{1+\sqrt{2}}$ // $= \underline{2'07}$

$$x = \frac{10 - \frac{(2+\sqrt{2})\cdot 5}{1+\sqrt{2}}}{2} = \frac{10+10\sqrt{2}-10-5\sqrt{2}}{2(1+\sqrt{2})} =$$

$$= \frac{5\sqrt{2}}{2(1+\sqrt{2})} \quad // \quad = \underline{\underline{1'46}}$$

$$A(0) = 0 \quad //$$

$$k \in \left[0, \frac{10}{1+\sqrt{2}}\right] \qquad A\left(\frac{10}{1+\sqrt{2}}\right) = 0 \quad //$$

3) a) Domini: $\ln x > 0 \Rightarrow x > 0$

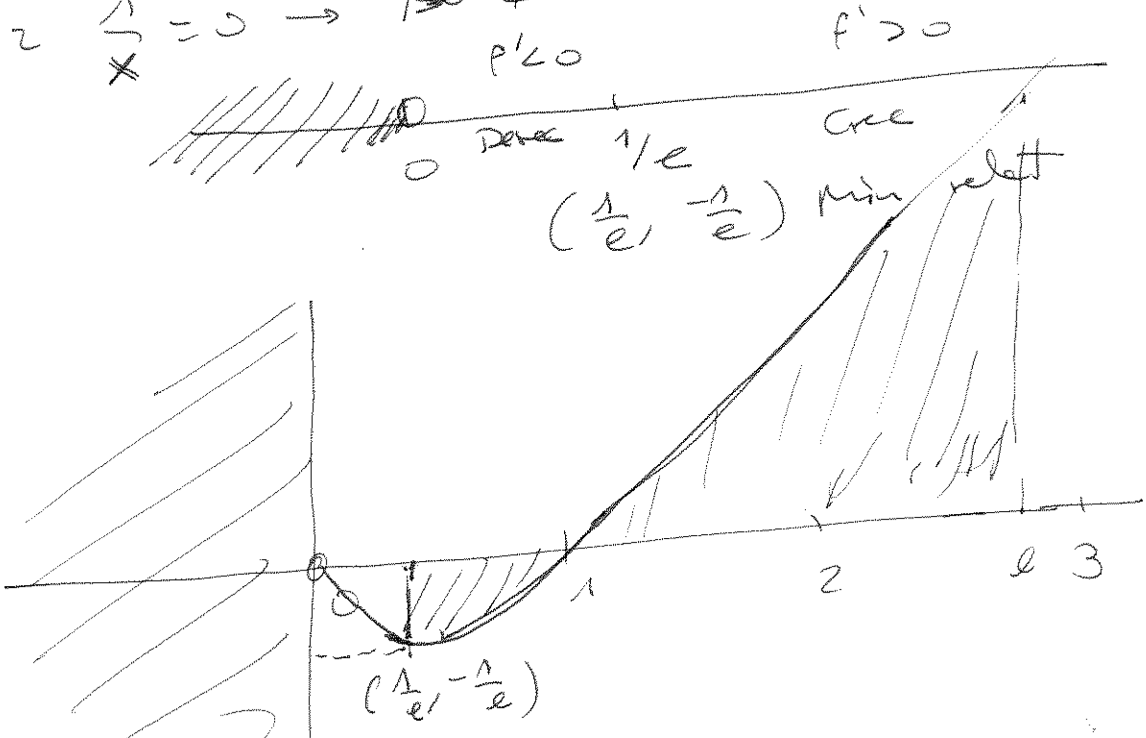
$D = (0, +\infty)$

$$y' = \ln x + x \cdot \frac{1}{x} = \ln x + 1 = 0 \Rightarrow \ln x = -1$$

$$\Rightarrow x = e^{-1} = \frac{1}{e}$$

$E_1\left(\frac{1}{e}, -\frac{1}{e}\right)$ min relat

$y'' = \frac{1}{x} = 0 \rightarrow$ ∄ sol. $\quad y''\left(\frac{1}{e}\right) = \frac{1}{\frac{1}{e}} = e > 0$

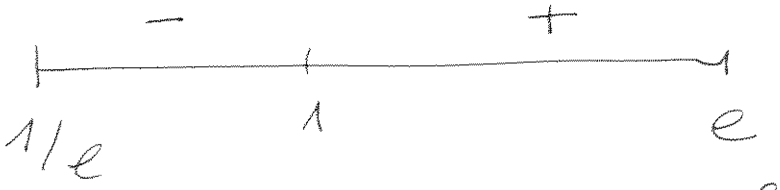


$$\lim_{x\to 0^+} x\cdot \ln x = \underset{x\to 0^+}{\ell} \frac{\ln x}{\frac{1}{x}} = \underset{x\to 0^+}{\ell} \frac{\frac{1}{x}}{-\frac{1}{x^2}} =$$

$$= \underset{x\to 0^+}{\ell} -x = 0$$

b) $y = x \cdot \ln x = 0 \nearrow x \cancel{= 0} \; //$

$\searrow \ln x = 0 \to x = 1 \; //$



$$A = -\int_{1/e}^{1} x \cdot \ln x \, dx + \int_{1}^{e} x \cdot \ln x \, dx$$

$u = \ln x \qquad v = \frac{x^2}{2}$

$J = \int x \cdot \ln x \, dx$

$du = \frac{dx}{x} \qquad dv = x\,dx$

$$J = \frac{x^2 \cdot \ln x}{2} - \int \frac{x^2 \cdot dx}{2x} = \frac{x^2 \ln x}{2} - \frac{1}{2}\int x\,dx =$$

$$= \frac{x^2 \ln x}{2} - \frac{1}{2} \cdot \frac{x^2}{2} = \frac{x^2 \ln x}{2} - \frac{x^2}{4} =$$

$$= \frac{2x^2 \ln x - x^2}{4} = \frac{x^2(2 - \ln x)}{4}$$

$$A = -\left[\frac{x^2(2 - \ln x)}{4}\right]_{1/e}^{1} + \left[\frac{x^2(2 - \ln x)}{4}\right]_{1}^{e} =$$

$$= -\left[\frac{1(2-0)}{4} - \frac{\frac{1}{e^2}(2-(-1))}{4}\right] + \left[\frac{e^2(2-1)}{4} - \frac{1(2-0)}{4}\right] =$$

$$= -\left[\frac{2}{4} - \frac{3}{4e^2}\right] + \left[\frac{e^2}{4} - \frac{2}{4}\right] =$$

$$= -\frac{2}{4} + \frac{3}{4e^2} + \frac{e^2}{4} - \frac{2}{4} = \frac{-2e^2 + 3 + e^4 - 2e^2}{4e^2} = \underline{\underline{\frac{e^4 - 4e^2 + 3}{4e^2}}}$$

