1 a) 21 equipos a doble vuelta $\Rightarrow$ 40 partidos

(20) x g 3p 2p.

(10) y em 1p. 1p

(10) z perd 0p 0p

$$\begin{cases} x+y+z=40 \\ 3x+y=70 \\ 2x+y=50 \end{cases} \quad \left(\begin{array}{ccc|c} 1 & 1 & 1 & 40 \\ 3 & 1 & 0 & 70 \\ 2 & 1 & 0 & 50 \end{array}\right)$$

$$\begin{array}{ccc} z & y & x \end{array}$$

$$\rightarrow \left(\begin{array}{ccc|c} 1 & 1 & 1 & 40 \\ 0 & 1 & 3 & 70 \\ 0 & 1 & 2 & 50 \end{array}\right) \rightarrow \left(\begin{array}{ccc|c} 1 & 1 & 1 & 40 \\ 0 & 1 & 3 & 70 \\ 0 & 0 & -1 & -20 \end{array}\right)$$

$$\left(\begin{array}{ccc|c} 1 & 1 & 1 & 40 \\ 0 & -2 & -3 & -80 \\ 0 & 0 & -1 & -10 \end{array}\right)$$

$$\boxed{\begin{array}{l} x=20 \\ y=10 \\ z=10 \end{array}}$$

b) i) $A=\begin{pmatrix} a^3 & a^2b & a^2b \\ ab & a^2 & b^2 \\ ab & b^2 & a^2 \end{pmatrix}$

$$|A| = a^2\begin{vmatrix} a & b & b \\ ab & a^2 & b^2 \\ ab & b^2 & a^2 \end{vmatrix} = a^3\begin{vmatrix} 1 & b & b \\ b & a^2 & b^2 \\ b & b^2 & a^2 \end{vmatrix} = a^3\begin{vmatrix} 1 & b & b \\ 0 & a^2-b^2 & 0 \\ 0 & 0 & a^2-b^2 \end{vmatrix} =$$

$$= a^3(a^2-b^2)^2.$$

ii) $b=-a$

$$A=\begin{pmatrix} a^3 & -a^3 & -a^3 \\ -a^2 & a^2 & a^2 \\ -a^2 & a^2 & a^2 \end{pmatrix} \rightarrow \begin{pmatrix} a^3 & -a^3 & -a^3 \\ 0 & 0 & 0 \\ & & \end{pmatrix} \Rightarrow \underline{\text{rango } A = 1}$$

2 a) $A=\begin{pmatrix} 2 & 0 \\ -1 & 1 \\ 1 & 0 \end{pmatrix}$ $B=\begin{pmatrix} 1 & -1 & 2 \\ 0 & 1 & 0 \end{pmatrix}$ $C=\begin{pmatrix} -1 & 2 & 1 \\ -1 & 3 & 1 \\ 2 & 1 & 0 \end{pmatrix}$

$$AB - CX = 2C \Rightarrow AB - 2C = CX \Rightarrow$$

$$\Rightarrow X = C^{-1}(AB-2C) = \boxed{\begin{pmatrix} 0 & -5/2 & 4 \\ -3 & 2 & -6 \\ 10 & -35/2 & 18 \end{pmatrix}}$$

$$A\cdot B = \begin{pmatrix} 2 & -2 & 4 \\ -1 & 2 & -2 \\ 1 & -1 & 2 \end{pmatrix} \qquad C^{-1} = \begin{pmatrix} 1/2 & -1/2 & 1/2 \\ -1 & 1 & 0 \\ 7/2 & -5/2 & 1/2 \end{pmatrix}$$

$$2C = \begin{pmatrix} -2 & 4 & 2 \\ -2 & 6 & 2 \\ 4 & 2 & 0 \end{pmatrix}$$

$$AB - 2C = \begin{pmatrix} 4 & -6 & 2 \\ 1 & -4 & -4 \\ -3 & -3 & 2 \end{pmatrix}$$

b) $\begin{vmatrix} a & b & c \\ p & q & r \\ x & y & z \end{vmatrix} = 7$

$$\begin{vmatrix} 3a & 3b & 3c \\ a+p & b+q & c+r \\ -x+a & -y+b & -z+c \end{vmatrix} = 3\overbrace{\begin{vmatrix} a & b & c \\ a & b & c \\ -x+a & -y+b & -z+c \end{vmatrix}}^{=0} + 3\begin{vmatrix} a & b & c \\ p & q & r \\ -x+a & -y+b & -z+c \end{vmatrix} =$$

$$= 3\begin{vmatrix} a & b & c \\ p & q & r \\ -x & -y & -z \end{vmatrix} + 3\overbrace{\begin{vmatrix} a & b & c \\ p & q & r \\ a & b & c \end{vmatrix}}^{=0} = -3\begin{vmatrix} a & b & c \\ p & q & r \\ x & y & z \end{vmatrix} = -3\cdot 7 = \underline{\underline{-21}}$$

3 $\begin{cases} 2mx - 2y - mz = 2 \\ mx - y + z = 5 \\ 3mx + 4y + (m-1)z = m-8 \end{cases}$

a) $\left(\begin{array}{ccc|c} 2m & -2 & -m & 2 \\ m & -1 & 1 & 5 \\ 3m & 4 & m-1 & m-8 \end{array}\right) \to \left(\begin{array}{ccc|c} m & -1 & 1 & 5 \\ 0 & 7 & m-4 & m-23 \\ 0 & 0 & -m-2 & 8 \end{array}\right)$

$|A| = -7m^2 - 14m = -7m(m+2) \quad -7m(m+2) = 0 \Rightarrow \begin{cases} m = 0 \\ m = -2 \end{cases}$

Si $m \neq 0, -2$ rang A = rang $A|B$ = nº incógnitas S.C.D.

Si $m = 0$

$\left(\begin{array}{ccc|c} 0 & -2 & 0 & 2 \\ 0 & 0 & 1 & 4 \\ 0 & 0 & 0 & 0 \end{array}\right)$ rang $A = 2$ = rang $A|B < 3$ = nº incógn. S.C.I.

Si $m = -2$

$\left(\begin{array}{ccc|c} -4 & -2 & 2 & 2 \\ 0 & 7 & -6 & 13 \\ 0 & 0 & 0 & 4 \end{array}\right)$ rang $A = 2 \neq 3$ = rang $A|B$ S.I.

b) resolverlo.

si $m = 0$ $\quad z = 4$ y $y = -1 \Rightarrow (\lambda, -1, 4)$

si $m \neq 0, -2$

$$x = \frac{\begin{vmatrix} 2 & -2 & -m \\ 5 & -1 & 1 \\ m-8 & 4 & m-1 \end{vmatrix}}{-7m(m+2)} = \frac{\overset{-m(m+6)}{-m^2 - 6m}}{-7m(m+2)} = \frac{m+6}{7(m+2)}$$

$$y = \frac{\begin{vmatrix} 2m & 2 & -m \\ m & 5 & 1 \\ 3m & m-8 & m-1 \end{vmatrix}}{-7m(m+2)} = \frac{-m^3 + 29m^2 + 14m}{-7m(m+2)} = \frac{m^2 - 29m - 14}{7(m+2)}$$

$$z = \frac{\begin{vmatrix} 2m & -2 & 2 \\ m & -1 & 5 \\ 3m & 4 & m-8 \end{vmatrix}}{-7m(m+2)} = \frac{-56m}{-7m(m+2)} = \frac{8}{m+2}$$

1) a) $500 \to x$, $200 \to y$, $100 \to z$

$$\left.\begin{array}{l} 500x + 200y + 100z = 6100 \\ 4x = 3z + 5 \\ 5y + 4 = 2(x+z) \end{array}\right\}$$

$$\left.\begin{array}{l} 5x + 2y + z = 61 \\ 4x \quad - 3z = 5 \\ -2x + 5y - 2z = -4 \end{array}\right\} \left(\begin{array}{ccc|c} 5 & 2 & 1 & 61 \\ 4 & 0 & -3 & 5 \\ -2 & 5 & -2 & -4 \end{array}\right) \sim$$

$$\sim \left(\begin{array}{ccc|c} 5 & 2 & 1 & 61 \\ 0 & +8 & +19 & +219 \\ 0 & 29 & -8 & 102 \end{array}\right) \sim \left(\begin{array}{ccc|c} 5 & 2 & 1 & 61 \\ 0 & +8 & +19 & +219 \\ 0 & 0 & 615 & +5535 \end{array}\right)$$

$$5x + 2y + z = 61 \quad 5x = 61 - 12 - 9 = 40; \ x = \frac{40}{5} = 8$$

$$8y + 19z = 219 \quad 8y + 171 = 219; \ y = \frac{48}{8} = 6$$

$$615z = 5535 \quad z = \frac{5535}{615} = 9$$

$$\underline{(8, 6, 9)}$$

b)

I) $$\begin{vmatrix} 3x & 3y & 3z \\ 5+1 & 0+1 & 3+1 \\ 1 & 1 & 1 \end{vmatrix} = \begin{vmatrix} 3x & 3y & 3z \\ 5 & 0 & 3 \\ 1 & 1 & 1 \end{vmatrix} + \begin{vmatrix} 3x & 3y & 3z \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{vmatrix}$$

$$= 3 \begin{vmatrix} x & y & z \\ 5 & 0 & 3 \\ 1 & 1 & 1 \end{vmatrix} = 3 \cdot 2 = 6$$

II) $$\begin{vmatrix} 5x & 5y & 5z \\ 1 & 0 & 3/5 \\ 1 & 1 & 1 \end{vmatrix} = 5 \cdot \frac{1}{5} \begin{vmatrix} x & y & z \\ 5 & 0 & 3 \\ 1 & 1 & 1 \end{vmatrix} = 1 \cdot 2 = 2$$

III) $$\begin{vmatrix} x & y & z \\ 2x & 2y & 2z \\ x+1 & y+1 & z+1 \end{vmatrix} + \begin{vmatrix} x & y & z \\ 5 & 0 & 3 \\ x+1 & y+1 & z+1 \end{vmatrix} = \begin{vmatrix} x & y & z \\ 5 & 0 & 3 \\ x & y & z \end{vmatrix} + \begin{vmatrix} x & y & z \\ 5 & 0 & 3 \\ 1 & 1 & 1 \end{vmatrix} =$$

$$= 2$$

2) a)

$$|A| = \begin{vmatrix} 1 & 0 & a^2 \\ 1 & a^2-1 & a \\ 1 & 2(a^2-1) & 2a-1 \end{vmatrix} = \begin{array}{l} (a^2-1)(2a-1)+2a^2(a^2-1) \\ -a^2(a^2-1)-2a(a^2-1) = \\ = (a^2-1)[2a-1+2a^2-a^2-2a] = \end{array}$$

$$= (a^2-1)(a^2-1) = \underline{\underline{(a^2-1)^2}} = \underline{\underline{a^4 - 2a^2 + 1}}$$

$a^2-1=0 \Rightarrow \underline{\underline{a = \pm 1}}$

$\underline{\underline{a=1}}$ $\begin{pmatrix} 1 & 0 & 1 \\ 1 & 0 & 1 \\ 1 & 0 & 1 \end{pmatrix}$ $\underline{\underline{\text{rango } 1}}$

$\underline{\underline{a=-1}}$ $\begin{pmatrix} 1 & 0 & 1 \\ 1 & 0 & -1 \\ 1 & 0 & -3 \end{pmatrix} \sim \begin{pmatrix} 1 & 0 & 1 \\ 0 & 0 & -2 \\ 0 & 0 & -4 \end{pmatrix} \sim \begin{pmatrix} 1 & 0 & 1 \\ 0 & 0 & -2 \\ 0 & 0 & 0 \end{pmatrix}$ $\underline{\underline{\text{rango } 2}}$

Si $a \neq \pm 1 \Rightarrow$ $\underline{\underline{\text{rango } 3}}$

$$A = \begin{pmatrix} 1 & 0 & 0 \\ 1 & -1 & 0 \\ 1 & -2 & -1 \end{pmatrix} \; ; \; |A| = 1$$

$$\text{Adj } A = \begin{pmatrix} 1 & 1 & -1 \\ 0 & -1 & 2 \\ 0 & 0 & -1 \end{pmatrix}$$

$$(\text{Adj } A)^t = \begin{pmatrix} 1 & 0 & 0 \\ 1 & -1 & 0 \\ -1 & 2 & -1 \end{pmatrix} , \quad A^{-1} = \begin{pmatrix} 1 & 0 & 0 \\ 1 & -1 & 0 \\ -1 & 2 & -1 \end{pmatrix}$$

b) I) $A^2 = \begin{pmatrix} 1 & 1 & 3 \\ 0 & 2 & 5 \\ 0 & -1 & -2 \end{pmatrix} \begin{pmatrix} 1 & 1 & 3 \\ 0 & 2 & 5 \\ 0 & -1 & -2 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 2 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{pmatrix}$ ②

$A^3 = \begin{pmatrix} 1 & 0 & 2 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{pmatrix} \begin{pmatrix} 1 & 1 & 3 \\ 0 & 2 & 5 \\ 0 & -1 & -2 \end{pmatrix} = \begin{pmatrix} 1 & -1 & -1 \\ 0 & -2 & -5 \\ 0 & 1 & 2 \end{pmatrix}$

$A^4 = \begin{pmatrix} 1 & -1 & -1 \\ 0 & -2 & -5 \\ 0 & 1 & 2 \end{pmatrix} \begin{pmatrix} 1 & 1 & 3 \\ 0 & 2 & 5 \\ 0 & -1 & -2 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$

II) TEÓRICO

III) $A^{23} = (A^4)^5 \cdot A^3 = (I_3)^5 \cdot A^3 = A^3 = \begin{pmatrix} 1 & -1 & -1 \\ 0 & -2 & -5 \\ 0 & 1 & 2 \end{pmatrix}$

