EXAMEN ÁLGEBRA

1

1) a)
$$\left.\begin{array}{l} x+y+z=20 \\ 0'2x+0'5y+0'25z=6 \\ 0'8x+0'9y+0'85z=17 \end{array}\right\} \left(\begin{array}{ccc|c} 1 & 1 & 1 & 20 \\ 0'2 & 0'5 & 0'25 & 6 \\ 0'8 & 0'9 & 0'85 & 17 \end{array}\right) \sim$$

$$\sim \left(\begin{array}{ccc|c} 1 & 1 & 1 & 20 \\ 0 & 0'3 & 0'05 & 2 \\ 0 & 0'1 & 0'05 & 1 \end{array}\right) \sim \left(\begin{array}{ccc|c} 1 & 1 & 1 & 20 \\ 0 & 0'3 & 0'05 & 2 \\ 0 & 0 & -0'1 & -1 \end{array}\right) \quad \left.\begin{array}{l} x+y+z=20 \\ 0'3y+0'05z=2 \\ -0'1z=-1 \end{array}\right\}$$

$$z=\frac{-1}{-0'1}=\underline{\underline{10}} \quad ; \quad 0'3y=2-0'5=1'5 ; \quad y=\frac{1'5}{0'3}=\underline{\underline{5}} \quad \underline{\underline{x=5}}$$

$$(5,5,10)$$

b) TEÓRICO

2) $XA = B.C$; $X=(B.C)A^{-1}$

$$B.C=\begin{pmatrix} 4 & 8 & -1 \\ 0 & 2 & 1 \\ -2 & -7 & -1 \end{pmatrix} \qquad |A| = 56+25+36-28-30-60=\underline{\underline{-1}}$$

$$A^t=\begin{pmatrix} 2 & 5 & 4 \\ 3 & 7 & 5 \\ 1 & 3 & 4 \end{pmatrix} \qquad Adj\,(A^t)=\begin{pmatrix} 13 & -7 & 2 \\ -8 & 4 & -1 \\ -3 & 2 & -1 \end{pmatrix}$$

$$A^{-1}=\begin{pmatrix} -13 & 7 & -2 \\ 8 & -4 & 1 \\ 3 & -2 & 1 \end{pmatrix}$$

$$X=\begin{pmatrix} 4 & 8 & -1 \\ 0 & 2 & 1 \\ -2 & -7 & -1 \end{pmatrix}\begin{pmatrix} -13 & 7 & -2 \\ 8 & -4 & 1 \\ 3 & -2 & 1 \end{pmatrix}=\underline{\underline{\begin{pmatrix} 9 & -2 & -1 \\ 19 & -10 & 3 \\ -33 & 16 & -4 \end{pmatrix}}}$$

b) $|2A| = 1 \;;\; |2A| = 16|A| = 1 \Rightarrow |A| = \frac{1}{16}$

i) $|3A| = 81 \cdot |A| = \frac{81}{16}$ //

ii) $|A^{-1}| = \frac{1}{|A|} = \frac{1}{\frac{1}{16}} = 16$ //

iii) $|A \cdot A^t| = |A|\,|A^t| = |A|\,|A| = |A|^2 = \frac{1}{256}$ //

c) $A^t = A^{-1} \Rightarrow |A^t| = |A^{-1}| \Rightarrow |A| = \frac{1}{|A|}$

$\Rightarrow |A|^2 = 1 \Rightarrow \boxed{|A| = \pm 1}$

3) $$\begin{vmatrix} a & 1 & -1 \\ 1 & a & -1 \\ 3 & 0 & 1-a \end{vmatrix} = a^2(1-a) - 3 + 3a - (1-a) =$$
$$= a^2 - a^3 - 3 + 3a - 1 + a = \boxed{-a^3 + a^2 + 4a - 4}$$
$$= \boxed{-(a-1)(a+2)(a-2)}$$

$$\begin{array}{r|rrrr} & -1 & 1 & 4 & -4 \\ 1 & & -1 & 0 & 4 \\ \hline & -1 & 0 & 4 & \boxed{0} \end{array}$$