4) a) $I = \int 2x \,\text{arc tg}\, x \, dx = 2 \int x \,\text{arctg}\, x \, dx$

$u = \text{arctg}\, x \qquad v = \frac{x^2}{2}$

$du \; \frac{dx}{1+x^2} \qquad dv = x\,dx$

$$I = 2\left[\frac{x^2 \,\text{arctg}\, x}{2} - \underbrace{\int \frac{x^2/2}{1+x^2} dx}_{I_1} \right]$$

$$\int \frac{x^2/2}{1+x^2} dx = \frac{1}{2} \int \frac{x^2}{1+x^2} dx =$$

$$= \frac{1}{2} \int \frac{(1+x^2)-1}{(1+x^2)} dx = \frac{1}{2} \int \left(1 - \frac{1}{1+x^2}\right) dx =$$

$$= \frac{1}{2} \int 1\,dx - \frac{1}{2} \int \frac{1}{1+x^2} dx = \frac{1}{2}\left[x - \text{arctg}\, x\right]$$

$$I = 2\left[\frac{x^2 \,\text{arctg}\, x}{2} - \frac{1}{2}(x - \text{arctg}\, x) \right] =$$

$$= \underline{\underline{x^2 \,\text{arctg}\, x - x + \text{arctg}\, x + cte}}$$

b) $I = \int \frac{x^2+x+1}{e^x} dx = \int (x^2+x+1)\, e^{-x} dx$

$v = -e^{-x}$

$u = x^2+x+1$

$du = (2x+1)\,dx \qquad dv = e^{-x} dx$

$$I = -(x^2+x+1)\,e^{-x} + \underbrace{\int (2x+1)\,e^{-x}\,dx}_{I_1}$$

$u_1 = 2x+1$

$du_1 = 2\,dx$

$v = -e^{-x}$

$dv = e^{-x}\,dx$

$$I_1 = -(2x+1)\,e^{-x} + \int 2\,e^{-x}\,dx =$$

$$= -(2x+1)\,e^{-x} - 2\,e^{-x}$$

$$I = -(x^2+x+1)\,e^{-x} - (2x+1)\,e^{-x} - 2\,e^{-x} =$$

$$= \underline{\underline{-\,e^{-x}(x^2+3x+4) + cte}}$$

1)

a) $f'(x) = -\frac{2}{x^2} + \frac{2x}{x^2} = \frac{-2+2x}{x^2} = 0$

$-2+2x=0 \Rightarrow x = 1$ //

$f(1) = \frac{2}{1} + \ln 1 = 2$; $\underline{\underline{M(1,2)}}$

$f''(x) = \frac{2x^2 - 2x(-2+2x)}{x^4} = \frac{2x^2+4x-4x^2}{x^4} =$

$= \frac{-2x^2+4x}{x^4} = \frac{-2x+4}{x^3} = 0$

$-2x+4=0 \Rightarrow x=2$ $\quad f(2) = \frac{2}{2} + \ln 4 = 1+\ln 4$

$\underline{\underline{I(2, 1+\ln 4)}}$

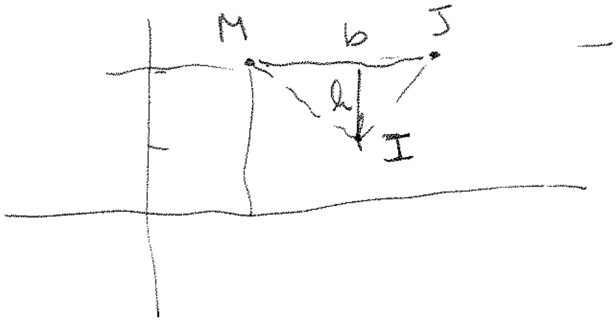
$f'(1)=0$; Tgte en M $\Rightarrow$ $y = 2$

$f'(2) = \frac{-2+4}{2^2} = \frac{2}{4} = \frac{1}{2} \Rightarrow y-(1+\ln 4) = \frac{1}{2}(x-2)$

$2-(1+\ln 4) = \frac{1}{2}(x-2)$; $4 - 2(1+\ln 2) = x-2$

$x = 4-2(1+\ln 2)+2 = 4-2-2\ln 2+2 = 4-2\ln 2$

$\underline{\underline{J(4-2\ln 2, 2)}}$



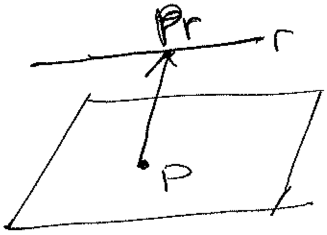
$h = 2-(1+\ln 4) =$
$= 1-\ln 4$

$b = 3-2\ln 2 = 3-\ln 4$

$A = \frac{(3-\ln 4)(1-\ln 4)}{2}$

GEOMETRÍA

1) a)



$P_r(-1, 1, 0)$

$\overrightarrow{PP_r}(1, -2, -4)$

$\overrightarrow{u_r}(2, 3, 1)$

$$\vec{n} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & -2 & -4 \\ 2 & 3 & 1 \end{vmatrix} = (10, -9, 7)$$

$10x - 9y + 7z + D = 0$

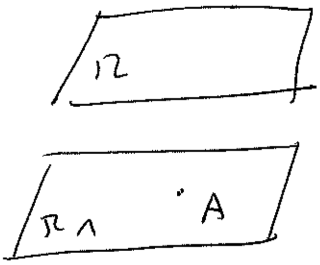
$-20 - 27 + 28 + D = 0$; $D = 19$

$$\boxed{\pi : 10x - 9y + 7z + 19 = 0}$$

$$d(r, P) = d(P_r, \pi) = \frac{|10 \cdot (-1) - 9 \cdot 1 + 7 \cdot 0 + 19|}{\sqrt{10^2 + (-9)^2 + 7^2}} =$$

$$= \frac{|-10 - 9 + 19|}{\sqrt{230}} = 0 //$$

b)



$$\vec{n} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & 2 & 0 \\ 1 & -3 & 1 \end{vmatrix} = (2, -1, -5)$$

$\overrightarrow{n_1} = \vec{n} = (2, -1, -5)$

$\pi_1 : 2x - y - 5z + D = 0$

$2 \cdot 3 - (-2) - 5 \cdot 4 + D = 0$; $6 + 2 - 20 + D = 0$; $\underline{\underline{D = 12}}$

$$\boxed{\pi_1 : 2x - y - 5z + 12 = 0}$$

$P_\pi(1, -3, 2)$

$2x - y - 5z + D = 0$

$2 + 3 - 10 + D = 0$; $D = 5$

$\pi : 2x - y - 5z + 5 = 0$

$$d(\pi_1, \pi) = d(A, \pi) = \frac{|2 \cdot 3 - (-2) - 5 \cdot 4 + 5|}{\sqrt{2^2 + (-1)^2 + (-5)^2}} = \frac{7}{\sqrt{30}} = \boxed{\frac{7\sqrt{30}}{30} \quad u}$$