IV) $AX = A^4;\ \underbrace{A^{-1}A}_{I_3}X = A^{-1}A^4$

$X = A^3 = \begin{pmatrix} 1 & -1 & -1 \\ 0 & -2 & -5 \\ 0 & 1 & 2 \end{pmatrix}$

3) $\begin{vmatrix} 2m+5 & 2m+10 & 6 \\ m^2 & 0 & -2 \\ 1 & 2 & 1 \end{vmatrix} = 12m^2 - 2(2m+10) + 4(2m+5) - m^2(2m+10) =$

$= 12m^2 - 4m - 20 + 8m + 20 - 2m^3 - 10m^2 =$

$= -2m^3 + 2m^2 + 4m = -2m(m^2 - m - 2) =$

$= -2m(m+1)(m-2)$ → $m = 0$, $m = -1$, $m = 2$

m = 0

$$\left(\begin{array}{ccc|c} 5 & 10 & 6 & 8 \\ 0 & 0 & -2 & 0 \\ 1 & 2 & 1 & 1 \end{array}\right) \sim \left(\begin{array}{ccc|c} 1 & 2 & 1 & 1 \\ 5 & 10 & 6 & 8 \\ 0 & 0 & -2 & 0 \end{array}\right) \sim$$

$$\sim \left(\begin{array}{ccc|c} 1 & 2 & 1 & 1 \\ 0 & 0 & 1 & 3 \\ 0 & 0 & -2 & 0 \end{array}\right) \sim \left(\begin{array}{ccc|c} 1 & 2 & 1 & 1 \\ 0 & 0 & 1 & 3 \\ 0 & 0 & 0 & 6 \end{array}\right) \longrightarrow \text{S.I}$$

m = −1

$$\left(\begin{array}{ccc|c} 3 & 8 & 6 & 8 \\ 1 & 0 & -2 & -1 \\ 1 & 2 & 1 & 1 \end{array}\right) \sim \left(\begin{array}{ccc|c} 1 & 2 & 1 & 1 \\ 1 & 0 & -2 & -1 \\ 3 & 8 & 6 & 8 \end{array}\right) \sim$$

$$\sim \left(\begin{array}{ccc|c} 1 & 2 & 1 & 1 \\ 0 & -2 & -3 & -2 \\ 0 & 2 & 3 & 5 \end{array}\right) \sim \left(\begin{array}{ccc|c} 1 & 2 & 1 & 1 \\ 0 & -2 & -3 & -2 \\ 0 & 0 & 0 & 3 \end{array}\right) \longrightarrow \text{S.I}$$

m = 2

$$\left(\begin{array}{ccc|c} 9 & 14 & 6 & 8 \\ 4 & 0 & -2 & 2 \\ 1 & 2 & 1 & 1 \end{array}\right) \sim \left(\begin{array}{ccc|c} 1 & 2 & 1 & 1 \\ 4 & 0 & -2 & 2 \\ 9 & 14 & 6 & 8 \end{array}\right) \sim$$

$$\sim \left(\begin{array}{ccc|c} 1 & 2 & 1 & 1 \\ 0 & -8 & -6 & -2 \\ 0 & -4 & -3 & -1 \end{array}\right) \sim \left(\begin{array}{ccc|c} 1 & 2 & 1 & 1 \\ 0 & -8 & -6 & -2 \\ 0 & 0 & 0 & 0 \end{array}\right) \longrightarrow \text{S.C.I}$$

$$\left.\begin{array}{r} x + 2y + z = 1 \\ 4y + 3z = 1 \end{array}\right\} \longrightarrow y = \frac{1-3z}{4}$$

$$x + \frac{2-6z}{4} + z = 1; \qquad 4x + 2 - 6z + 4z = 4$$

$$4x = 2 + 2z; \qquad x = \frac{1+z}{2} \qquad \underline{\underline{\left(\frac{1+\lambda}{2}, \frac{1-3\lambda}{4}, \lambda\right)}}$$

Si $m \neq 0, -1, 2 \longrightarrow$ S.C.D

$$x = \frac{\begin{vmatrix} 8 & 2m+10 & 6 \\ m & 0 & -2 \\ 1 & 2 & 1 \end{vmatrix}}{-2m(m+1)(m-2)} = \frac{-\cancel{2}\,\cancel{(m-2)}(m+3)}{-\cancel{2}m(m+1)\cancel{(m-2)}} = \frac{m+3}{m(m+1)}$$

$12m - 4m - 20 + 32 - 2m^2 - 10m =$
$= -2m^2 - 2m + 12 = -2(m^2 + m - 6)$

$$y = \frac{\begin{vmatrix} 2m+5 & 8 & 6 \\ m^2 & m & -2 \\ 1 & 1 & 1 \end{vmatrix}}{-2m(m+1)(m-2)} = \frac{3\cancel{(m-2)}}{-2m(m+1)\cancel{(m-2)}} = \frac{3}{-2m(m+1)}$$

$2m^2 + 5m + 6m^2 - 16 - 6m + 4m + 10 - 8m^2 =$
$= 3m - 6$

$$z = \frac{\begin{vmatrix} 2m+5 & 2m+10 & 8 \\ m^2 & 0 & m \\ 1 & 2 & 1 \end{vmatrix}}{-2m(m+1)(m-2)} = \frac{-\cancel{2}m^{\cancel{2}}\cancel{(m-2)}}{-\cancel{2}\cancel{m}(m+1)\cancel{(m-2)}} = \frac{m}{m+1}$$

$16m^2 + 2m^2 + 10m - 4m^2 - 10m - 2m^3 - 10m^2$
$= -2m^3 + 4m^2$

1) a) $\bar{u} \wedge \vec{v} = \begin{vmatrix} \bar{i} & \bar{j} & \vec{k} \\ 1 & 1 & 1 \\ 2 & 2 & a \end{vmatrix} = (a-2, -a+2, 0)$

$\bar{u} \cdot \vec{v} = 1(a-2) + 1(-a+2) + 1 \cdot 0 = 0$

Sí es ~~cierto~~. $\bar{u} \wedge \vec{v} \perp \vec{u}$

b) $\begin{vmatrix} 1 & 1 & 1 \\ 2 & 2 & a \\ 0 & a & 2 \end{vmatrix} = \cancel{4} + 2a - a^2 - 4 = -a^2 + 2a$

$-a^2 + 2a = 0 \Rightarrow -a(a-2) = 0$ $\begin{cases} a = 0 \,// \\ a = 2 \,// \end{cases}$

c) $\bar{u} + \bar{v} = (3, 3, a+1)$ $\quad \bar{u} - \overline{w} = (1, 1-a, -1)$

$(\bar{u}+\bar{v}) \cdot (\bar{u} - \overline{w}) = 3 + 3(1-a) - (a+1) = 3 + 3 - 3a - a - 1 =$

$= -4a + 5 = 0; \quad a = 5/4 \,//$

d) $(1, 2, -3) = \alpha(1,1,1) + \beta(2,2,1) + \gamma(0,1,2)$

$\left.\begin{aligned} 1 &= \alpha + 2\beta \\ 2 &= \alpha + 2\beta + \gamma \\ -3 &= \alpha + \beta + 2\gamma \end{aligned}\right\}$ $\left(\begin{array}{ccc|c} 1 & 2 & 0 & 1 \\ 1 & 2 & 1 & 2 \\ 1 & 1 & 2 & -3 \end{array}\right) \sim \left(\begin{array}{ccc|c} 1 & 2 & 0 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & -1 & 2 & -4 \end{array}\right)$

$\left(\begin{array}{ccc|c} 1 & 2 & 0 & 1 \\ 0 & -1 & 2 & -4 \\ 0 & 0 & 1 & 1 \end{array}\right)$ $\begin{aligned} &\alpha + 12 = 1; \quad \alpha = -11 \,// \\ &-\beta + 2 = -4; \quad \beta = 6 \,// \\ &\gamma = 1 \,// \end{aligned}$

2) $\overline{u_r}(2, a+1, 1)$; $\overline{n_\pi} = (2, b, 1)$; $P_r(-2, 4, -1)$

a) $r \perp \pi \Rightarrow u_r \,//\, \overrightarrow{n_\pi}$

$0 \in r \Rightarrow \frac{2}{2} = \frac{-4}{a+1} = 1; \quad -4 = a + 1; \quad \boxed{a = -5}$

$\overline{u_r}(2, -4, 1) \qquad \frac{2}{2} = \frac{b}{-4} = \frac{1}{1} \Rightarrow \boxed{b = -4}$

$$r: \begin{cases} x = -2+2t \\ y = 4-4t \\ z = -1+t \end{cases} \qquad \pi: 2x-4y+z+1=0$$

$$2(-2+2t)-4(4-4t)+(-1+t)+1=0$$

$$-4+4t-16+16t-1+t+1=0 ; \quad 21t=20$$

$$t=\frac{20}{21} \qquad I\left(\frac{-2}{21}, \frac{4}{21}, \frac{-1}{21}\right)$$

b) $P\in r \quad \longrightarrow \quad \frac{4}{2}=\frac{-2}{a+1}=2, \quad \frac{-2}{a+1}=2; \quad -2=2a+2$

$r \parallel \pi \qquad 2a=-4; \quad a=-\frac{4}{2}; \quad \boxed{a=-2}$

$\vec{u_r} \perp \vec{n_\pi} \qquad \vec{u_r}(2,-1,1) \quad \vec{n_\pi}(2,b,1)$

$4-b+1=0; \quad \boxed{b=5}$

$$d(r,\pi)=d(P,\pi)=\frac{|2.2+5.2+1+1|}{\sqrt{2^2+5^2+1^2}}=$$

$$=\frac{|4+10+1+1|}{\sqrt{4+25+1}}=\frac{16}{\sqrt{30}}=\frac{16\sqrt{30}}{30}=\frac{8\sqrt{30}}{15}\,u$$

c) $r\subset\pi; \quad \vec{u_r}(2,a+1,1) \quad \vec{n_\pi}(2,b,1)$

$\vec{u_r}\perp\vec{n_\pi} \qquad 4+b(a+1)+1=0; \quad b(a+1)=-5$

$P_r\in\pi; \qquad P_r(-2,4,-1)$

$2.(-2)+b.4+(-1)+1=0 ; \quad -4+4b=0$

$\boxed{b=1} \qquad 1.(a+1)=-5; \quad a+1=-5; \quad \boxed{a=-6}$

2

3) a) $M_{AB} = (2, 1, 0)$

$r \perp \pi \Rightarrow \vec{u_r} = \vec{n_\pi} = (2, 3, -1)$

$$r: \begin{cases} x = 2 + 2t \\ y = 1 + 3t \\ z = -t \end{cases}$$

b) $$r_\perp = \begin{cases} x = 1 + 2t \\ y = 2 + 3t \\ z = 1 - t \end{cases} \qquad \pi: 2x + 3y - z = 0$$

$2(1+2t) + 3(2+3t) - (1-t) = 0; \quad 2+4t+6+9t-1+t=0$

$14t = -7; \quad t = -1/2 \qquad P\left(0, \frac{1}{2}, \frac{3}{2}\right)$

$d = \frac{|2+6-1|}{\sqrt{4+9+1}} = \frac{7}{\sqrt{14}} = \frac{\sqrt{14}}{2} u$

c) $\vec{u_r} = \vec{PQ} = (1, -1, 3-2\alpha); \quad \vec{n_\pi}(2, 3, -1)$

si $\vec{u_r} \perp \vec{n_\pi}$ $\quad 2-3-3+2\alpha = 0; \quad 2\alpha = 4; \quad \alpha = 2$

si $\alpha = 2 \Rightarrow r: \begin{cases} x = t \\ y = -t \\ z = 2-t \end{cases} \qquad P_r(0, 0, 2)$

$2 \cdot 0 + 3 \cdot 0 - 2 \neq 0; \qquad \alpha = 2 \Rightarrow$ paralelos no coincidentes

si $\alpha \neq 2 \Rightarrow$ Secantes

4) a) $\overrightarrow{u_{r_{AB}}} = \overrightarrow{AB} = (2, 0, 3)$

$$r_{AB}: \begin{cases} x = 1 + 2t \\ y = 0 \\ z = -1 + 3t \end{cases}$$

b) $\overrightarrow{u_r} = (1, 2, -1)$; $\overrightarrow{n_\pi} = (1, -2, 1)$

$$\operatorname{sen} \alpha = \frac{|1 \cdot 1 + 2 \cdot (-2) + (-1) \cdot 1|}{\sqrt{6} \cdot \sqrt{6}} = \frac{4}{6} = \frac{2}{3}$$

$\alpha = 41'81^\circ$

c) $\overrightarrow{u_r} = (1, 2, -1)$

$$\overrightarrow{u_s} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & 1 & -1 \\ 2 & 0 & 1 \end{vmatrix} = (1, -3, -2)$$

$P_r(-1, 1, 0)$ $P_s(0, 1, 1)$

$\overrightarrow{P_rP_s}(1, 0, 1)$

$$\overrightarrow{u_r} \wedge \overrightarrow{u_s} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & 2 & -1 \\ 1 & -3 & -2 \end{vmatrix} = (-7, 1, -5)$$

$$[\overrightarrow{u_r}, \overrightarrow{u_s}, \overrightarrow{P_rP_s}] = \begin{vmatrix} 1 & 2 & -1 \\ 1 & -3 & -2 \\ 1 & 0 & 1 \end{vmatrix} = -3 - 4 - 3 - 2 = -12$$

$$|\overrightarrow{u_r} \wedge \overrightarrow{u_s}| = \sqrt{49 + 1 + 25} = \sqrt{75}$$

$$d(r, s) = \frac{12}{\sqrt{75}} = \frac{12\sqrt{75}}{75} = \frac{60\sqrt{3}}{75} = \frac{4\sqrt{3}}{5} \, u$$

d) ~~[illegible]~~

$\pi_1 \perp r \Rightarrow \vec{n}_{\pi_1} = \vec{u}_r = (1, 2, -1)$

$\pi_1: x + 2y - z + d = 0$

$$d(A, \pi_1) = \frac{|1 + 2\cdot 0 - (-1) + d|}{\sqrt{1+4+1}} = \frac{|2+d|}{\sqrt{6}} = \frac{\sqrt{2}}{2}$$