$-a^2 + 4 = 0 \Rightarrow a = \pm 2$

$\boxed{a = 1}$ $$\left(\begin{array}{ccc|c} 1 & 1 & -1 & 1 \\ 1 & 1 & -1 & 1 \\ 3 & 0 & 0 & 3 \end{array}\right) \sim \left(\begin{array}{ccc|c} 1 & 1 & -1 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & -3 & 3 & 0 \end{array}\right)$$

$$\left.\begin{array}{l} x + y - z = 1 \\ -3y + 3z = 0 \end{array}\right\} \begin{array}{l} x = 1 \\ y = z \end{array} \longrightarrow \underline{\underline{(1, \lambda, \lambda)\ \lambda \in \mathbb{R}}}$$

$\boxed{a = 2}$ $$\left(\begin{array}{ccc|c} 2 & 1 & -1 & 1 \\ 1 & 2 & -1 & 2 \\ 3 & 0 & -1 & 4 \end{array}\right) \sim \left(\begin{array}{ccc|c} 2 & 1 & -1 & 1 \\ 0 & -3 & 1 & -3 \\ 0 & -6 & 2 & -2 \end{array}\right)$$

$$\sim \left(\begin{array}{ccc|c} 2 & 1 & -1 & 1 \\ 0 & -3 & 1 & -3 \\ 0 & 0 & 0 & 4 \end{array}\right) \longrightarrow \underline{\underline{S.I}}$$

$\boxed{a=-2}$ $\left(\begin{array}{ccc|c} -2 & 1 & -1 & 1 \\ 1 & -2 & -1 & -2 \\ 3 & 0 & 3 & 0 \end{array}\right) \sim \left(\begin{array}{ccc|c} -2 & 1 & -1 & 1 \\ 0 & -3 & -3 & -3 \\ 0 & 6 & 6 & 6 \end{array}\right)$

$\sim \left(\begin{array}{ccc|c} -2 & 1 & -1 & 1 \\ 0 & -3 & -3 & -3 \\ 0 & 0 & 0 & 0 \end{array}\right)$ $\left.\begin{array}{l} -2x+y-z=1 \\ -3y-3z=-3 \end{array}\right\}$

$y=1-z$

$-2x+1-z-z=1;\ -2x=2z;\ x=-z$ $\qquad (-\lambda, 1-\lambda, \lambda)_{\lambda\in\mathbb{R}}$

$\boxed{a\neq 1,2,-2}$ $\begin{vmatrix} 1 & 1 & -1 \\ a & a & -1 \\ a+2 & 0 & 1-a \end{vmatrix} = \begin{array}{l} a(1-a)-(a+2)+a(a+2) \\ -a(1-a) = -(a+2)+a(a+2) \\ = (a-1)(a+2) \end{array}$

$x = \dfrac{\cancel{(a-1)}\cancel{(a+2)}}{-\cancel{(a-1)}\cancel{(a+2)}(a-2)} = -\dfrac{1}{a-2}$

$\begin{vmatrix} a & 1 & -1 \\ 1 & a & -1 \\ 3 & a+2 & 1-a \end{vmatrix} = \begin{array}{l} a^2(1-a)-(a+2)-3+3a+a(a+2)-(1-a) \\ = a^2-a^3-a-2-3+3a+a^2+2a-1+a = \\ = -a^3+2a^2+5a-6 = -(a-1)(a+2)(a-3) \end{array}$

$\begin{array}{r|rrrr} & -1 & 2 & 5 & -6 \\ 1 & & -1 & 1 & 6 \\ \hline & -1 & 1 & 6 & \boxed{0} \end{array}$ $\quad -a^2+a+6=0;\ a^2-a-6=0 \begin{array}{l} 3 \\ -2 \end{array}$

$y = \dfrac{-\cancel{(a-1)}\cancel{(a+2)}(a-3)}{-\cancel{(a-1)}\cancel{(a+2)}(a-2)} = \dfrac{a-3}{a-2}$

$\begin{vmatrix} a & 1 & 1 \\ 1 & a & a \\ 3 & 0 & a+2 \end{vmatrix} = \begin{array}{l} a^2(a+2)+\cancel{3a}-\cancel{3a}-(a+2) = (a+2)(a^2-1) = \\ = (a+2)(a-1)(a+1) \end{array}$