2) a) $\overrightarrow{u_r} = (1, 2, 1)$

$$\overline{u_s} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 4 & 5 & 0 \\ 0 & 3 & -4 \end{vmatrix} = (-20, 16, 12) \sim (-5, 4, 3)$$

$P_r(3, -1, 2) \qquad P_s\left(2, -3, \frac{m+2}{-4}\right)$

$-9 - 4z + 7 - m = 0; \quad -4z = m + 2; \quad z = \frac{m+2}{-4}$

$P_rP_s\left(-1, -2, \frac{m+2}{-4} - 2\right) = \left(-1, -2, \frac{m+10}{-4}\right)$

$$[\overline{u_r}, \overline{u_s}, \overline{P_rP_s}] = \begin{vmatrix} 1 & 2 & 1 \\ -5 & 4 & 3 \\ -1 & -2 & \frac{m+10}{-4} \end{vmatrix} =$$

$$= -m - \cancel{10} + \cancel{10} - \cancel{6} + 4 + \cancel{6} + \frac{10m+100}{-4} =$$

$$= -m + 4 + \frac{10m+100}{-4} = \frac{+4m - 16 + 10m + 100}{-4} =$$

$$\frac{14m+84}{-4} = 0; \quad 14m + 84 = 0; \quad m = \frac{-84}{14} = \underline{\underline{-6}}$$

b) $$\overline{n} = \overline{u_r} \wedge \overline{u_s} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & 2 & 1 \\ -5 & 4 & 3 \end{vmatrix} = (2, -8, 14) \sim (1, -4, 7)$$

$x - 4y + 7z + D = 0$

$3 - 4(-1) + 7 \cdot 2 + D = 0; \quad 3 + 4 + 14 + D = 0; \quad D = -21$

$$\boxed{\pi: x - 4y + 7z - 21 = 0}$$

$$r: \begin{cases} x = 3 + t_1 \\ y = -1 + 2t_1 \\ z = 2 + t_1 \end{cases} \qquad s: \begin{cases} x = 2 - 5t_2 \\ y = -3 + 4t_2 \\ z = 1 + 3t_2 \end{cases}$$

$$\left.\begin{array}{l} 3 + t_1 = 2 - 5t_2 \\ -1 + 2t_1 = -3 + 4t_2 \\ 2 + t_1 = 1 + 3t_2 \end{array}\right\} \left.\begin{array}{l} t_1 + 5t_2 = -1 \\ 2t_1 - 4t_2 = -2 \\ t_1 - 3t_2 = -1 \end{array}\right\} \begin{array}{l} t_1 = -1 \\ \\ t_2 = 0 \end{array}$$

$$\underline{\underline{I = (2, -3, 1)}}$$

3) a) $2m \cdot 1 + (m+1) \cdot (-1) - 3(m+1) \cdot 2 + 2m + 4 = 0$

$2m - m - 1 - 6m - 6 + 2m + 4 = 0$

$-3m - 3 = 0;$ $\boxed{m = -1}$ $\boxed{\pi: -2x + 2 = 0}$

b) $$\overline{u_r} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & 0 & 3 \\ 0 & 1 & -5 \end{vmatrix} = (-3, 5, 1) \qquad \vec{n}\,(2m, m+1, -3(m+1))$$

$\pi \perp \vec{u_r} \Rightarrow \vec{n} \parallel \vec{u_r}$

$$\underbrace{\frac{2m}{-3} = \frac{m+1}{5}} = \frac{-3(m+1)}{1}$$

$10m = -3m - 3;$ $m = \frac{-3}{13}$

$2m = 9m + 9;$ $m = -\frac{9}{7}$

no existe se plano

4)a) Eje Ox : $\begin{cases} x = t \\ y = 0 \\ z = 0 \end{cases}$ $\overline{u}_{ox} = (1, 0, 0)$

$P_x\ (1, 0, 0)$

$$\overline{u_r} = \begin{vmatrix} \overline{i} & \overline{j} & \overline{k} \\ 2 & -3 & 0 \\ 2 & -3 & -1 \end{vmatrix} = (3, 2, 0)$$

$P_r\ (2, 0, 4)$

$r : \begin{cases} x = 2 + 3t \\ y = 2t \\ z = 4 \end{cases}$

$\overrightarrow{P_xP_r}\ (1, 0, 4)$

$$d(Ox, r) = \frac{|[\overline{u}_{ox}, \overline{u_r}, \overline{P_xP_r}]|}{|\overrightarrow{u_{ox}} \wedge \overline{u_r}|}$$

$$\begin{vmatrix} 1 & 0 & 0 \\ 3 & 2 & 0 \\ 1 & 0 & 4 \end{vmatrix} = 8$$

$$\overline{u_{ox}} \wedge \overline{u_r} = \begin{vmatrix} \overline{i} & \overline{j} & \overline{k} \\ 1 & 0 & 0 \\ 3 & 2 & 0 \end{vmatrix} = (0, 0, 2)$$

$$d(Ox, r) = \frac{|8|}{2} = \underline{\underline{4\ u}}$$

b) $P_r\ (2, 3, 4)$ $\overline{u_r}\ (4, 4, 2)$

$\overrightarrow{PP_r} = (1, 1, 1)$

$$\overline{PP_r} \wedge \overline{u_r} = \begin{vmatrix} \bar{i} & \bar{j} & \bar{k} \\ 1 & 1 & 1 \\ 4 & 4 & 2 \end{vmatrix} = (-2, +2, 0)$$

$$d(P,r) = \frac{\sqrt{(-2)^2+2^2+0^2}}{\sqrt{4^2+4^2+2^2}} = \frac{\sqrt{8}}{6} = \frac{\sqrt{2}}{3}\,u$$

$\bar{n} = \overline{u_r} = (4,4,2) \simeq (2,2,1)$

$2x + 2y + z + D = 0;\quad 2 \cdot 1 + 2 \cdot 2 + 3 + D = 0$

$2 + 4 + 3 + D = 0;\quad D = -9$

$2x + 2y + z - 9 = 0$

$$r' \begin{cases} x = 2 + 2t \\ y = 3 + 2t \\ z = 4 + t \end{cases}$$

$2(2+2t) + 2(3+2t) + (4+t) - 9 = 0$

$4 + 4t + 6 + 4t + 4 + t - 9 = 0;\quad 9t = -5;\quad \underline{\underline{t = \frac{-5}{9}}}$

$$M = \left(2 - \frac{10}{9},\ 3 - \frac{10}{9},\ 4 - \frac{5}{9}\right) = \left(\frac{8}{9}, \frac{17}{9}, \frac{31}{9}\right)$$