$2|2+d| = \sqrt{12}$; $2 \cdot |2+d| = 2\sqrt{3}$

$|2+d| = \sqrt{3}$; $2+d = \sqrt{3} \longrightarrow d_1 = \sqrt{3} - 2$ //

$2+d = -\sqrt{3} \longrightarrow d_2 = -\sqrt{3} - 2$ //

e) $r_{e_\perp}: \begin{cases} x = 2 + t \\ y = t \\ z = 1 - t \end{cases}$ $\pi_s: x + y - z + 2 = 0$

$(2+t) + t - (1-t) + 2 = 0$; $2 + t + t - 1 + t + 2 = 0$

$3t = -3$; $t = -1$ //

~~$I(1, -1, 2)$~~

$C'(x, y, z)$; $\dfrac{x+2}{2} = 1 \to x = 0$

$\dfrac{y+0}{2} = -1 \to y = -2$

$\dfrac{z+1}{2} = 2 \to z = 3$

$C'(0, -2, 3)$

1) a) $a^2+2b^2=1$

$2ab+b^2=0$

$b(2a+b)=0$

$b=0$; $b=-2a$

$\underline{b=0} \Rightarrow a^2=1,\ a=\pm 1$ → $a=1$; $b=0$; $a=-1$; $b=0$

$\underline{b=-2a}$ $a^2+2\cdot 4a^2=1$; $9a^2=1$; $a^2=\frac{1}{9}$; $a=\pm\frac{1}{3}$

$a=1/3$ $b=-2/3$; $a=-1/3$; $b=2/3$

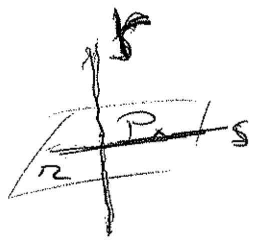
b) $\begin{vmatrix} a & 1 & 1 \\ 1 & a & 1 \\ 1 & 1 & a \end{vmatrix} = a^3+1+1-a-a-a = a^3-3a+2$

$$\begin{array}{c|cccc} & 1 & 0 & -3 & 2 \\ 1 & & 1 & 1 & -2 \\ \hline & 1 & 1 & -2 & 0 \end{array}$$

$a^2+a-2=0$ → $a=-2$; $a=1$

$\boxed{a \neq 1, -2}$

2) a) $\overline{u_r} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 3 & -1 & 1 \\ 1 & -1 & 0 \end{vmatrix} = (1,1,-2)$



$x+y-2z+d=0$; $2-1+d=0$; $d=-1$

π: $x+y-2z-1=0$; $P_r(0,-1,-1)$; $P_s(2,-1,0)$

r: $\begin{cases} x=t \\ y=-1+t \\ z=-1-2t \end{cases}$

$t-1+t+2+4t-1=0$

$6t=0$; $t=0$

$\left.\begin{array}{l} t_1=2+2t_2 \\ -1+t_1=-1 \\ -1-2t_1=1-t_2 \end{array}\right\} \rightarrow \begin{array}{l} t_1=0 \\ t_2=2 \end{array}$

$Q(0,-1,-1)$

$\overline{u_s} = \overrightarrow{P_rP_s} = (2,0,1)$

s: $\begin{cases} x=2+2t \\ y=-1 \\ z=t \end{cases}$

$$\frac{x+2}{2}=0 \ ; \quad \frac{y-1}{2}=-1 \ ; \quad \frac{z+0}{2}=-1$$

$$\underline{\underline{P'(-2,-1,-2)}}$$

b) TEÓRICO

3) a) i) $d(P,\pi) = \dfrac{|2-3\cdot 1+(-1)+13|}{\sqrt{1^2+(-3)^2+1^2}} = \dfrac{11}{\sqrt{11}} = \underline{\underline{\sqrt{11}\ u}}$

ii) $P_r(1,0,-2)$ $\quad \overline{u_r}(2,2,-1)$

$\overrightarrow{PP_r}(-1,-1,-1)$

$$\overline{u_r} \wedge \overline{PP_r} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 2 & 2 & -1 \\ -1 & -1 & -1 \end{vmatrix} = (-3,3,0)$$

$$d(P,r) = \frac{\sqrt{(-3)^2+3^2+0^2}}{\sqrt{2^2+2^2+(-1)^2}} = \frac{\sqrt{18}}{\sqrt{9}} = \underline{\underline{\sqrt{2}\ u}}$$

b) $r: \begin{cases} x = 1+2t \\ y = 2t \\ z = -2-t \end{cases}$

$(1+2t)-3(2t)+(-2-t)+13=0$

$1+2t-6t-2-t+13=0$

$-5t=-12 \ ; \quad t=\frac{12}{5}$

$$P_{r\pi} = \left(\frac{29}{5}, \frac{24}{5}, -\frac{22}{5}\right)$$

$$A = \frac{\left| \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 2 & 1 & -1 \\ 29/5 & 24/5 & -22/5 \end{vmatrix} \right|}{2} = \frac{\left|\left(\frac{2}{5}, 3, \frac{19}{5}\right)\right|}{2} =$$

$$= \frac{\sqrt{\frac{4}{25}+9+\frac{361}{25}}}{2} = \frac{\sqrt{\frac{590}{25}}}{2} = \underline{\underline{\frac{\sqrt{590}}{10}\ u}}$$

c) $P_r(1,0,-2)$; $\overrightarrow{PP_r}(-1,-1,-1)$; $\overrightarrow{u_r}(2,2,-1)$

$P(2,1,-1)$ $\quad \vec{n} = \overrightarrow{u_r} \wedge \overrightarrow{PP_r} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 2 & 2 & -1 \\ -1 & -1 & -1 \end{vmatrix} = (-3, 3, 0)$

$-3x + 3y + d = 0$; $-6 + 3 + d = 0$, $d = 3$

$-3x + 3y + 3 = 0$; $\boxed{\pi: x - y - 1 = 0}$

4) a) $\overrightarrow{u_r}(3,-2,1)$; $\overrightarrow{u_s}(2,-1,2)$

$P_r(2,1,0)$; $P_s(-1,-2,1)$; $\overrightarrow{P_rP_s}(-3,-3,1)$

$\frac{2}{3} \neq \frac{-1}{-2} \neq \frac{2}{1}$ — no son PARALELAS

$$\begin{vmatrix} 3 & -2 & 1 \\ 2 & -1 & 2 \\ -3 & -3 & 1 \end{vmatrix} = -3 - 6 + 12 - 3 + 18 + 4 = 22 \neq 0 \rightarrow \underline{\text{SE CRUZAN}}$$

b)

$$\overrightarrow{u_r} \wedge \overrightarrow{u_s} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 3 & -2 & 1 \\ 2 & -1 & 2 \end{vmatrix} = (-3, -4, +1)$$

$$|\overrightarrow{u_r} \wedge \overrightarrow{u_s}| = \sqrt{(-3)^2 + (-4)^2 + 1^2} = \sqrt{9+16+1} = \sqrt{26}$$

$$d(r,s) = \frac{22}{\sqrt{26}} = \frac{22\sqrt{26}}{26} = \underline{\frac{11\sqrt{26}}{13}\ u}$$

$$\begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ -3 & -4 & 1 \\ 3 & -2 & 1 \end{vmatrix} = (-2, 6, 18) \qquad \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ -3 & -4 & 1 \\ 2 & -1 & 2 \end{vmatrix} = (-7, 8, 11)$$

$-2x + 6y + 18z + d = 0$

$-4 + 6 + d = 0, \quad \underline{d = -2}$

$-2x + 6y + 18z - 2 = 0$

$\underline{x - 3y - 9z + 1 = 0}$

$-7x + 8y + 11z + d = 0$

$7 - 16 + 11 + d = 0, \quad d = -$

$-7x + 8y + 11z - 2 = 0$

$7x - 8y - 11z + 2 = 0$

$$t: \begin{cases} x - 3y - 9z + 1 = 0 \\ 7x - 8y - 11z + 2 = 0 \end{cases}$$

$$\begin{cases} x - 3y = -1 \\ 7x - 8y = -2 \end{cases} \qquad \begin{aligned} -7x + 21y &= 7 \\ 7x - 8y &= -2 \\ y &= \frac{5}{13} \end{aligned}$$

$x = -1 + \frac{15}{13} = \frac{2}{13}$

$$t: \begin{cases} x = \frac{2}{13} + 3t \\ y = \frac{5}{13} + 4t \\ z = -t \end{cases}$$

1) (1)

a) $$\lim_{x\to -1}\left(\frac{(x^2+1)+(x+1)}{(x^2+1)}\right)^{\frac{kx^2}{x+1}} =$$

$$= \lim_{x\to -1}\left(1+\frac{(x+1)}{(x^2+1)}\right)^{\frac{kx^2}{x+1}} = \lim_{x\to -1}\left(1+\frac{1}{\frac{x^2+1}{x+1}}\right)^{\frac{kx^2}{x+1}} =$$

$$= \lim_{x\to -1}\left(1+\frac{1}{\frac{x^2+1}{x+1}}\right)^{\frac{kx^2}{x+1}\cdot\frac{x^2+1}{x+1}\cdot\frac{x+1}{x^2+1}} =$$

$$= e^{\lim_{x\to -1}\frac{kx^2}{\cancel{(x+1)}}\cdot\frac{\cancel{(x+1)}}{x^2+1}} = e^{\lim_{x\to -1}\frac{kx^2}{x^2+1}} =$$

$$= e^{\frac{k}{2}} = \sqrt{e} = e^{1/2} \Rightarrow \frac{k}{2} = \frac{1}{2} \Rightarrow \boxed{k=1}$$

b) $$\lim_{x\to +\infty}\frac{\left(\sqrt{4x^4-3x^3-2}-\sqrt{4x^4-5x^3+1}\right)\left(\sqrt{4x^4-3x^3-2}+\sqrt{4x^4-5x^3+1}\right)}{(2x+1)\left(\sqrt{4x^4-3x^3-2}+\sqrt{4x^4-5x^3+1}\right)} =$$

$$= \lim_{x\to +\infty}\frac{(4x^4-3x^3-2)-(4x^4-5x^3+1)}{(2x+1)\left(\sqrt{4x^4-3x^3-2}+\sqrt{4x^4-5x^3+1}\right)} = \lim_{x\to +\infty}\frac{2x^3-3}{(2x+1)(2x^2+2x^2)} =$$

$$= \lim_{x\to +\infty}\frac{2x^3}{8x^3} = \frac{2}{8} = \boxed{\frac{1}{4}}$$

c) $$\lim_{x\to 2}\left(\frac{(x-2)(x^2-x-2)-(x-2)(x^2+x-6)}{(x^2+x-6)(x^2-x-2)}\right) =$$

$$= \lim_{x\to 2}\left(\frac{(x-2)(x-2)(x+1)-(x-2)(x-2)(x+3)}{(x-2)(x+3)(x-2)(x+1)}\right) =$$

$$= \lim_{x\to 2}\frac{\cancel{(x-2)^2}(x+1-x-3)}{\cancel{(x-2)^2}(x+3)(x+1)} = \lim_{x\to 2}\frac{-2}{(x+3)(x+1)} =$$

$$= \frac{-2}{5\cdot 3} = \frac{-2}{15}$$

2) a) $2-x=0 \Rightarrow x=2$

$+ \qquad \underset{2}{|} \qquad -$

i) $$f(x)=\begin{cases} \dfrac{x+a}{2} & \text{si } x<2 \\[2mm] \dfrac{x+a}{2x-2} & \text{si } x\geq 2 \end{cases}$$

ii) FUNCIONES: No hay problema.