$z = \dfrac{\cancel{(a-1)}\cancel{(a+2)}(a+1)}{-\cancel{(a-1)}\cancel{(a+2)}(a-2)} = -\dfrac{a+1}{a-2}$

DISCUSIÓN

Si $a \neq 1, -2, 2 \rightarrow$ S.CD $\rightarrow \left(\frac{-1}{a-2}, \frac{a-3}{a-2}, -\frac{a+1}{a-2}\right)$

Si $a = 1 \rightarrow$ S.C.I $\rightarrow (1, \lambda, \lambda)$ $\lambda \in \mathbb{R}$

Si $a = -2 \rightarrow$ S.C.I $\rightarrow (-\lambda, 1-\lambda, \lambda)$ $\lambda \in \mathbb{R}$

Si $a = 2 \rightarrow$ S.I

EXAMEN ÁLGEBRA (RECUPERACIÓN) (1)

1) a) $A \to x$ (24€) $\quad x+y+z=600$

$S \to y$ (28€) $\quad 24x+28y+40z=21120$

$N \to z$ (40€) $\quad x+y=\frac{z}{2} \to x+y-2z=0$

$$\left(\begin{array}{ccc|c} 1 & 1 & 1 & 600 \\ 24 & 28 & 40 & 21120 \\ 2 & 2 & -1 & 0 \end{array}\right) \sim \left(\begin{array}{ccc|c} 1 & 1 & 1 & 600 \\ 0 & 4 & 16 & 6720 \\ 0 & 0 & -3 & -1200 \end{array}\right)$$

$$\left.\begin{array}{l} x+y+z=600 \\ 4y+16z=6720 \\ -3z=-1200 \end{array}\right\} \begin{array}{l} x=600-80-400=120 \\ 4y=6720-6400=320;\ y=\frac{320}{4}=80 \\ \to z=400 \end{array}$$

b) TEÓRICO

2) $A\times B - C = D$; $\quad A\times B = D+C$

$X = A^{-1}(D+C)B^{-1}$; $\quad |A| = 2-2-1=-1$

$|B| = 8-6=2$

$$A^t = \begin{pmatrix} 1 & 1 & -1 \\ 1 & 2 & 0 \\ -1 & 0 & 1 \end{pmatrix} \quad Adj(A^t) = \begin{pmatrix} 2 & -1 & 2 \\ -1 & 0 & -1 \\ 2 & -1 & 1 \end{pmatrix}$$

$$A^{-1} = \begin{pmatrix} -2 & 1 & -2 \\ 1 & 0 & 1 \\ -2 & 1 & -1 \end{pmatrix}$$

$$B^t = \begin{pmatrix} 2 & 3 \\ 2 & 4 \end{pmatrix} \quad Adj(B^t) = \begin{pmatrix} 4 & -2 \\ -3 & 2 \end{pmatrix}$$

$$B^{-1} = \begin{pmatrix} 2 & -1 \\ -\frac{3}{2} & 1 \end{pmatrix}; \quad D+C = \begin{pmatrix} 3 & -1 \\ -1 & 1 \\ 3 & 0 \end{pmatrix}$$

$$X = \begin{pmatrix} -2 & 1 & -2 \\ 1 & 0 & 1 \\ -2 & 1 & -1 \end{pmatrix} \begin{pmatrix} 3 & -1 \\ -1 & 1 \\ 3 & 0 \end{pmatrix} \begin{pmatrix} 2 & -1 \\ -\frac{3}{2} & 1 \end{pmatrix} =$$

$$= \begin{pmatrix} -13 & 3 \\ 6 & -1 \\ -10 & 3 \end{pmatrix} \begin{pmatrix} 2 & -1 \\ -\frac{3}{2} & 1 \end{pmatrix} = \underline{\begin{pmatrix} -\frac{61}{2} & 16 \\ \frac{27}{2} & -7 \\ -\frac{49}{2} & 13 \end{pmatrix}}$$

b) i) $$A^2 = \begin{pmatrix} -2 & 1 \\ 1 & k \end{pmatrix} \begin{pmatrix} -2 & 1 \\ 1 & k \end{pmatrix} = \begin{pmatrix} 5 & -2+k \\ -2+k & 1+k^2 \end{pmatrix}$$