$P'(x, y, z)$

$\frac{x+1}{2} = \frac{8}{9} \rightarrow 9x + 9 = 16 \rightarrow x = \frac{7}{9}$

$\frac{y+2}{2} = \frac{17}{9} \rightarrow 9y + 18 = 34 \rightarrow y = \frac{16}{9}$

$\frac{z+3}{2} = \frac{31}{9} \rightarrow 9z + 27 = 62 \rightarrow z = \frac{35}{9}$

$$P'\left(\frac{7}{9}, \frac{16}{9}, \frac{35}{9}\right)$$

ÁLGEBRA

1)
$$\left.\begin{array}{l} x+y+z=7'2 \\ x=1'4y \\ z=2(x+y) \end{array}\right\} \quad \left.\begin{array}{l} x+y+z=7'2 \\ x-1'4y\quad=0 \\ -2x-2y+z=0 \end{array}\right\}$$

$$\left(\begin{array}{ccc|c} 1 & 1 & 1 & 7'2 \\ 1 & -1'4 & 0 & 0 \\ -2 & -2 & 1 & 0 \end{array}\right) \underset{2F_1+F_3\to F_3}{\overset{F_1-F_2\to F_2}{\sim}} \left(\begin{array}{ccc|c} 1 & 1 & 1 & 7'2 \\ 0 & 2'4 & 1 & 7'2 \\ 0 & 0 & 3 & 14'4 \end{array}\right)$$

$$\left.\begin{array}{r} x+y+z=7'2 \\ 2'4y+z=7'2 \\ 3z=14'4 \end{array}\right\} \quad \begin{array}{l} x=7'2-4'8-1=\underline{1'4} \\ 2'4y=7'2-4'8=2'4 \rightarrow \underline{\underline{y=1}} \\ z=\dfrac{14'4}{3}=4'8 // \end{array}$$

$$(1'4,\ 1,\ 4'8)$$

2) a) $A \cdot X \cdot B + C = D$; $\quad A X B = D - C$

$$\underbrace{A^{-1}A}_{I_2} X \underbrace{BB^{-1}}_{I_3} = A^{-1}(D-C)B^{-1}$$

$$\boxed{X = A^{-1}(D-C)B^{-1}}$$

$$A=\begin{pmatrix} 1 & 0 \\ -1 & 1 \end{pmatrix} \qquad |A|=1; \qquad A^t=\begin{pmatrix} 1 & -1 \\ 0 & 1 \end{pmatrix}$$

$$\mathrm{Adj}(A^t)=\begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix} \qquad \underline{A^{-1}=\frac{\mathrm{Adj}(A^t)}{|A|}=\begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix}}$$

$$B=\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -1 & 1 \end{pmatrix} \qquad |B|=1; \qquad B^t=\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{pmatrix}$$

$$\mathrm{Adj}(B^t)=\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 1 \end{pmatrix} \qquad B^{-1}=\frac{\mathrm{Adj}(B^t)}{|B|}=\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 1 \end{pmatrix}$$

$$X = \begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix} \begin{pmatrix} 2 & -1 & -1 \\ 1 & -1 & 3 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 1 \end{pmatrix}$$

$$X = \begin{pmatrix} 2 & -1 & -1 \\ 3 & -2 & 2 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 1 \end{pmatrix} = \begin{pmatrix} 2 & -2 & -1 \\ 3 & -4 & 2 \end{pmatrix}$$

$$\underline{\underline{X = \begin{pmatrix} 2 & -2 & -1 \\ 3 & 0 & 2 \end{pmatrix}}}$$

b) $$|A| = x \begin{vmatrix} x & y & 0 \\ 0 & x & y \\ 0 & 0 & x \end{vmatrix} - y \begin{vmatrix} y & 0 & 0 \\ x & y & 0 \\ 0 & x & y \end{vmatrix} =$$

$$= x \cdot x^3 - y\, y^3 = x^4 - y^4 =$$

$$= (x^2+y^2)(x^2-y^2)$$

$x^2+y^2=0 \Rightarrow$ $x=0$ // , $y=0$ //

$x^2-y^2=0 \Rightarrow \underline{\underline{x = \pm y}}$

3) a)

$$A = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} - \begin{pmatrix} 0 & 2 \\ -1/4 & 3/2 \end{pmatrix} =$$

$$= \begin{pmatrix} 1 & -2 \\ 1/4 & -1/2 \end{pmatrix}$$

$$A^2 = \begin{pmatrix} 1 & -2 \\ 1/4 & -1/2 \end{pmatrix} \begin{pmatrix} 1 & -2 \\ 1/4 & -1/2 \end{pmatrix} = \begin{pmatrix} 1/2 & -1 \\ 1/8 & -1/4 \end{pmatrix}$$

$$\boxed{A^2 = \frac{1}{2} A .} \qquad \underline{\underline{k = \frac{1}{2}}}$$

~~$A^3 = \begin{pmatrix} 1/2 & -1 \\ 1/8 & -1/4 \end{pmatrix} \begin{pmatrix} 1 & -2 \\ 1/4 & -1/2 \end{pmatrix}$~~

$$A^3 = A^2 \cdot A = \frac{1}{2} \cdot A \cdot A = \frac{1}{2} A^2 = \frac{1}{2} \cdot \frac{1}{2} \cdot A = \frac{1}{4} A$$

$$A^4 = A^2 \cdot A^2 = \frac{1}{2} \cdot A \cdot \frac{1}{2} A = \frac{1}{4} A^2 = \frac{1}{4} \cdot \frac{1}{2} A = \frac{1}{8} A$$

$$\boxed{A^n = \frac{1}{2^{n-1}} \cdot A}$$

$$A^n = \begin{pmatrix} \frac{1}{2^{n-1}} & \frac{-2}{2^{n-1}} \\ \frac{1}{4 \cdot 2^{n-1}} & \frac{-1}{2 \cdot 2^{n-1}} \end{pmatrix}$$

b)

$$\begin{vmatrix} 1 & 1 & 1 & x \\ 2 & x & 2 & 2 \\ 3 & 5 & x & 3 \\ 4 & 4 & 4 & x+3 \end{vmatrix} \begin{array}{c} C_2-C_1\to C_2 \\ C_3-C_1\to C_3 \\ = \\ C_4-C_1\to C_4 \end{array} \begin{vmatrix} 1 & 0 & 0 & x-1 \\ 2 & x-2 & 0 & 0 \\ 3 & 2 & \boxed{x-3} & 0 \\ 4 & 0 & 0 & x-1 \end{vmatrix} =$$

$$= (x-3) \begin{vmatrix} 1 & 0 & x-1 \\ 2 & x-2 & 0 \\ 4 & 0 & x-1 \end{vmatrix} =$$