CONEXIONES

$$\left.\begin{array}{l} \lim\limits_{x\to 2^-} f(x)=\dfrac{2+a}{2} \\[2mm] \lim\limits_{x\to 2^+} f(x)=\dfrac{2+a}{2} \\[2mm] f(2)=\dfrac{2+a}{2} \end{array}\right\}$$ CONTINUA

iii) $$f'(x)=\begin{cases} \dfrac{1}{2} & \text{si } x<2 \\[2mm] \dfrac{(2x-2)-2(x+a)}{(2x-2)^2}=\dfrac{-2-2a}{(2x-2)^2} & \text{si } x>2 \end{cases}$$

FUNCIONES: no hay problema

CONEXIONES

$$\lim_{x\to 2^-} f'(x)=\frac{1}{2}$$

$$\lim_{x\to 2^+} f'(x)=\frac{-2-2a}{4}$$

$$\frac{1}{2}=\frac{-2-2a}{4};\quad 4=-4-4a;\quad 8=-4a \Rightarrow \boxed{a=-2}$$

b) TEÓRICO

3) i) FUNCIONES : no hay problema

CONEXIONES

$$\lim_{x \to -1^-} f(x) = \frac{-a-1}{-2}$$

$$\lim_{x \to -1^+} f(x) = b+1$$

$$f(-1) = \frac{-a-1}{-2}$$

$$\frac{-a-1}{-2} = b+1$$

$$-a-1 = -2b-2$$

$$\boxed{a-2b=1}$$

$$f'(x) = \begin{cases} \dfrac{a(x-1)-(ax-1)}{(x-1)^2} = \dfrac{-a+1}{(x-1)^2} & \text{si } x < -1 \\[2ex] \dfrac{-b}{(x+2)^2} + \dfrac{1}{2\sqrt{x+2}} & \text{si } x > -1 \end{cases}$$

FUNCIONES : no hay problema

CONEXIONES

$$\lim_{x \to -1^-} f'(x) = \frac{-a+1}{4}$$

$$\lim_{x \to -1^+} f'(x) = -b + \frac{1}{2}$$

$$\frac{-a+1}{4} = -b + \frac{1}{2} \; ; \; -a+1 = -4b+2 \; ; \; \boxed{a-4b=-1}$$

$$\left.\begin{aligned} a-2b &= 1 \\ a-4b &= -1 \end{aligned}\right\} \quad 2b = 2 \Rightarrow b = 1 \; ; \; a = 3 \qquad \boxed{\begin{aligned} a &= 3 \\ b &= 1 \end{aligned}}$$

ii) $x=-1$ $\quad P_1(-1, 2)$ $\quad f'(-1) = -\frac{1}{2}$

$$\boxed{y-2 = -\frac{1}{2}(x+1)} \qquad y = -\frac{1}{2}x + \frac{3}{2}$$

$x=-2$ $\quad P_2\left(-2, \frac{7}{3}\right)$ $\quad f'(-2) = -\frac{2}{9}$

$$\boxed{y-\frac{7}{3} = \frac{-2}{9}(x+2)} \qquad y = \frac{-2}{9}x + \frac{17}{9}$$

$x=2$ $\quad P_3\left(2, \frac{9}{4}\right)$ $\quad f'(2) = \frac{-1}{16} + \frac{1}{4} = \frac{3}{16}$

$$\boxed{y-\frac{9}{4} = \frac{3}{16}(x-2)} \qquad y = \frac{3}{16}x + \frac{30}{16}$$

4) a) $\ln y = \ln (x+1)^{1/x^2} = \frac{1}{x^2} \cdot \ln(x+1)$

$$\frac{y'}{y} = \frac{-2}{x^3} \ln(x+1) + \frac{1}{x^2} \cdot \frac{1}{x+1} = \frac{-2 \ln(x+1)}{x^3} + \frac{1}{x^2(x+1)}$$

$$\boxed{y' = (x+1)^{1/x^2} \cdot \left(\frac{-2 \ln(x+1)}{x^3} + \frac{1}{x^2(x+1)} \right)}$$

$P_1(1, 2)$ $\quad y'(1) = 2\left(-2 \ln 2 + \frac{1}{2}\right) \simeq -1'77$

$$\boxed{y - 2 = -1'77\,(x-1)}$$

$y = -1'77x + 3'77$

b) $y = \ln \sqrt{\frac{2+x}{2-x}} = \ln \left(\frac{2+x}{2-x}\right)^{1/2} = \frac{1}{2} \ln \left(\frac{2+x}{2-x}\right) =$

$= \frac{1}{2} \left[\ln(2+x) - \ln(2-x) \right]$

$$y' = \frac{1}{2} \left[\frac{1}{2+x} - \frac{-1}{2-x} \right] = \frac{1}{2} \left[\frac{(2-x) + (2+x)}{(2+x)(2-x)} \right] =$$

$$= \frac{1}{2} \cdot \frac{4}{4-x^2} = \frac{2}{4-x^2}, \quad \boxed{y' = \frac{2}{4-x^2}}$$

$P_2(1, \underbrace{\ln \sqrt{3}}_{0'54})$ $\quad y'(1) = \frac{2}{3}$

$$\boxed{y - \ln\sqrt{3} = \frac{2}{3}(x-1)}$$

$y = \frac{2}{3}x - 0'12$

c) $y' = \frac{1}{1+x^2} - \frac{1}{1+\left(\frac{1}{x}\right)^2} \cdot \frac{-1}{x^2} = \frac{1}{1+x^2} - \frac{1}{\frac{x^2+1}{\cancel{x^2}}} \cdot \frac{-1}{\cancel{x^2}}$

$$= \frac{1}{1+x^2} + \frac{1}{x^2+1} = \frac{2}{x^2+1}; \quad \boxed{y' = \frac{2}{x^2+1}}$$

$P_3(1, 0)$ $\quad y'(1) = 1$

$$\boxed{y = x - 1}$$

1) a) $L = \lim_{x\to\frac{\pi}{2}} (1+2\cos x)^{\frac{1}{\cos x}}$

$\ln L = \lim_{x\to\frac{\pi}{2}} \ln(1+2\cos x)^{\frac{1}{\cos x}} = \lim_{x\to\frac{\pi}{2}} \frac{1}{\cos x}\cdot \ln(1+2\cos x) =$

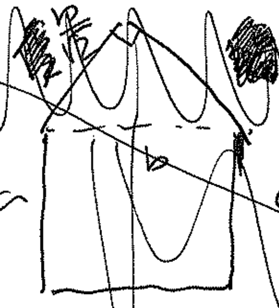
$\overset{\text{l'Hôpital}}{=} \lim_{x\to\frac{\pi}{2}} \frac{\frac{-2\sin x}{1+2\cos x}}{-\sin x} = \lim_{x\to\frac{\pi}{2}} \frac{2}{1+2\cos x} = 2$

$L = e^2$ //

b) $\lim_{x\to 0}\frac{e^x+e^{-x}-x^2-2}{\sin^2 x - x^2} \overset{\text{l'Hôp}}{=} \lim_{x\to 0} \frac{e^x-e^{-x}-2x}{2\sin x\cos x - 2x} =$

$\overset{\text{l'Hôp}}{=} \lim_{x\to 0} \frac{e^x+e^{-x}-2}{2(\cos^2x-\sin^2x)-2} \overset{\text{l'Hôp}}{=} \lim_{x\to 0} \frac{e^x-e^{-x}}{\underbrace{2(-2\cos x\sin x - 2\sin x\cos x)}_{-8\sin x\cos x}} =$

$\overset{\text{l'Hôp}}{=} \lim_{x\to 0} \frac{e^x+e^{-x}}{-8(\cos^2x-\sin^2x)} = \frac{2}{-8} = -\frac{1}{4}$ //

2) 

$A = ab + \frac{b}{4} \to$ máximo

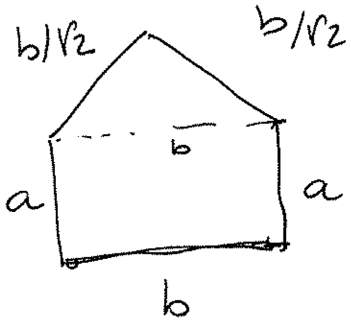
$2a + 2\sqrt{b/2} + b = 6$

$a = \frac{6-b-2\sqrt{b/2}}{2}$

$A = \left(\frac{6-b-2\sqrt{b/2}}{2}\right)b + \frac{b}{4} = \frac{6b-b^2-2b\sqrt{b/2}}{2} + \frac{b}{4} =$

$= \frac{12b-2b^2-4b\sqrt{b/2}+b}{4} = \frac{-2b^2+13b-\frac{4}{\sqrt2}b^{3/2}}{4}$

$A' = \frac{-4b+13-\frac{6}{\sqrt2}b^{1/2}}{4} = 0 \qquad -4b+13 = \frac{6}{\sqrt2}b^{1/2}$



$A = ab + \frac{b^2}{4}$ — Maksimi

$2a + \frac{2}{\sqrt{2}} b + b = 6$

$2a + (\sqrt{2}+1) b = 6$, $a = \frac{6-(\sqrt{2}+1)b}{2}$

$$A = \frac{(6-(\sqrt{2}+1)b)b}{2} + \frac{b^2}{4} = \frac{6b-(\sqrt{2}+1)b^2}{2} + \frac{b^2}{4} =$$

$$= \frac{12b - 2(\sqrt{2}+1)b^2 + b^2}{4} = \frac{12b - 2\sqrt{2}b^2 - b^2}{4}$$ — maks

$$A' = \frac{12 - 4\sqrt{2}b - 2b}{4} = 0 \qquad 12 - 4\sqrt{2}b - 2b = 0$$

$12 = (4\sqrt{2}+2)b$; $\quad b = \frac{12}{4\sqrt{2}+2} = \frac{6}{2\sqrt{2}+1}$

$$a = \frac{6 - \frac{6(\sqrt{2}+1)}{2\sqrt{2}+1}}{2} = \frac{\frac{12\sqrt{2}+\cancel{6}-6\sqrt{2}-\cancel{6}}{2\sqrt{2}+1}}{2} = \frac{6\sqrt{2}}{2(2\sqrt{2}+1)} = \frac{3\sqrt{2}}{2\sqrt{2}+1}$$

$\sqrt{2}b + b = 6$

$b = \frac{6}{\sqrt{2}+1}$

2) $\underline{D} = \mathbb{R}$

$\underline{C.E}$
- $OX: y = 0 \rightarrow \ln(x^2+1) = 0; \quad x^2+1 = 1 \Rightarrow x = 0$
- $OY: x = 0 \rightarrow \ln(0^2+1) = \ln 1 = 0 \quad \longrightarrow (0,0)$

$\underline{AH}$ $\quad \lim_{x\to+\infty} f(x) = +\infty \qquad \lim_{x\to-\infty} f(x) = +\infty$, NO TIENE

$\underline{A.V}$ $\quad x^2+1 = 0 \Rightarrow x^2 = -1 \rightarrow \nexists$ sol ; NO TIENE

$\underline{A.O}$; $\quad \lim_{x\to\infty} \dfrac{\ln(1+x^2)}{x} \overset{L'HôP}{=} \lim_{x\to\infty} \dfrac{\frac{2x}{1+x^2}}{1} = 0$, NO TIENE

$\underline{\text{Extrems relat}}$ $\qquad y' = \dfrac{2x}{x^2+1} = 0 \Rightarrow 2x = 0 \Rightarrow x = 0$

$E_1(0,0)$ — min relat

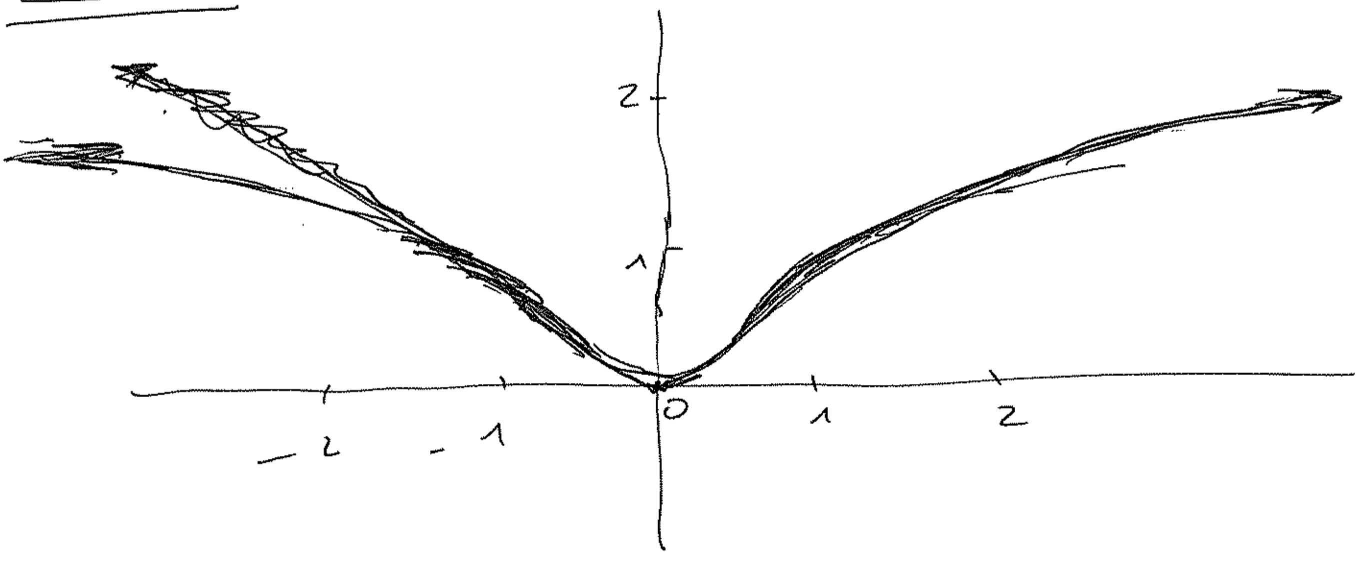
$$y'' = \frac{2(x^2+1) - 2x\cdot 2x}{(x^2+1)^2} = \frac{2-2x^2}{(x^2+1)^2} = 0$$

$2 - 2x^2 = 0 \Rightarrow 2 = 2x^2; \; x^2 = 1 \Rightarrow x = \pm 1$

$y''(0) = \dfrac{2}{1} > 0$

$I_1(1, \ln 2) \qquad I_2(-1, \ln 2)$

$\underline{\text{Simetria}}$ $\quad f(-x) = \ln(1+(-x)^2) = \ln(1+x^2) = f(x)$ PAR



3) a) $\sqrt{x} = t$; $\frac{1}{2\sqrt{x}} dx = dt$; $\frac{dx}{\sqrt{x}} = 2dt$

$$\int 2 \, \text{arctg}\, t \, dt = 2 \int \text{arctg}\, t \, dt =$$

$\text{arctg}\, t = u$ $\qquad$ $v = t$

$\frac{dt}{1+t^2} = du$ $\qquad$ $dv = dt$

$$= 2 \left[t \,\text{arctg}\, t - \int \frac{t\,dt}{1+t^2} \right] =$$

$1+t^2 = z$

$2t\,dt = dz$

$t\,dt = \frac{dz}{2}$

$$\int \frac{t\,dt}{1+t^2} = \int \frac{dz/2}{z} = \frac{1}{2} \ln z = \frac{1}{2} \ln(1+t^2)$$

$$= 2 \left[t \,\text{arctg}\, t - \frac{1}{2} \ln(1+t^2) \right] = 2t \,\text{arctg}\, t - \ln(1+t^2) =$$

$$= \boxed{2\sqrt{x}\,\text{arctg}\sqrt{x} - \ln(1+x) + cte}$$

b) $e^x = t$ $\qquad$ $\displaystyle\int \frac{t+1}{t^2+t-2}\, dt$

$e^x dx = dt$

$t^2+t-2 = 0 \quad \begin{cases} -2 \\ 1 \end{cases}$

$$\frac{t+1}{t^2+t-2} = \frac{A}{(t+2)} + \frac{B}{(t-1)} = \frac{A(t-1)+B(t+2)}{(t+2)(t-1)}$$

$t = 1 \Rightarrow 2 = 3B \Rightarrow B = 2/3$

$t = -2 \Rightarrow -1 = -3A \Rightarrow A = \frac{1}{3}$

$$\int \frac{1/3}{t+2}\,dt + \int \frac{2/3}{t-1}\,dt = \frac{1}{3}\ln(t+2) + \frac{2}{3}\ln(t-1) =$$

$$= \boxed{\frac{1}{3}\ln(e^x+2) + \frac{2}{3}\ln(e^x-1) + cte}$$

4) a) $\ln x = t$; $\frac{dx}{x} = dt$

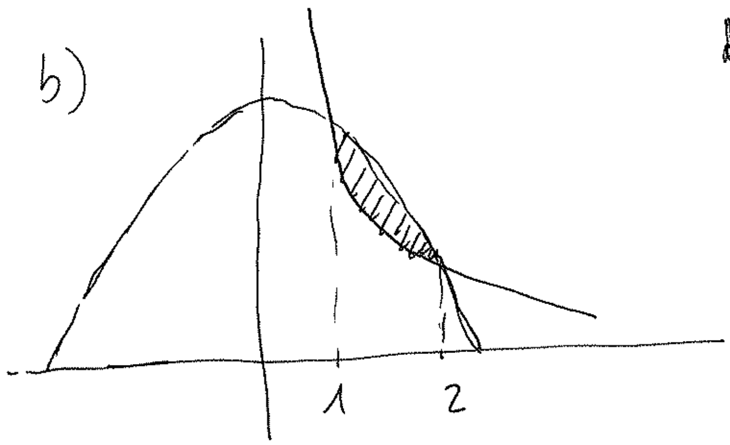
$$\int \frac{\ln t}{t}\,dt = \qquad \ln t = z \, ; \quad \frac{dt}{t} = dz$$

$$= \int z\,dz = \frac{z^2}{2} = \frac{(\ln t)^2}{2} = \left[\frac{(\ln(\ln x))^2}{2}\right]_e^{e^e} =$$

$$= \left[\frac{(\ln(\ln e^e))^2}{2} - \frac{(\ln(\ln e))^2}{2}\right] =$$

$$= \frac{(\ln e)^2}{2} - \frac{(\ln 1)^2}{2} = \frac{1^2}{2} - \frac{0^2}{2} = \frac{1}{2}$$

b)



$$\cancel{H}(x) = \frac{4}{x^2} + x^2 - 5 = \frac{4 + x^4 - 5x^2}{x^2}$$

$x^4 - 5x^2 + 4 = 0 \qquad x^2 = t$

$t^2 - 5t + 4 = 0. \quad t = \begin{cases} 4 \\ 1 \end{cases} \qquad x = \pm\sqrt{4} \begin{cases} +2 \\ -2 \end{cases} \quad x = \pm\sqrt{1} \begin{cases} +1 \\ -1 \end{cases}$

2

1

$$A = -\int_1^2 \left(\frac{4}{x^2} + x^2 - 5\right) = -\left[-\frac{4}{x} + \frac{x^3}{3} - 5x\right]_1^2 =$$

$$= -\left[\left(-2 + \frac{8}{3} - 10\right) - \left(-4 + \frac{1}{3} - 5\right)\right] = -\left[\left(\frac{-28}{3}\right) - \left(\frac{-26}{3}\right)\right] =$$

$$= -\left[-\frac{2}{3}\right] = \boxed{\frac{2}{3}\,u^2}$$

1

1)

a)

$$f'(x) = \begin{cases} \dfrac{-m}{(x-2)^2} \cdot e^{\frac{m}{x-2} - n} & \text{si } x < 1 \\ -\ln x - 1 & \text{si } x \geq 1 \end{cases}$$

i)

$$\lim_{x \to 1^-} f(x) = e^{-m-n} \; ; \quad \lim_{x \to 1^+} f(x) = 1 \; ; \quad f(1) = 1$$

$$e^{-m-n} = 1 \Rightarrow -m-n = 0 \Rightarrow m+n=0$$

$$\lim_{x \to 1^-} f'(x) = -m \cdot e^{-m-n} \; ; \quad \lim_{x \to 1^+} f'(x) = -1$$

$$-m \cdot \underbrace{e^{-m-n}}_{1} = -1 \; ; \Rightarrow \underline{\underline{m = 1}} \; ; \quad \underline{\underline{n = -1}}$$

ii) $P(1, 1)$ $\quad m = -1$; $\quad y - 1 = -1(x-1)$

$$\boxed{y = -x + 2}$$

b)

$$f'(x) = \frac{1/2}{\sqrt{1 - \frac{x^2}{4}}} - \frac{\cancel{2}x}{\cancel{2}\sqrt{4-x^2}} = \frac{1/\cancel{2}}{\frac{\sqrt{4-x^2}}{\cancel{2}}} - \frac{x}{\sqrt{4-x^2}} =$$

$$= \frac{1-x}{\sqrt{4-x^2}} \; //$$

2) a)

i) $\lim_{x\to 1}\left(\frac{(3x^2-2)+(x^3-3x^2-2x+4)}{(3x^2-2)}\right)^{\frac{x^2-3x}{x^2+x-2}} =$

$= \lim_{x\to 1}\left(1+\frac{1}{\frac{3x^2-2}{x^3-3x^2-2x+4}}\right)^{\frac{x^2-3x}{x^2+x-2}\cdot\frac{3x^2-2}{x^3-3x^2-2x+4}\cdot\frac{x^3-3x^2-2x+4}{3x^2-2}}$

$= e^{\lim_{x\to 1}\frac{(x^2-3x)(x^3-3x^2-2x+4)}{(x^2+x-2)(3x^2-2)}} =$

$= e^{\lim_{x\to 1}\frac{(x^2-3x)\cancel{(x-1)}(x^2-2x-4)}{\cancel{(x-1)}(x+2)(3x^2-2)}} = e^{\frac{(-2)(-5)}{3\cdot 1}} = e^{\frac{10}{3}}$ //

	1	-3	-2	4
1		1	-2	-4
	1	-2	-4	0

$x^2-2x-4=0$

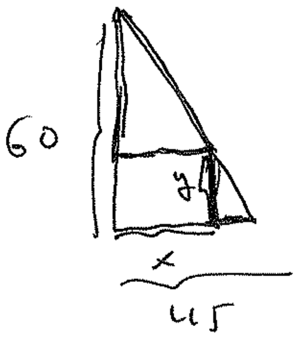
$x = 2 \pm \sqrt{4+16}$

ii) $\lim_{x\to 1}\left(\frac{x\ln x - x + 1}{(x-1)\ln x}\right) \overset{\text{L'HOSPITAL}}{=}$

$= \lim_{x\to 1}\left(\frac{\ln x + \cancel{1} - \cancel{1}}{\ln x + \frac{x-1}{x}}\right) = \lim_{x\to 1}\left(\frac{x\ln x}{x\ln x + x - 1}\right) =$

$= \lim_{x\to 1}\left(\frac{\ln x + 1}{\ln x + 1 + x}\right) = \frac{1}{2}$ //

b)



$$\frac{60 \times 45}{2} = 1350$$

(2)

$$C_1 = 2500xy + 500(1350 - xy) =$$

$$= 2000xy + 675.000$$

$$\frac{60}{y} = \frac{45}{45-x} \; ; \qquad 2700 - 60x = 45y$$

$$y = \frac{2700 - 60x}{45} \; ;$$

$$C = 2000x\left(\frac{2700-60x}{45}\right) + 675.000 =$$

$$= \frac{5400000x - 120000x^2}{45} + 675.000$$

$$C' = \frac{5400000 - 240000x}{45} = 0 \qquad x = \frac{5400000}{240000} = 22'5$$

$$x = 22'5 \text{ m} \; ; \qquad y = \frac{2700 - 1350}{45} = \frac{1350}{45} = 30 \text{ m}$$

3) a) $$f'(x) = -e^{-x}(x^2+6x+9) + e^{-x}(2x+6) =$$

$$= e^{-x}(-x^2-4x-3)$$

$$f''(x) = -e^{-x}(-x^2-4x-3) + e^{-x}(-2x-4) =$$

$$= e^{-x}(x^2+2x-1)$$

$e^{-x}(-x^2-4x-3)=0 \Rightarrow -x^2-4x-3=0 \quad x^2+4x+3=0$

$$x=\frac{-4\pm\sqrt{16-12}}{2}=\frac{-4\pm 2}{2}=\begin{cases}-1\\-3\end{cases}$$

$f''(-1)=e^{-1}((-1)^2+2(-1)-1)=e^{-1}(-2)<0$ – Máx

$f''(-3)=e^{-3}((-3)^2+2(-3)-1)=e^{-3}\cdot 2>0 \rightarrow$ Mínimo

$f'<0$	-3	$f'>0$	-1	$f'<0$
DeCrec	$\downarrow$ Mín relat	Crec	$\downarrow$ Máx relat	DeCrec

$f''(x)=e^{-x}(x^2+2x-1)=0 \Rightarrow x^2+2x-1=0$

$$x=\frac{-2\pm\sqrt{4+4}}{2}=\frac{-2\pm2\sqrt{2}}{2}=\begin{cases}-1+\sqrt{2}\rightarrow 0'41\\-1-\sqrt{2}\rightarrow -2'41\end{cases}$$

b) $I=\int e^{-x}(x^2+6x+9)\,dx$

$u=x^2+6x+9 \quad v=-e^{-x}$

$du=(2x+6)\,dx \quad dv=e^{-x}$

$$I=-e^{-x}(x^2+6x+9)+\underbrace{\int e^{-x}(2x+6)\,dx}_{I_1}$$

$I_1=-(2x+6)e^{-x}+\int e^{-x}\cdot 2\,dx=$

$u=2x+6 \quad v=-e^{-x}$

$du=2dx \quad dv=e^{-x}$

$=-(2x+6)e^{-x}-2e^{-x}=$

$=e^{-x}(-2x-8)$

$$I = e^{-x}(-x^2-6x-9-2x-8) =$$

$$= \underline{\underline{e^{-x}(-x^2-8x-17) + cte}}$$

4) a) $J = \int (x^2+1)\ln(x+1)\,dx$

$u = \ln(x+1) \qquad v = \frac{x^3}{3}+x$

$du = \frac{1}{x+1}dx \qquad dv = x^2+1$

$$I = \left(\frac{x^3}{3}+x\right)\ln(x+1) - \underbrace{\int \frac{\frac{x^3}{3}+x}{x+1}\,dx}_{J_1}$$

$$J_1 = \int \frac{\frac{x^3}{3}+x}{x+1}\,dx = \int \frac{x^3+3x}{3(x+1)}\,dx =$$

$$= \frac{1}{3}\underbrace{\int \frac{x^3+3x}{x+1}\,dx}$$

$$\begin{array}{l|l} x^3+0x^2+3x+0 & x+1 \\ \hline -x^3-x^2 & x^2-x+4 \\ \hline -x^2+3x+0 & \\ +x^2+x & \\ \hline / \quad 4x+0 & \\ -4x-4 & \\ \hline / \quad -4 & \end{array}$$

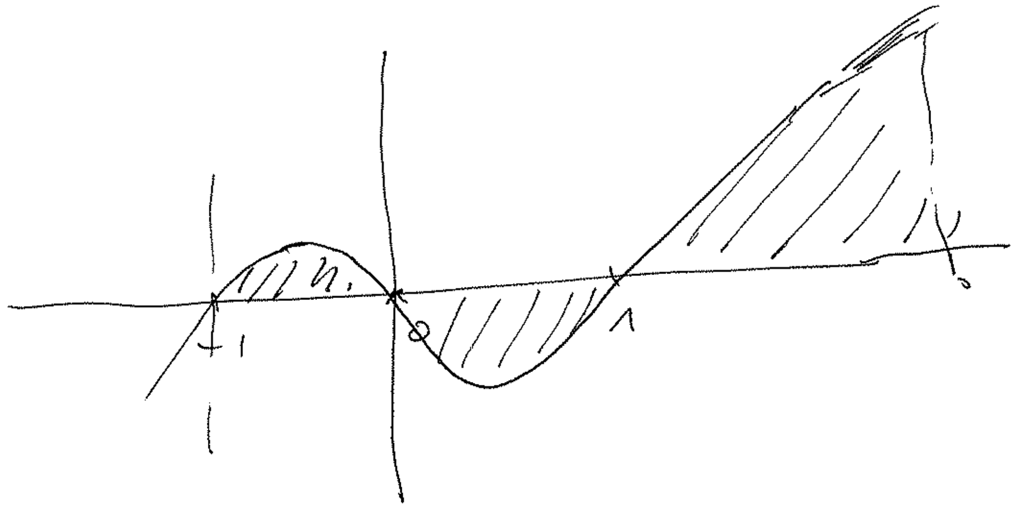
$$\int \frac{(x^2-x+4)(x+1)-4}{(x+1)}\,dx =$$

$$= \int (x^2-x+4)\,dx - \int \frac{4}{x+1}\,dx =$$

$$= \frac{1}{3}\left(\frac{x^3}{3} - \frac{x^2}{2} + 4x - 4\ln(x+1)\right)$$

$$\underline{\underline{I = \left(\frac{x^3}{3}+x\right)\ln(x+1) - \frac{1}{3}\left(\frac{x^3}{3} + \frac{x^2}{2} - 4x + 4\ln(x+1)\right) + cte}}$$