$$= (x-3) \left[\underbrace{(x-2)(x-1) - 4(x-2)(x-1)}_{-3(x-1)(x-2)} \right] =$$

$$= -3(x-1)(x-2)(x-3)$$

4)

$$\begin{vmatrix} a & -3 & -2 \\ -1 & 5+a & 1 \\ 2 & 3 & 4 \end{vmatrix} = 4a(5+a) + 6 - 6 + 4(5+a) - 3a - 12 =$$

$$= 4a^2 + 20a + 20 + 4a - 3a - 12 =$$

$$= 4a^2 + 21a + 8 = 0$$

$$a = \frac{-21 \pm \sqrt{441-128}}{8} = \frac{-21 \pm \sqrt{313}}{8} = \begin{cases} -0'41 \\ -4'83 \end{cases}$$

$$\text{Si } a \neq \begin{cases} \frac{-21+\sqrt{313}}{8} \\ \frac{-21-\sqrt{313}}{8} \end{cases} \Rightarrow \text{S.C.D. Sol } (0,0,0)$$

$$\text{Si } a = \begin{cases} \frac{-21+\sqrt{313}}{8} \\ \frac{-21-\sqrt{313}}{8} \end{cases} \Rightarrow \text{S.C.I.}$$

$\underline{\underline{a = -0'41}}$

$$\left(\begin{array}{ccc|c} -0'41 & -3 & -2 & 0 \\ -1 & 4'59 & 1 & 0 \\ 2 & 3 & 4 & 0 \end{array}\right) \sim \left(\begin{array}{ccc|c} -0'41 & -3 & -2 & 0 \\ 0 & -4'88 & -2'41 & 0 \\ 0 & -4'88 & -2'41 & 0 \end{array}\right)$$

$$\left(\begin{array}{ccc|c} -0'41 & -3 & -2 & 0 \\ 0 & -4'88 & -2'41 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right)$$

S. C. $\underline{I}$

$\underline{\underline{a = -4'83}}$

$$\left(\begin{array}{ccc|c} -4'83 & -3 & -2 & 0 \\ -1 & 0'17 & 1 & 0 \\ 2 & 3 & 4 & 0 \end{array}\right) \sim \left(\begin{array}{ccc|c} -4'83 & -3 & -2 & 0 \\ 0 & -3'82 & -6'83 & 0 \\ 0 & 3'34 & 6 & 0 \end{array}\right)$$

$$\left(\begin{array}{ccc|c} -4'83 & -3 & -2 & 0 \\ 0 & -3'82 & -6'83 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right)$$

S. C. $\underline{I}$

1a) $f(x) = \begin{cases} ax^2+1 & \text{si } x<2 \\ e^{2-x}+2 & \text{si } x \geq 2 \end{cases}$

$$\lim_{x\to 2^-} f(x) = \lim_{x\to 2^-} ax^2+1 = 4a+1$$

$$\| \Rightarrow 4a+1=3 \Rightarrow 4a=2 \Rightarrow \boxed{a=\frac{1}{2}}$$

$$\lim_{x\to 2^+} f(x) = \lim_{x\to 2^+} e^{2-x}+2 = 3 \quad ; \quad f(2)=3$$

b) $f(x) = \begin{cases} \frac{1}{2}x^2+1 & \text{si } x<2 \\ e^{2-x}+2 & \text{si } x\geq 2 \end{cases}$

f(x) continua en ℝ

$f'(x) = \begin{cases} x & \text{si } x<2 \\ -e^{2-x} & \text{si } x>2 \end{cases}$

$\lim_{x\to 2^-} f'(x) = f'(2^-) = 2$ $\neq$ $\Rightarrow$ f(x) no derivable en $x=2$ $\Rightarrow$ f(x) derivable en $\mathbb{R}-\{2\}$

$\lim_{x\to 2^+} f'(x) = f'(2^+) = -1$

c) $f(0) = \frac{1}{2}0^2+1 = 1 \Rightarrow$ lo toca en (0,1)

$f(x)=0 \Rightarrow \frac{1}{2}x^2+1 \neq 0$

$e^{2-x}+2=0 \Rightarrow e^{2-x} \neq -2$ No existe ningún punto

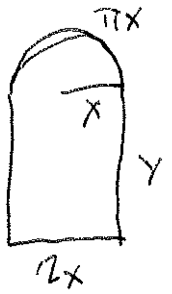
2 a) $$\lim_{x\to+\infty} (\sqrt{ax^2+x} - \sqrt{bx^2+3x}) = \lim_{x\to\infty} \frac{ax^2+x-bx^2-3x}{\sqrt{ax^2+x}+\sqrt{bx^2+3x}}$$

para que sea finito $a=b$ $\Rightarrow \lim_{x\to\infty} \frac{-2x}{\sqrt{ax^2+x}+\sqrt{bx^2+3x}} = \frac{-2}{\sqrt{a}+\sqrt{b}} = \frac{-2}{2\sqrt{a}} =$

$= \frac{-1}{\sqrt{a}} \Rightarrow \frac{-1}{\sqrt{a}} = \frac{-\sqrt{2}}{2} \Rightarrow \frac{-\sqrt{a}}{a} = \frac{-\sqrt{2}}{2} \Rightarrow a=b=2$

b) $$\lim_{x\to 0} \frac{mx^2-1+\cos x}{\operatorname{sen} x^2} = \frac{0}{0} \underset{\text{L'Hop.}}{=} \lim_{x\to 0} \frac{2mx - \operatorname{sen} x}{2x\cos x^2} = \frac{0}{0} \underset{\text{L'Hop}}{=} \lim_{x\to 0} \frac{2m-\cos x}{2\cos x^2 \cancel{+} (2x)^2 \operatorname{sen} x^2} = \frac{2m-1}{2}$$

$$\Rightarrow \frac{2m-1}{2} = 0 \Rightarrow 2m-1=0 \Rightarrow m=\frac{1}{2}$$



$P = 2x+2y+\pi x$

$S = 2m^2 = 2xy + \frac{\pi x^2}{2} \Rightarrow \frac{2-\pi x^2}{2x} = y$

$P = (2+\pi)x + \cancel{2}\,\frac{2-\pi x^2}{\cancel{2}x}$ mínimo.

$P' = 2+\pi + \frac{-2\pi x\cdot x-(2-\pi x^2)}{x^2} = 0 \Rightarrow$

$-(2+\pi)x^2 = 2\pi x^2.$ $\frac{(2+\pi)x^2-2\pi x^2-2+\pi x^2}{x^2}=0$

$\frac{2x^2+\pi x^2-2\pi x^2-2+\pi x^2}{x^2}=0$

$2x^2=2 \Rightarrow \boxed{x=1}$

$4 = 4xy+\pi x^2 \Rightarrow 4-\pi x^2 = 4xy \Rightarrow y = \frac{4-\pi x^2}{4x}$

$P = 2x + \frac{4-\pi x^2}{2x} + \pi x$

$P' = (2+\pi) + \frac{-2\pi x(2x)-(4-\pi x^2).2}{4x^2} = \frac{8x^2+\cancel{4\pi x^2}-\cancel{4\pi x^2}-8+2\pi x^2}{4x^2} = 0$

$8x^2+2\pi x^2 = 8 \Rightarrow x^2 = \frac{8}{8+2\pi} = \frac{4}{4+\pi} \Rightarrow \underline{\underline{x = \sqrt{\frac{4}{4+\pi}}}}$ mínimo.