4)



$$x\sqrt[3]{x^2-1}=0 \Rightarrow \begin{cases} x=0 \\ x^2-1=0 \; \begin{cases} x=-1 \\ x=1 \end{cases} \end{cases}$$

$$\begin{array}{ccccccc} & + & & - & & + & \\ -1 & & 0 & & 1 & & 3 \end{array}$$

$$\frac{1}{2}\int x\sqrt[3]{x^2-1}\,dx = \int \sqrt[3]{t}\cdot\frac{dt}{2} = \qquad x^2-1=t,\quad 2x\,dx=dt,\quad x\,dx=\frac{dt}{2}$$

$2x\,(x^2-1)^{1/3}$

$\frac{1}{2}\,\frac{(x^2-1)^{4/3}}{4/3}$

$$=\frac{1}{2}\int t^{1/3}\,dt = \frac{1}{2}\,\frac{t^{4/3}}{4/3} = \frac{3t^{4/3}}{8} =$$

$$=\frac{3\sqrt[3]{(x^2-1)^4}}{8}$$

$$A = \left[\frac{3\sqrt[3]{(x^2-1)^4}}{8}\right]_{-1}^{0} - \left[\frac{3\sqrt[3]{(x^2-1)^4}}{8}\right]_{0}^{1} +$$

$$+\left[\frac{3\sqrt[3]{(x^2-1)^4}}{8}\right]_{1}^{3} = \left[\frac{3}{8}-0\right] - \left[0-\frac{3}{8}\right] +$$

$$+\left[6-0\right] = \frac{3}{8}+\frac{3}{8}+6 = \frac{54}{8} = \frac{27}{4}\,u^2$$

(1)

1) $1^{o} \to x$ $\quad x+y+z=180$
$2^{o} \to y$ $\quad 4x=3y$
$3^{o} \to z$ $\quad \frac{30(x+y)}{100}=z+2$

$$\left.\begin{array}{l} x+y+z=180 \\ 4x-3y=0 \\ 30x+30y-100z=200 \end{array}\right\} \quad \left(\begin{array}{ccc|c} 1 & 1 & 1 & 180 \\ 4 & -3 & 0 & 0 \\ 30 & 30 & -100 & 200 \end{array}\right)$$

$$\left(\begin{array}{ccc|c} 1 & 1 & 1 & 180 \\ 0 & 7 & 4 & 720 \\ 0 & 0 & 130 & 5200 \end{array}\right) \to z=\frac{5200}{130}=40$$

$$7y+160=720, \quad y=\frac{720-160}{7}=\frac{560}{7}=80$$

$$x=180-40-80=60 \qquad (60,80,40)$$

2) a) $A\cdot B=\begin{pmatrix} 1+\lambda & 3\lambda \\ 1 & 2\lambda \end{pmatrix}$

$$|AB|=2\lambda(1+\lambda)-3\lambda=2\lambda^2+2\lambda-3\lambda=$$
$$=2\lambda^2-\lambda=0, \quad \lambda(2\lambda-1)=0 \begin{cases} \lambda=0 \\ \lambda=1/2 \end{cases}$$

b) $A\cdot B=\begin{pmatrix} 3 & 6 \\ 1 & 4 \end{pmatrix}$

$$(A\cdot B)^{-1}=\begin{pmatrix} \frac{4}{6} & -\frac{6}{6} \\ -\frac{1}{6} & \frac{3}{6} \end{pmatrix}=\begin{pmatrix} \frac{2}{3} & -1 \\ -\frac{1}{6} & \frac{1}{2} \end{pmatrix}$$

c)

$$X = \begin{pmatrix}0 & 1\\ 1 & 0\\ 1 & -1\end{pmatrix} (AB)^{-1}$$

$$X = \begin{pmatrix}0 & 1\\ 1 & 0\\ 1 & -1\end{pmatrix} \begin{pmatrix}\frac{4}{6} & -\frac{6}{6}\\ -1/6 & 3/6\end{pmatrix} = \begin{pmatrix}-1/6 & 3/6\\ 4/6 & -6/6\\ 5/6 & -9/6\end{pmatrix}$$

3) $X = \begin{pmatrix}2 & a\\ b & c\end{pmatrix}$ $\quad \begin{pmatrix}2 & -2\\ 6 & 0\end{pmatrix}\begin{pmatrix}2 & a\\ b & c\end{pmatrix} + \begin{pmatrix}2 & a\\ b & c\end{pmatrix}\begin{pmatrix}-1 & -1\\ 11 & -1\end{pmatrix}$

$$\begin{pmatrix}4-2b & 2a-2c\\ 12 & 6a\end{pmatrix} + \begin{pmatrix}-2+11a & -2-a\\ -b+11c & -b-c\end{pmatrix} =$$

$$= \begin{pmatrix}2+11a-2b & a-2c-2\\ -b+11c+12 & 6a-b-c\end{pmatrix} = \begin{pmatrix}0 & 0\\ 0 & 0\end{pmatrix}$$

$$\left.\begin{aligned} 11a - 2b &= -2\\ a \quad - 2c &= 2\\ -b + 11c &= -12\\ 6a - b - c &= 0 \end{aligned}\right\} \begin{pmatrix}11 & -2 & 0 & | & -2\\ 1 & 0 & -2 & | & 2\\ 0 & -1 & 11 & | & -12\\ 6 & -1 & -1 & & 0\end{pmatrix} \sim$$

$$\begin{pmatrix}11 & -2 & 0 & | & -2\\ \cancel{0} & \cancel{-2} & \cancel{22} & | & \cancel{-24}\\ \cancel{0} & \cancel{-1} & \cancel{11} & | & \cancel{-12}\\ 0 & 1 & -11 & & 12\end{pmatrix}$$

$$\left.\begin{array}{r} 11a - 7b = -2 \\ b - 11c = 12 \end{array}\right\} \quad b = 12 + 11c$$

$$11a - 74 - 77c = -2; \quad a = \frac{22 + 77c}{11} = 2 + 7c$$

$$\underline{(2 + 7c,\ 12 + 11c,\ c)}$$

b) TEÓRICO

4) $$A = \left(\begin{array}{ccc|c} 1 & m-1 & -1 & 0 \\ m-1 & 3 & 1 & m \\ 0 & 1 & 1 & 1 \end{array}\right)$$

$$|A| = 3 - m + 1 - 1 - m^2 - 1 + 2m = -m^2 + m + 2 = \boxed{-(m-2)(m+1)}$$

$$m^2 - m - 2 = 0; \quad m = \frac{1 \pm \sqrt{1+8}}{2} = \frac{1 \pm 3}{2} = \begin{cases} 2 \\ -1 \end{cases}$$

$\underline{m = -1}$

$$\left(\begin{array}{ccc|c} 1 & -2 & -1 & 0 \\ -2 & 3 & 1 & -1 \\ 0 & 1 & 1 & 1 \end{array}\right) \sim \left(\begin{array}{ccc|c} 1 & -2 & -1 & 0 \\ 0 & -1 & -1 & -1 \\ 0 & 1 & 1 & 1 \end{array}\right)$$

S.C.I. $$\left.\begin{array}{r} x - 2y - z = 0 \\ y + z = 1 \end{array}\right\} \quad \begin{array}{l} x = 2 - 2z + z = 2 - z \\ y = 1 - z \end{array}$$

$$\boxed{(2 - \lambda,\ 1 - \lambda,\ \lambda) \quad \lambda \in \mathbb{R}}$$

$\underline{m = 2}$

$$\left(\begin{array}{ccc|c} 1 & 1 & -1 & 0 \\ 1 & 3 & 1 & 2 \\ 0 & 1 & 1 & 1 \end{array}\right) \sim \left(\begin{array}{ccc|c} 1 & 1 & -1 & 0 \\ 0 & 2 & 2 & 2 \\ 0 & 1 & 1 & 1 \end{array}\right)$$

$$\left.\begin{array}{r} x + y - z = 0 \\ y + z = 1 \end{array}\right\} \quad \begin{array}{l} x = -1 + z + z = -1 + 2z \\ y = 1 - z \end{array}$$

$$\boxed{(-1 + 2\lambda,\ 1 - \lambda,\ \lambda) \ \lambda \in \mathbb{R}}$$

$m \neq -1, 2$

$$\begin{vmatrix} 0 & m-1 & -1 \\ m & 3 & 1 \\ 1 & 1 & 1 \end{vmatrix} = -m + m - 1 + 3 - m^2 + m = -m^2 + m + 2 = -(m+1)(m-2)$$

$$x = \frac{-(m+1)(m-2)}{-(m+1)(m-2)} = 1$$

$$\begin{vmatrix} 1 & 0 & -1 \\ m-1 & m & 1 \\ 0 & 1 & 1 \end{vmatrix} = m - m + 1 - 1 = 0$$

$$y = 0$$

$$\begin{vmatrix} 1 & m-1 & 0 \\ m-1 & 3 & m \\ 0 & 1 & 1 \end{vmatrix} = 3 - m - m^2 - 1 + 2m = -m^2 + m + 2$$

$$z = 1$$

3

1) $\overline{u_r} = \overline{AB} = (-3, 0, -3) \simeq (1, 0, 1)$

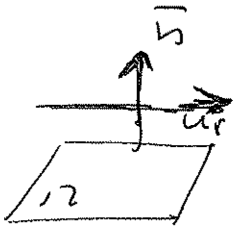
$$r: \begin{cases} x = 2 + t \\ y = 2 \\ z = 4 + t \end{cases}$$

$\overline{CD} = (0, -1, -1)$ $\quad \overline{CE} = (2, 0, -1)$

$$\overline{n} = \overline{CD} \wedge \overline{CE} = \begin{vmatrix} \overline{i} & \overline{j} & \overline{k} \\ 0 & -1 & -1 \\ 2 & 0 & -1 \end{vmatrix} = (1, -2, 2)$$

$x - 2y + 2z + D = 0$, $\quad D = -3$

$$\pi: \; x - 2y + 2z - 3 = 0$$



a) $\overline{u_r} = (1, 0, 1)$ $\quad \overline{n} = (1, -2, 2)$

$\overline{u_r} \cdot \overline{n} = 1 + 0 + 2 = 3 \neq 0$ no es paralela

b) $(2+t) - 2 \cdot 2 + 2(4+t) - 3 = 0$

$2 + t - 4 + 8 + 2t - 3 = 0$; $3t = -3$; $t = -1$

$$I = (1, 2, 3)$$

$$\operatorname{sen}(\widehat{r, \pi}) = \frac{|\overline{u_r} \cdot \overline{n}|}{|\overline{u_r}| \cdot |\overline{n}|} = \frac{|1 + 0 + 2|}{\sqrt{2}\,\sqrt{9}} = \frac{3}{3\sqrt{2}} =$$

$$= \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2} \longrightarrow (\widehat{r, \pi}) = 45^\circ$$

c) $$\frac{|(2+t) - 2 \cdot 2 + 2(4+t) - 3|}{\sqrt{1+4+4}} = \frac{|2 + t - 4 + 8 + 2t - 3|}{3} =$$

$$= \frac{|3t+3|}{3} = 4 \; ; \; |3t+3| = 12$$

$3t+3=12 \;-\; t_1 = 3$

$3t+3=-12 \;-\; t_2 = -5$

$\underline{\underline{S(5,2,7)}}$ $\qquad$ $\underline{\underline{T(-3,\ 2,\ -1)}}$

2) a) $\overline{u_r} = \begin{vmatrix} \bar{i} & \bar{j} & \vec{k} \\ 1 & -1 & 0 \\ 1 & 1 & -1 \end{vmatrix} = (1,\ 1,\ 2)$

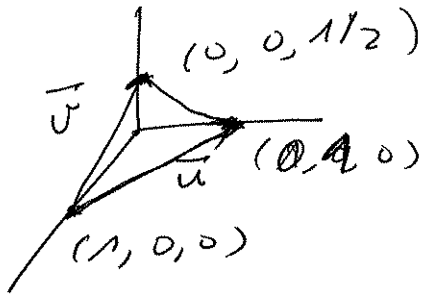
$\bar{n} = \overrightarrow{u_r}$ $\qquad$ $x+y+2z+D=0$

$1+2-2+D=0$; $D=-1$; $\boxed{x+y+2z-1=0}$

b) $x=y=0$; $z=1/2$

$x=z=0$; $y=1$

$y=z=0$; $x=1$



$\bar{u} = (-1,1,0)$; $\bar{v}(-1,0,1/2)$

$\bar{u} \wedge \bar{v} = \begin{vmatrix} \bar{i} & \bar{j} & \vec{k} \\ -1 & 1 & 0 \\ -1 & 0 & 1/2 \end{vmatrix} = (1/2,\ 1/2,\ 1)$

$A = \frac{\sqrt{\frac{1}{4}+\frac{1}{4}+1}}{2} = \frac{\sqrt{3/2}}{2} = \frac{\sqrt{3}}{2\sqrt{2}} = \underline{\underline{\frac{\sqrt{6}}{4}\, u^2}}$