$- \searrow \sqrt{\frac{4}{4+\pi}} \nearrow +$

$y = \frac{4-\pi\frac{4}{4+\pi}}{4\sqrt{\frac{4}{4+\pi}}} = \frac{4+\cancel{\pi}-\cancel{\pi}}{\sqrt{\frac{4}{4+\pi}}} = \frac{4\sqrt{4+\pi}}{2}$

$\underline{\underline{y = 2\sqrt{4+\pi}}}$

3 a) $f(x) = (x^2+a)e^x + bx$

$f'(x) = 2xe^x + (x^2+a)e^x + b = (x^2+2x+a)e^x + b$

$f''(x) = 2e^x + 2xe^x + 2xe^x + (x^2+a)e^x$

$f''(0) = 0 \Rightarrow 2 + a = 0 \Rightarrow \underline{\underline{a = -2}}$

$f'(1) = 2e + (1+a)e + b = 0 \Rightarrow 2e - e + b = 0 \Rightarrow$

$\Rightarrow \underline{\underline{b = -e}}$

b) $f(x) = 2x^3 + 12x^2 + ax + b \quad // \quad f'(x) = 6x^2 + 24x + a$

$f'(-2) = 2$

$y = 2(-2) + 3 = -1$ punto $(-2,-1)$

$f(-2) = -1$

$f'(-2) = 24 - 48 + a = 2 \Rightarrow \underline{\underline{a = 26}}$

$f(-2) = -16 + 48 - 2a + b = -1 \Rightarrow b = -1 + 16 - 48 + 52 \Rightarrow \underline{\underline{b = 19}}$

$+3 \quad +4$

4 a) $\displaystyle\int \frac{\ln x + 1}{x(\ln^2 x - 4)}\,dx = \int \frac{t+1}{t^2-4}\,dt = \int \frac{A}{t+2}\,dt + \int \frac{B}{t-2}\,dt =$

$\ln x = t$

$\frac{1}{x}\,dx = dt$

$= A\ln|t+2| + B\ln|t-2| + K = A\ln|\ln x + 2| + B\ln|\ln x - 2| + K$

$\underline{\underline{= \frac{1}{4}\ln|\ln x + 2| + \frac{3}{4}\ln|\ln x - 2| + K}}$

$t + 1 = A(t-2) + B(t+2)$

$t=2 \quad 3 = 4B \Rightarrow B = 3/4$

$t=-2 \quad -1 = -4A \Rightarrow A = 1/4$

b) $\int 2x e^{x^2+1} sen(x^2+1)\,dx = \int e^t sen\,t\,dt = e^t sen\,t - \int e^t \cos t\,dt =$

$x^2+1 = t$

$2x\,dx = dt$

$u = sen\,t \Rightarrow du = \cos t$

$dv = e^t \Rightarrow v = e^t$

$u = \cos t \Rightarrow du = -sen\,t$

$dv = e^t \Rightarrow v = e^t$

$= e^t sen\,t - e^t \cos t - \int e^t sen\,t\,dt \Rightarrow$

$2I = e^t sen\,t - e^t \cos t$

$I = \frac{e^t sen\,t - e^t \cos t}{2} + k_1$

$$\int 2x e^{x^2+1} sen(x^2+1)\,dx = \underline{\underline{\frac{e^{x^2+1} sen(x^2+1) - e^{x^2+1}\cos(x^2+1)}{2} + K}}$$

c) $\ln x = \ln(2x-1) \Rightarrow x = 2x-1 \Rightarrow \underline{1 = x}$

$$\int_1^e \left(\ln(2x-1) - \ln x\right)dx = \left[\frac{1}{2}(2x-1)\ln(2x-1) - \frac{1}{2}(2x-1) - x\ln x + x\right]_1^2 =$$

$$= \left(\left(\frac{3}{2}\ln 3 - \frac{3}{2} - 2\ln 2 + 2\right) - \left(\frac{1}{2}\overset{0}{\ln 1} - \frac{1}{2} - 1\overset{0}{\ln 1} + 1\right)\right) =$$

$\int \ln x\,dx = x\ln x - x$

$u = \ln x \Rightarrow du = \frac{1}{x}$

$dv = dx \Rightarrow v = x$

$$\left(\frac{3}{2}\ln 3 - \frac{3}{2} - 2\ln 2 + 2 + \frac{1}{2} - 1\right) = \underline{\underline{\frac{3}{2}\ln 3 - 2\ln 2}} \simeq 0{,}261624073$$

-1

$\int \ln(2x-1)\,dx = x\ln(2x-1) - \int \frac{2x}{2x-1}\,dx = x\ln(2x-1) - x - \int \frac{1}{2x-1} =$

$u = \ln(2x-1) \Rightarrow du = \frac{2}{2x-1}$

$dv = dx \Rightarrow v = x$

$2x \;|\underline{2x-1}$

$\underline{-2x+1}\quad 1$

$+1$

$= \underline{x\ln(2x-1) - x - \frac{1}{2}\ln(2x-1)}$

$\int \ln(2x-1)\,dx = \frac{1}{2}\int \ln t\,dt = \frac{1}{2}(t\ln t - t) = \frac{1}{2}(2x-1)\ln(2x-1) - \frac{1}{2}(2x-1)$

$2x-1 = t$

$2dx = dt$

①

1) a) 1) $\begin{vmatrix} 0 & 1 & 3 \\ 2-a & 1 & a \\ 3 & 3 & a \end{vmatrix} = 9(2-a) + 3a - 9 - a(2-a) =$

$= 18 - 9a + 3a - 9 + a^2 - 2a =$

$= a^2 - 8a + 9 = 0$

$a = \frac{8 \pm \sqrt{64-36}}{2} = \frac{8 \pm \sqrt{28}}{2} = \begin{cases} 4+\sqrt{7} \\ 4-\sqrt{7} \end{cases}$

Si $a \neq 4+\sqrt{7}, 4-\sqrt{7}$ rang $= 3$

$a = 4+\sqrt{7}$

$a = 4-\sqrt{7} \Rightarrow$ rang $= 2$

2) $P \cdot X = Q \Rightarrow X = P^{-1} Q$

$P = \begin{pmatrix} 0 & 1 & 3 \\ 1 & 1 & 1 \\ 3 & 3 & 1 \end{pmatrix}$ $\quad |P| = 2$

$P^t = \begin{pmatrix} 0 & 1 & 3 \\ 1 & 1 & 3 \\ 3 & 1 & 1 \end{pmatrix}$ $\quad \text{Adj}(P^t) = \begin{pmatrix} -2 & 8 & -2 \\ 2 & -9 & 3 \\ 0 & 3 & -1 \end{pmatrix}$

$P^{-1} = \begin{pmatrix} -1 & 4 & -1 \\ 1 & -9/2 & 3/2 \\ 0 & 3/2 & -1/2 \end{pmatrix}$

$X = \begin{pmatrix} -1 & 4 & -1 \\ 1 & -9/2 & 3/2 \\ 0 & 3/2 & -1/2 \end{pmatrix} \begin{pmatrix} 1 & 0 & 1 \\ 0 & 2 & -1 \\ 1 & 1 & 2 \end{pmatrix} = \begin{pmatrix} -2 & 7 & -7 \\ 5/2 & -15/2 & 13/2 \\ -1/2 & 5/2 & -5/2 \end{pmatrix}$

b) $$\begin{vmatrix} 1 & 0 & 1 \\ 1 & m & -1 \\ -m & -1 & -1 \end{vmatrix} = -m - 1 + m^2 - 1 =$$

~~$= m^2 - m = 0 \rightarrow m = 0 \;//\; \rightarrow m = 1 \;//$~~

~~Si $m \neq 0, 1 \rightarrow$ S.C.D~~

~~Si $m = 0$~~ [illegible]

$$= m^2 - m - 2 = 0 \begin{array}{l} \rightarrow m = -1 \\ \rightarrow m = 2 \end{array}$$

Si $m \neq -1, 2 \rightarrow$ S.C.D

Si $m = -1$

$$\left(\begin{array}{ccc|c} 1 & 0 & 1 & 2 \\ 1 & -1 & -1 & 4 \\ 1 & -1 & -1 & -5 \end{array}\right) \sim \left(\begin{array}{ccc|c} 1 & 0 & 1 & 2 \\ 0 & -1 & -2 & 2 \\ 0 & -1 & -2 & -7 \end{array}\right)$$

$$\sim \left(\begin{array}{ccc|c} 1 & 0 & 1 & 2 \\ 0 & -1 & -2 & 2 \\ 0 & 0 & 0 & -9 \end{array}\right) \rightarrow \text{S.I}$$

Si $m = 2$

$$\left(\begin{array}{ccc|c} 1 & 0 & 1 & 2 \\ 1 & 2 & -1 & 4 \\ -2 & -1 & -1 & -5 \end{array}\right) \sim \left(\begin{array}{ccc|c} 1 & 0 & 1 & 2 \\ 0 & 2 & -2 & 2 \\ 0 & -1 & 1 & -1 \end{array}\right) \sim$$

$$\left(\begin{array}{ccc|c} 1 & 0 & 1 & 2 \\ 0 & 2 & -2 & 2 \\ 0 & 0 & 0 & 0 \end{array}\right) \rightarrow \text{S.C.I}$$

$$x = 2 - z$$

$$\left.\begin{array}{l} x + z = 2 \\ 2y - 2z = 2 \end{array}\right\} \rightarrow y - z = 1 \Rightarrow y = 1 + z$$

$(2 - z, 1 + z, z)$

$(2 - \lambda, 1 + \lambda, \lambda)\ \lambda \in \mathbb{R}$

b $m=-2$ ②

$$\left(\begin{array}{ccc|c} 1 & 0 & 1 & 2 \\ 1 & -2 & -1 & 4 \\ 2 & -1 & -1 & -5 \end{array}\right) \sim \left(\begin{array}{ccc|c} 1 & 0 & 1 & 2 \\ 0 & -2 & -2 & 2 \\ 0 & -1 & -3 & -9 \end{array}\right)$$

$$\left(\begin{array}{ccc|c} 1 & 0 & 1 & 2 \\ 0 & -2 & -2 & 2 \\ 0 & 0 & 4 & 20 \end{array}\right) \quad \left.\begin{array}{r} x + z = 2 \\ -2y-2z = 2 \\ 4z = 20 \end{array}\right\} \begin{array}{l} \to \cancel{x} = -3 \\ \to y = -6 \\ \to z = 5 \end{array}$$

$(-3, -6, +5)$

2) a)

I) $\overline{u_r} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & 1 & 0 \\ 0 & 1 & -1 \end{vmatrix} = (-1, 1, 1)$

$\overline{u_s} = \overrightarrow{AB} = (-1, -1, -1)$

$P_r(0, 2, 2)$ $\quad P_s = A = (2, 1, 0)$

$\overline{P_rP_s}\,(2, -1, -2)$

$[\overline{u_r}, \overline{u_s}, \overline{P_rP_s}] = \begin{vmatrix} -1 & 1 & 1 \\ -1 & -1 & -1 \\ 2 & -1 & -2 \end{vmatrix} = -2+1-2+2+1-2 =$
$= -2 \neq 0$

Se cruzan no paralelas

II) $r: \begin{cases} x = -t \\ y = 2+t \\ z = 2+t \end{cases}$ $\quad \overline{CA} = (2+t, -1-t, -2-t)$
$\quad \overline{CB} = (1+t\ -2-t\ -3-t)$

$\overline{CA}\cdot\overline{CB} = (2+t)(1+t) + (-1-t)(-2-t) + (-2-t)(-3-t) =$

$= 2+2t+t+t^2+2+t+2t+t^2+6+2t+3t+t^2 =$

$= 3t^2+11t+10 = 0$

$t = \dfrac{-11 \pm \sqrt{121-120}}{6} = \dfrac{-11 \pm 1}{6} = \Big< \begin{array}{l} \frac{-10}{6} = -5/3 \\ -2 \end{array}$