4

3)

a) $$\overline{u_r} = \begin{vmatrix} \bar{i} & \bar{j} & \vec{k} \\ 4 & 2 & -1 \\ 1 & -1 & 2 \end{vmatrix} = (3, -9, -6) \simeq (1, -3, -2)$$

$$\overline{u_s} = \begin{vmatrix} \bar{i} & \bar{j} & \vec{k} \\ 1 & 1 & -1 \\ a & -2 & 0 \end{vmatrix} = (-2, -a, -2-a)$$

$$\frac{-2}{+1} = \frac{-a}{-3} = \frac{-2-a}{-2} \Rightarrow \boxed{a = -6}$$

$P_r(0, 6, 3) \qquad P_s(0, 1, 1)$

$$\overrightarrow{P_s P_r} = (0, 5, 2)$$

$$\overline{n} = \begin{vmatrix} \bar{i} & \bar{j} & \vec{k} \\ 1 & -3 & -2 \\ 0 & 5 & 2 \end{vmatrix} = (4, -2, 5)$$

$$4x - 2y + 5z + D = 0$$

$$-2 + 5 + D = 0 \; ; \; D = -3$$

$$\boxed{\pi : 4x - 2y + 5z - 3 = 0}$$

$$r: \begin{cases} x = t_1 \\ y = t_1 \\ z = t_1 \end{cases} \qquad s: \begin{cases} x = 1+t_2 \\ y = 3+t_2 \\ z = -t_2 \end{cases}$$

$$\overline{u_r} = (1, 1, 1) \qquad \overline{u_s} = (1, 1, -1)$$

$$\overline{P_rP_s} = (1+t_2-t_1,\ 3+t_2-t_1,\ -t_2-t_1)$$

$$\left.\begin{array}{l} 1+t_2-t_1+3+t_2-t_1-t_2-t_1=0 \\ 1+t_2-t_1+3+t_2-t_1+t_2+t_1=0 \end{array}\right\}$$

$$\left.\begin{array}{l} -3t_1+t_2=-4 \\ -t_1+3t_2=-4 \end{array}\right\} \quad \left.\begin{array}{l} 9t_1 - 3t_2 = 12 \\ -t_1+3t_2=-4 \end{array}\right\}$$

$$8t_1 = 8; \quad \underline{\underline{t_1 = 1 \Rightarrow t_2 = -1}}$$

$$I_r(1, 1, 1) \qquad I_s(0, 2, 1)$$

$$\overline{I_rI_s} = (-1, 1, 0)$$

$$t: \begin{cases} x = 1-t \\ y = 1+t \\ z = 1 \end{cases}$$

$$\left.\begin{array}{l} x+y-2z=0 \\ x+y+2z-4=0 \end{array}\right\}$$

1) a)

$$f(x)=\begin{cases}\dfrac{x+a}{x+1}+x & \text{si } 0\le x<2\\[2mm] \dfrac{x-b}{x-1}-x & \text{si } 2\le x\end{cases}$$

$$f'(x)=\begin{cases}\dfrac{1-a}{(x+1)^2}+1 & \text{si } 0<x<2\\[2mm] \dfrac{-1+b}{(x-1)^2}-1 & \text{si } 2<x\end{cases}$$

i)

$\lim\limits_{x\to 2^-} f(x)=\dfrac{2+a}{3}+2$; $\lim\limits_{x\to 2^+} f(x)=\dfrac{2-b}{1}-2$

$f(2)=\dfrac{2-b}{1}-2$; $\dfrac{2+a}{3}+2=\cancel{2}-b-\cancel{2}$

$2+a+6=-3b$; $\boxed{a+3b=-8}$

$\lim\limits_{x\to 2^-} f'(x)=\dfrac{1-a}{9}+1$; $\lim\limits_{x\to 2^+} f'(x)=\dfrac{-1+b}{1}-1$

$\dfrac{1-a}{9}+1=\dfrac{-1+b}{1}-1$; $1-a+9=9b-18$

$\boxed{a+9b=28}$

$$\left\{\begin{array}{l}a+3b=-8\\ a+9b=28\end{array}\right. \qquad \boxed{\begin{array}{l}a=-26\\ b=6\end{array}}$$

ii) si $a=b=-2 \Rightarrow \lim\limits_{x\to 2^-} f(x)=2$; $\lim\limits_{x\to 2^+} f(x)=2$. $f(2)=2$

CONTINUA en $\mathbb{R}$

$\lim\limits_{x\to 2^-} f'(x)=\dfrac{4}{3}$ $\quad \lim\limits_{x\to 2^+} f(x)=-4$

NO DERIVABLE en $x=2$

b) $f'(x) = \frac{1}{\sqrt{1+tg^2x}} \cdot \frac{1}{2\sqrt{1+tg^2x}} \cdot 2\,tg\,x\,(1+tg^2x) =$

$= \frac{\cancel{2}\,tg\,x\,\cancel{(1+tg^2x)}}{\cancel{2}\,\cancel{(1+tg^2x)}} = tg\,x$

2) a) $L = \lim_{x\to 0}(1+\operatorname{sen}x-\cos x)^x$; $\ln L = \ln \lim_{x\to 0}(1+\operatorname{sen}x-\cos x)^x =$

$= \lim_{x\to 0} \ln(1+\operatorname{sen}x-\cos x)^x = \lim_{x\to 0} x \cdot \ln(1+\operatorname{sen}x-\cos x) =$

$= \lim_{x\to 0} \frac{\ln(1+\operatorname{sen}x-\cos x)}{\frac{1}{x}} = \lim_{x\to 0} \frac{\frac{\cos x+\operatorname{sen}x}{1+\operatorname{sen}x-\cos x}}{-\frac{1}{x^2}} =$

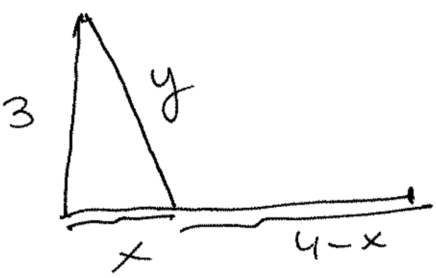
$= \lim_{x\to 0} -\frac{x^2(\cos x+\operatorname{sen}x)}{1+\operatorname{sen}x-\cos x} = \lim_{x\to 0} -\frac{2x(\cos x+\operatorname{sen}x)+x^2(-\operatorname{sen}x+\cos x)}{\cos x+\operatorname{sen}x} = 0$

$L = e^0 = 1$ //

b) $\lim_{x\to 0} \frac{e^{2x}+e^{-2x}-4x^2-2}{e^x-e^{-x}-2x} = \lim_{x\to 0} \frac{2e^{2x}-2e^{-2x}-8x}{e^x+e^{-x}-2} =$

$= \lim_{x\to 0} \frac{4e^{2x}+4e^{-2x}-8}{e^x-e^{-x}} = \lim_{x\to 0} \frac{8e^{2x}-8e^{2x}}{e^x+e^{-x}} = 0$ //

c)



$C_1 = 3000y + 1000(4-x)$

$y^2 = x^2+9$, $y = \sqrt{x^2+9}$

$C_1 = 3000\sqrt{x^2+9} + 4000 - 1000x$

$C_1' = 3000 \cdot \frac{x}{\sqrt{x^2+9}} - 1000 = \frac{3000x - 1000\sqrt{x^2+9}}{\sqrt{x^2+9}} = 0$

$3000x = 1000\sqrt{x^2+9}$; $3x = \sqrt{x^2+9}$; $9x^2 = x^2+9 \cdot 8x^2 = 9$

$x = \sqrt{\frac{9}{8}} = \frac{3}{2\sqrt{2}} = \frac{3\sqrt{2}}{4}$ //

$T \to \frac{16-3\sqrt{2}}{4}$ //

$A \to \frac{\sqrt{162}}{4} = \frac{9\sqrt{2}}{4}$ //

3) i) $D = \mathbb{R}$

ii) Corte ejes $\begin{cases} OX: y=0 \to (0,0) \\ OY: x=0 \to (0,0) \end{cases}$

iii) Asíntotas

H: $\lim_{x\to+\infty} x \cdot e^{-x^2/2} = 0$ $\quad \lim_{x\to-\infty} x e^{-x^2/2} = 0$

$\boxed{y=0}$ Dos lados

V: no tiene

O: $\lim_{x\to\infty} \frac{x e^{-x^2/2}}{x} = \lim_{x\to\infty} e^{-x^2/2} = 0$ no tiene

iv) Extremos relativos

$f'(x) = 1 \cdot e^{-x^2/2} + x \cdot e^{-x^2/2} \cdot (-x) = e^{-x^2/2}(1-x^2)$

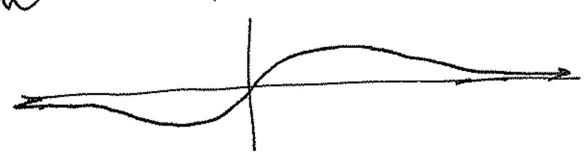
$e^{-x^2/2}(1-x^2) = 0$; $\quad x^2 - 1 = 0 \begin{cases} x=-1 \to (-1, \frac{-1}{\sqrt{e}}) \\ x=1 \to (1, \frac{+1}{\sqrt{e}}) \end{cases}$

$f''(x) = -e^{-x^2/2} \cdot x(1-x^2) + e^{-x^2/2} \cdot (-2x) =$

$= -e^{-x^2/2}(x - x^3 + 2x) = +e^{-x^2/2}(x^3 - 3x)$

$x^3 - 3x = 0 \begin{cases} x=0 \to (0,0) \\ x=-\sqrt{3} \to (-\sqrt{3}, \frac{-\sqrt{3}}{e^{3/2}}) \\ x=\sqrt{3} \to (\sqrt{3}, \frac{\sqrt{3}}{e^{3/2}}) \end{cases}$

$f'<0$		$f'>0$		$f'<0$
$-\sqrt{3}$	-1	0	1	$\sqrt{3}$
Inflex	Mínimo	Inflex	Máx	inflex



4) $\operatorname{sen} x = t$ $\qquad \displaystyle\int \frac{t^3+1}{t^3-2t^2-3t}$

$\cos x\,dx = dt$

$$\begin{array}{l|l} t^3 + 0t^2 + 0t + 1 & t^3 - 2t^2 - 3t \\ \hline -t^3 + 2t^2 + 3t & 1 \\ \hline \quad 2t^2 + 3t + 1 & \end{array}$$

$$t^3 + 1 = (t^3 - 2t^2 - 3t) + (2t^2 + 3t + 1)$$

$$\int \frac{t^3+1}{t^3-2t^2-3t}\,dt = \int 1\,\cancel{dt} + \int \frac{2t^2+3t+1}{t^3-2t^2-3t}\,dt$$

$$t^3 - 2t^2 - 3t = 0 \begin{cases} t = 0 \\ t = -1 \\ t = 3 \end{cases}$$

$$\frac{2t^2+3t+1}{t^3-2t^2-3t} = \frac{A}{t} + \frac{B}{t+1} + \frac{C}{t-3} =$$

$$= \frac{A(t+1)(t-3) + Bt(t-3) + Ct(t+1)}{t(t+1)(t-3)}$$

$t = 0 \Rightarrow -3A = 1 \Rightarrow A = -1/3$

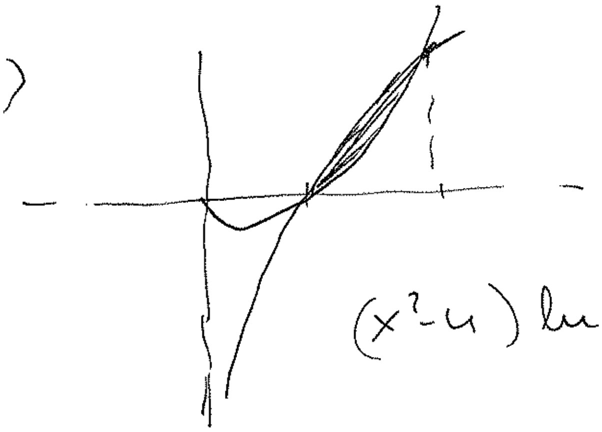
$t = -1 \Rightarrow -4B = 0 \Rightarrow B = 0$

$t = 3 \Rightarrow 12C = 28; \quad C = \frac{28}{12} = \frac{7}{3}$

$$I = t - 1/3 \ln t + \frac{7}{3} \ln(t-3) =$$

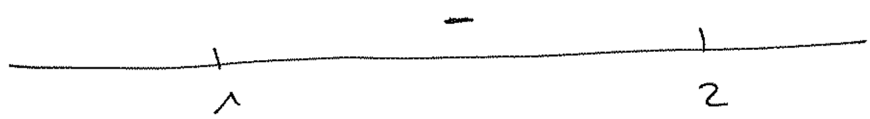
$$= \operatorname{sen} x - \frac{1}{3} \ln \operatorname{sen} x + \frac{7}{3} \ln(\operatorname{sen} x - 3) + cte$$

b)



$$h(x) = x^2 \ln x - 4 \ln x = (x^2-4)\ln x$$

$$(x^2-4)\ln x = 0 \;<\; \begin{array}{l} x^2-4=0 \to x=\cancel{-2},\ x=2 \\ \ln x = 0 \to x = 1 \end{array}$$



$$A = -\int_1^2 (x^2-4)\ln x \, dx$$

$$u = \ln x \qquad v = \frac{x^3}{3} - 4x$$

$$\int (x^2-4)\ln x \, dx =$$

$$du = \frac{dx}{x} \qquad dv = x^2-4$$

$$= \left(\frac{x^3}{3} - 4x\right)\ln x - \int \left(\frac{x^2}{3} - 4\right) dx =$$

$$= \left(\frac{x^3-12x}{3}\right)\ln x - \left(\frac{x^3}{9} - 4x\right) = \left(\frac{x^3-12x}{3}\right)\ln x - \left(\frac{x^3-36x}{9}\right)$$

$$A = -\left[\left(\frac{x^3-12x}{3}\right)\ln x - \left(\frac{x^3-36x}{9}\right)\right]_1^2 =$$

$$= -\left[\left(-\frac{16}{3}\ln 2 + \frac{64}{9}\right) - \left(0 + \frac{35}{9}\right)\right] = -\left[-\frac{16}{3}\ln 2 + \frac{29}{9}\right]$$

$$= \left(\frac{16}{3}\ln 2 - \frac{29}{9}\right) u^2$$

1)

a) $\underbrace{\begin{vmatrix} a & b & c \\ -a & -b & -c \\ c+a & b-a & c+b \end{vmatrix}}_{0} + \begin{vmatrix} a & b & c \\ c & -a & b \\ c+a & b-a & c+b \end{vmatrix} =$

$= \begin{vmatrix} a & b & c \\ c & -a & b \\ c & b & c \end{vmatrix} + \begin{vmatrix} a & b & c \\ c & -a & b \\ a & -a & b \end{vmatrix} =$