$t=-2 \Rightarrow \underline{P(2,0,0)}$

b) 1) $\vec{n}_{\pi}(1,3,1)$

$\vec{n}_{2}(1,0,0)$

$$\cos\alpha = \frac{|1\cdot1+3\cdot0+1\cdot0|}{\sqrt{11}\cdot\sqrt{1}} = \frac{1}{\sqrt{11}} = \frac{\sqrt{11}}{11}$$

$$\alpha = \arccos\frac{\sqrt{11}}{11} = \underline{72'45^{\circ}}$$

2) $x=y=0 \longrightarrow (0,0,4)$

$x=z=0 \longrightarrow (0,4/3,0)$

$y=z=0 \longrightarrow (4,0,0)$

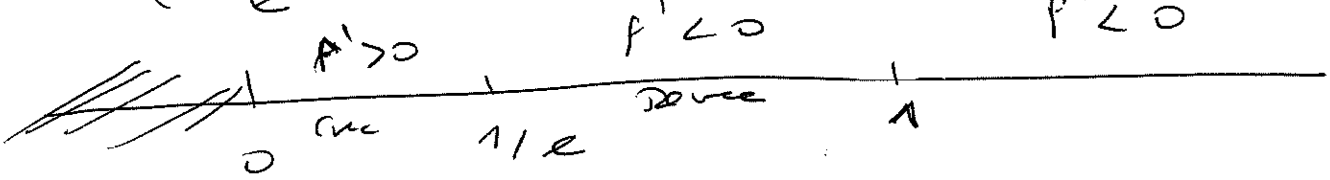
$$V_{Tet} = \frac{\left|\begin{vmatrix} 0 & 0 & 4 \\ 0 & 4/3 & 0 \\ 4 & 0 & 0 \end{vmatrix}\right|}{6} = \frac{64}{18} = \underline{\frac{32}{9}\,u^3}$$

3) a) 1) $D=(0,+\infty)-\{1\}$

2) $$y' = \frac{-(\ln x+1)}{(x\ln x)^2} = 0 \Rightarrow \ln x+1=0$$

$\Rightarrow \ln x=-1 \Rightarrow x=e^{-1}=\frac{1}{e}$

$P\left(\frac{1}{e},-e\right) \longrightarrow$ máximo.



Crec $(0, 1/e)$

Decrec $(1/e, +\infty)$

3) A.H: $\lim_{x \to +\infty} \frac{1}{x \ln x} = 0 \Rightarrow \boxed{y = 0}$ Dcha

A.V; $x \ln x = 0$ ~~$\ln x = 1 \Rightarrow x = e$ A.V~~

$x = 0 \rightarrow$ No puede ser

$\ln x = 0 ; x = 1 \Rightarrow \boxed{x = 1}$ A.V

Oblic: No tiene

Corte ejes < OX: No corte
OY: No corte

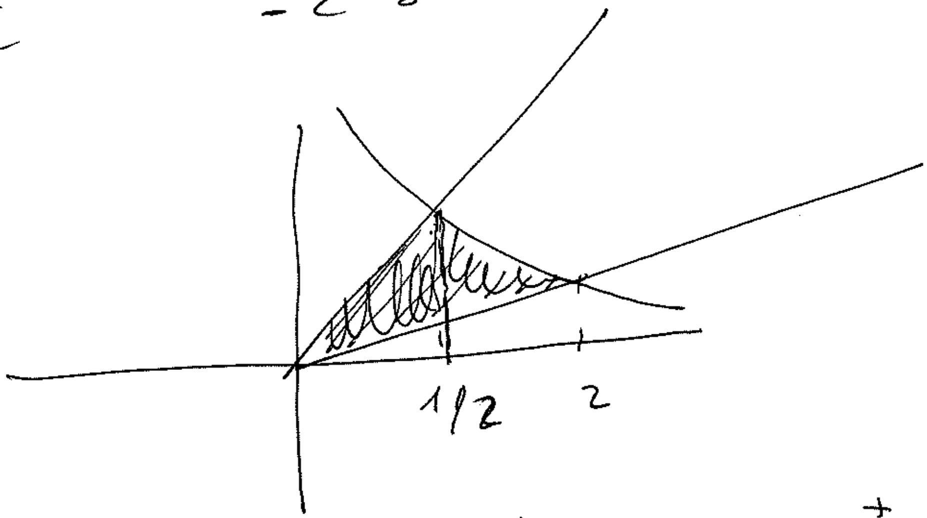
b) $\lim_{x \to \frac{\pi}{2}} \frac{\operatorname{sen} x (1 - \operatorname{sen} x)}{(\cos x)^2} =$

$= \lim_{x \to \frac{\pi}{2}} \frac{\cos x (1 - \operatorname{sen} x) - \operatorname{sen} x \cdot \cos x}{-2 \cos x \cdot \operatorname{sen} x} =$

$= \lim_{x \to \frac{\pi}{2}} \frac{(1 - \operatorname{sen} x) - \operatorname{sen} x}{-2 \operatorname{sen} x} =$

$= \lim_{x \to \frac{\pi}{2}} \frac{1 - 2\operatorname{sen} x}{-2 \operatorname{sen} x} = +\frac{1}{2}$ //

4) a)



$4x = \frac{1}{x} \Rightarrow 4x^2 - 1 \Rightarrow x = \pm \frac{1}{2}$

$\frac{x}{4} = \frac{1}{x} ; \quad x^2 = 4 \Rightarrow x = \pm 2$

$$A = \int_0^{1/2} \left(4x - \frac{x}{4}\right) dx + \int_{1/2}^{2} \left(\frac{1}{x} - \frac{x}{4}\right) dx =$$

$$= \left[2x^2 - \frac{x^2}{8}\right]_0^{1/2} + \left[\ln x - \frac{x^2}{8}\right]_{1/2}^{2} =$$

$$= \left[\left(\frac{2}{4} - \frac{1}{32}\right) - (0 - 0)\right] + \left[\left(\ln 2 - \frac{4}{8}\right) - \left(\ln\frac{1}{2} - \frac{1}{32}\right)\right] =$$

$$= \frac{15}{32} + \ln 2 - \ln\frac{1}{2} + \frac{1}{32} = \ln 2 - \ln\frac{1}{2} + \frac{1}{2} =$$

$$= \underline{\underline{2\ln 2 + \frac{1}{2}}}$$

b) $$\int \frac{e^{2x} \cdot e^{x}}{e^{2x} - 3e^{x} + 2}\, dx = \int \frac{t^2}{t^2 - 3t + 2}\, dt$$

$$= \int \frac{(t^2 - 3t + 2) + (3t - 2)}{t^2 - 3t + 2}\, dt =$$

$$= \int 1\, dt + \int \frac{3t - 2}{t^2 - 3t + 2}\, dt$$

$$t^2 - 3t + 2 = 0 \begin{cases} t = 1 \\ t = 2 \end{cases}$$

$$\frac{3t - 2}{t^2 - 3t + 2} = \frac{A}{(t-1)} + \frac{B}{t-2} = \frac{A(t-2) + B(t-1)}{(t-1)(t-2)}$$

$t=1 \Rightarrow 1 = -A \Rightarrow A = -1$

$t=2 \Rightarrow 4 = B$

$$\int \frac{3t-2}{t^2-3t+2}\,dt = \int \frac{-1}{t-1}\,dt + \int \frac{+4}{t-2}\,dt =$$

$$= -\ln|t-1| + 4\ln|t-2|$$

$$I = t - \ln|t-1| + 4\ln|t-2| + cte$$

$$\underline{\underline{I = e^x - \ln|e^x-1| + 4\ln|e^x-2| + cte}}$$