$= \begin{vmatrix} a & b & c \\ c & -a & b \\ c-a & 0 & 0 \end{vmatrix} + \begin{vmatrix} a & b & c \\ c & -a & b \\ a-c & 0 & 0 \end{vmatrix} =$

$= (c-a)(b^2+ac) + (a-c)(b^2+ac) = 0$ //

b) $\begin{vmatrix} a & 2 & 3 \\ 1 & a & 3 \\ 2 & 2+a & 6 \end{vmatrix} = 6a^2 + 6 + 3a + 12 - 6a - 6a - 3a^2 - 12 = 3a^2 - 9a + 6 =$

$= 3(a-1)(a-2)$

<u>Si a = 1</u>

$\left(\begin{array}{ccc|c} 1 & 2 & 3 & 1 \\ 1 & 1 & 3 & 2 \\ 2 & 3 & 6 & 3 \end{array}\right) \sim \left(\begin{array}{ccc|c} 1 & 2 & 3 & 1 \\ 0 & -1 & 0 & 1 \\ 0 & -1 & 0 & 1 \end{array}\right) \sim \left(\begin{array}{ccc|c} 1 & 2 & 3 & 1 \\ 0 & -1 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{array}\right)$

S.C.I

$\begin{cases} x + 2y + 3z = 1 \\ -y = 1 \end{cases}$ $\quad y = -1$; $\quad x + 3z = 3$; $\quad x = 3 - 3z$

$(3-3\lambda, -1, \lambda) \quad \lambda \in \mathbb{R}$

Si $a = 2$

$$\left(\begin{array}{ccc|c} 2 & 2 & 3 & 1 \\ 1 & 2 & 3 & 2 \\ 2 & 4 & 6 & 3 \end{array}\right) \sim \left(\begin{array}{ccc|c} 2 & 2 & 3 & 1 \\ 0 & -2 & -3 & -3 \\ 0 & 2 & 3 & 2 \end{array}\right) \sim$$

$$\sim \left(\begin{array}{ccc|c} 2 & 2 & 3 & 1 \\ 0 & -2 & -3 & -3 \\ 0 & 0 & 0 & -1 \end{array}\right) \longrightarrow \text{S.I}$$

Si $a \neq 1, 2 \longrightarrow$ S.C.D

$$6a+12+6a+18-9a-6-3a-24 =$$

$$x = \frac{\begin{vmatrix} 1 & 2 & 3 \\ 2 & a & 3 \\ 3 & 2+a & 6 \end{vmatrix}}{3(a-1)(a-2)} = \frac{0}{3(a-1)(a-2)} = 0$$

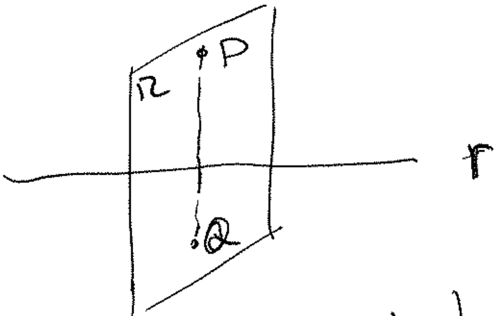
$$12a+9+6-12-9a-6 =$$

$$y = \frac{\begin{vmatrix} a & 1 & 3 \\ 1 & 2 & 3 \\ 2 & 3 & 6 \end{vmatrix}}{3(a-1)(a-2)} = \frac{3a-3}{3(a-1)(a-2)} = \frac{\cancel{3}\cancel{(a-1)}}{\cancel{3}\cancel{(a-1)}(a-2)} = \frac{1}{a-2}$$

$$6a^2+6+3a+12-6a-6a-3a^2-12 = 3a^2-9a+6 = 3(a^2-3a+2)$$

$$z = \frac{\begin{vmatrix} a & 2 & 3 \\ 1 & a & 3 \\ 2 & 2+a & 6 \end{vmatrix}}{3(a-1)(a-2)} = \frac{\cancel{3}\cancel{(a-1)}\cancel{(a-2)}}{\cancel{3}\cancel{(a-1)}\cancel{(a-2)}} = 1$$

a)



$$\vec{u_r} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 3 & 1 & -1 \\ 1 & -2 & 1 \end{vmatrix} = (-1, -4, -7) \to (1, 4, 7)$$

$x + 4y + 7z + D = 0$; $3 + 16 - 7 + D = 0$;

$12 + D = 0$; $D = -12$; $\pi: x + 4y + 7z - 12 = 0$

$P_r(0, -3, -6)$

$$r: \begin{cases} x = t \\ y = -3 + 4t \\ z = -6 + 7t \end{cases}$$

$t + 4(-3 + 4t) + 7(-6 + 7t) - 12 = 0$

$t - 12 + 16t - 42 + 49t - 12 = 0$; $66t - 66 = 0$

$t = 1$; $M(1, 1, 1)$

$\frac{x+3}{2} = 1$; $x = -1$ $\qquad$ $\frac{y+4}{2} = 1$; $y = -2$ $\qquad$ $\frac{z-1}{2} = 1$; $z = 3$

$\boxed{Q(-1, -2, 3)}$

b) $P_r(0, 2, 4-k)$ $\qquad$ $P_s(0, -2k-3, 5+2k)$

$$\vec{u_r} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & 1 & 0 \\ 1 & -2 & 1 \end{vmatrix} = (1, -1, -3)$$

$\overrightarrow{P_rP_s}(0, -2k-5, 1+3k)$

$$\vec{u_s} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 3 & -1 & 0 \\ 1 & 1 & 1 \end{vmatrix} = (-1, -3, 4)$$

$$\begin{vmatrix} 1 & -1 & -3 \\ -1 & -3 & 4 \\ 0 & -2k-5 & 1+3k \end{vmatrix} = -3 - 9k - 6k - 15 + 8k + 20 - 1 - 3k =$$

$$= -10k + 1 = 0 \rightarrow k = \frac{1}{10}$$

si $k \neq \frac{1}{10}$ se cruzan

si $k = \frac{1}{10}$ se cortan.

$$r \equiv \begin{cases} x = t_1 \\ y = 2 - t_1 \\ z = \frac{39}{10} - 3t_1 \end{cases} \qquad s \equiv \begin{cases} x = -t_2 \\ y = \frac{-32}{10} - 3t_2 \\ z = \frac{52}{10} + 4t_2 \end{cases}$$

$t_1 = -t_2$; $2 - t_1 = \frac{-32}{10} - 3t_2$. $2 - t_1 = \frac{-32}{10} + 3t_1$

$20 - 10t_1 = -32 + 30t_1$; $40t_1 = 52$; $t_1 = \frac{52}{40} = \frac{13}{10}$

$$I\left(\frac{13}{10}, \frac{7}{10}, 0\right)$$

3) a) $\lim_{x\to 0^-} f(x) = \lim_{x\to 0^-} \dfrac{\ln(1+x) - x}{x^2} =$

$= \lim_{x\to 0^-} \dfrac{\frac{1}{1+x} - 1}{2x} = \lim_{x\to 0^-} \dfrac{-x}{2x+2x^2} =$

$= \lim_{x\to 0^-} \dfrac{-1}{2+2x} = -\dfrac{1}{2}$

$\lim_{x\to 0^+} f(x) = -\dfrac{1}{2}$; $f(0) = -\dfrac{k}{2} \Rightarrow \boxed{k=1}$

b) $f'(x) = 2x\, e^{1/x} + x^2 \cdot \dfrac{-1}{x^2} e^{1/x} =$

$= e^{1/x}(2x-1) = 0 \Rightarrow x = \dfrac{1}{2}$

$f''(x) = -e^{1/x} \cdot \dfrac{2x-1}{x^2} + 2e^{1/x} =$

$= e^{1/x}\left(2 + \dfrac{1-2x}{x^2}\right) = e^{1/x}\left(\dfrac{2x^2-2x+1}{x^2}\right)$

$2x^2 - 2x + 1 = 0$. $x = \dfrac{2 \pm \sqrt{4-8}}{4} \rightarrow \nexists$ sol

$f'<0$, $f'>0$

$1/2$

$\left(1/2, \dfrac{\sqrt{e}}{4}\right)$ min relat

c)

x $2x$ y

$S = \left(\frac{x}{4}\right)^2 + \left(\frac{x}{2}\right)^2 + \left(\frac{y}{4}\right)^2 = \dfrac{x^2}{16} + \dfrac{x^2}{4} + \dfrac{y^2}{16} =$

$= \dfrac{5x^2+y^2}{16}$ $3x+y=140$. $y = 140-3x$

$S(x) = \dfrac{5x^2+(140-3x)^2}{16} = \dfrac{5x^2+19600+9x^2-840x}{16} =$

$$= \frac{14x^2 - 840x + 19600}{16}$$

$$S'(x) = \frac{28x - 840}{16} = 0; \quad 28x - 840 = 0$$

$$x = \frac{840}{28} = 30 \qquad \underline{\underline{(30, 60, 50)}}$$

$$S''(x) = \frac{28}{16} < 0 \quad \text{minimo.}$$

$$x \in \left[0, \frac{140}{3}\right]$$

4) a) $I = \int (x^2 - x) \operatorname{sen} 2x \, dx$

$u = x^2 - x \quad v = \frac{-\cos 2x}{2}$

$du = (2x-1)dx \quad dv = \operatorname{sen} 2x$

$$I = \frac{-(x^2-x)\cos 2x}{2} + \int \frac{(2x-1)\cos 2x}{2} dx =$$

$$= \frac{-(x^2-x)\cos 2x}{2} + \frac{1}{2} \underbrace{\int (2x-1)\cos 2x \, dx}_{I_1}$$

$u = 2x-1 \quad v = \frac{\operatorname{sen} 2x}{2}$

$du = 2dx \quad dv = \cos 2x$

$$I_1 = \frac{(2x-1)\operatorname{sen} 2x}{2} - \int \frac{\operatorname{sen} 2x}{\cancel{2}} \cdot \cancel{2} \, dx =$$

$$= \frac{(2x-1)\operatorname{sen} 2x}{2} + \frac{\cos 2x}{2}$$

$$I = \frac{-(x^2-x)\cos 2x}{2} + \frac{(2x-1)\operatorname{sen} 2x}{4} + \frac{\cos 2x}{4} =$$

$$= \left.\frac{(-2x^2-2x+1)\cos 2x + (2x-1)\operatorname{sen} 2x}{4}\right]_0^{\pi} =$$

$$= \left(\frac{(-2\pi^2-2\pi+1)\cdot 1 + (2\pi-1)\cdot 0}{4}\right) - \left(\frac{1+0}{4}\right) =$$

$$= \frac{-2\pi^2-2\pi}{4} = -\frac{2\pi(\pi+1)}{4} = -\frac{\pi(\pi+1)}{2}$$

b) $\ln x = t$ $\qquad I = \displaystyle\int \frac{t+1}{t^2-t}\,dt$

$\frac{1}{x}dx = dt$

$t^2 - t = 0 \quad \begin{matrix} t=0 \\ t=1 \end{matrix}$

$$\frac{t+1}{t^2-t} = \frac{A}{t} + \frac{B}{t-1} = \frac{A(t-1)+Bt}{t^2-t}$$

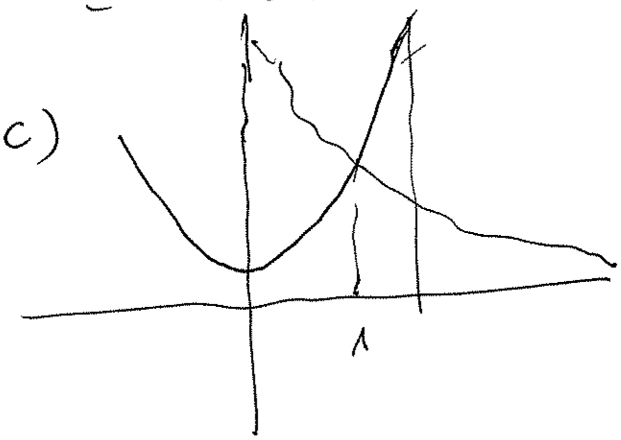
$t=0 \Rightarrow -A = 1 \Rightarrow A = -1$

$t=1 \Rightarrow B = 2$

$$\int \frac{t+1}{t^2-t}\,dt = \int \frac{-1}{t}\,dt + \int \frac{2}{t-1}\,dt =$$

$$= -\ln t + 2\ln(t-1) = \underline{\underline{-\ln(\ln x) + 2\ln(\ln x - 1) + \text{cte}}}$$

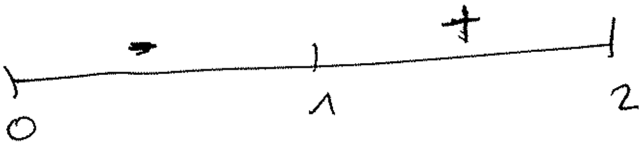
c)



$x^2 + 1 = \frac{2}{x}$

$x^3 + x - 2 = 0$

$$\begin{array}{c|cccc} & 1 & 0 & 1 & -2 \\ 1 & & 1 & 1 & 2 \\ \hline & 1 & 1 & 2 & \boxed{0} \end{array}$$



$$f(x) = x^2 + 1 - \frac{2}{x} = \frac{x^3+x-2}{x}$$

$$A = -\int_0^1 \left(x^2 + 1 - \frac{2}{x}\right)dx + \int_1^2 \left(x^2 + 1 - \frac{2}{x}\right)dx =$$

$$= -\left[\frac{x^3}{3} + x - 2\ln x\right]_0^1 + \left[\left(x^2 + 1 - \frac{2}{x}\right)\right]_1^2 =$$

$$= -\left[\left(\frac{1}{3} + 1 - 0\right) - (0 + 0 + \infty)\right] + \left[(4+1-1) - (1+1-2)\right] = \infty + 4 = \underline{\underline{\infty}}$$