$$I_2 - 2A = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} - \begin{pmatrix} -4 & 2 \\ 2 & 2k \end{pmatrix} = \begin{pmatrix} 5 & -2 \\ -2 & 1-2k \end{pmatrix}$$

$$\left.\begin{array}{l} -2+k = -2 \rightarrow k = 0 \\ 1+k^2 = 1-2k \rightarrow k^2+2k = 0 \end{array}\right\} \; \begin{array}{l} k=0 \\ \cancel{k=-2} \end{array}$$

ii) $$A^2 = \begin{pmatrix} 5 & -2 \\ -2 & 1 \end{pmatrix} ; \quad A^4 = \begin{pmatrix} 5 & -2 \\ -2 & 1 \end{pmatrix} \begin{pmatrix} 5 & -2 \\ -2 & 1 \end{pmatrix} = \begin{pmatrix} 29 & -12 \\ -12 & 5 \end{pmatrix}$$

$$\begin{pmatrix} 29 & -12 \\ -12 & 5 \end{pmatrix} = p \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} + q \begin{pmatrix} -2 & 1 \\ 1 & 0 \end{pmatrix} = \begin{pmatrix} p-2q & q \\ q & p \end{pmatrix}$$

$$\left.\begin{array}{r} p-2q = 29 \\ q = -12 \\ p = 5 \end{array}\right\} \quad \boxed{\begin{array}{l} p = 5 \\ q = -12 \end{array}}$$

iii) TEÓRICO

3) $\begin{vmatrix} 1 & a & 1 \\ 1 & 1 & a \\ a & 1 & 1 \end{vmatrix} = 1+1+a^3-a-a-a =$
$= a^3-3a+2 = \underline{\underline{(a-1)^2(a+2)}}$

$a^2+a-2=0 \to 1, -2$

$$\begin{array}{c|cccc} & 1 & 0 & -3 & 2 \\ 1 & & 1 & 1 & -2 \\ \hline & 1 & 1 & -2 & 0 \end{array}$$

Si $a=1$

$$\left(\begin{array}{ccc|c} 1 & 1 & 1 & 3 \\ 1 & 1 & 1 & -4 \\ 1 & 1 & 1 & 1 \end{array}\right) \sim \left(\begin{array}{ccc|c} 1 & 1 & 1 & 3 \\ 0 & 0 & 0 & -7 \\ 0 & 0 & 0 & -2 \end{array}\right) \to S.I$$

Si $a=-2$

$$\left(\begin{array}{ccc|c} 1 & -2 & 1 & 0 \\ 1 & 1 & -2 & 2 \\ -2 & 1 & 1 & -2 \end{array}\right) \sim \left(\begin{array}{ccc|c} 1 & -2 & 1 & 0 \\ 0 & 3 & -3 & 2 \\ 0 & -3 & 3 & -2 \end{array}\right) \sim$$

$$\sim \left(\begin{array}{ccc|c} 1 & -2 & 1 & 0 \\ 0 & 3 & -3 & 2 \\ 0 & 0 & 0 & 0 \end{array}\right)$$

$x-2y+z=0$
$3y-3z=2$; $y=\frac{2+3z}{3}$

$x-\frac{4+6z}{3}+z=0$; $3x-4-6z+3z=0$

$3x=4+5z$; $x=\frac{4+5z}{3}$; $\left(\frac{4+3\lambda}{3}, \frac{2+3\lambda}{3}, \lambda\right)$

Si $a \neq 1,-2$

$\begin{vmatrix} a+2 & a & 1 \\ -2(a+1) & 1 & a \\ a & 1 & 1 \end{vmatrix} = (a+2)-2(a+1)+a^3-a-a(a+2)+2a(a+1) =$

$= \cancel{a+2}-2a-\cancel{2}+a^3-a-a^2-2a+2a^2+2a =$

$= a^3+a^2-2a = a(a^2+a-2) = a(a-1)(a+2)$

$x = \frac{a\cancel{(a-1)}\cancel{(a+2)}}{(a-1)^{\cancel{2}}\cancel{(a+2)}} = \frac{a}{a-1}$ //

$$\begin{vmatrix} 1 & a+2 & 1 \\ 1 & -2(a+1) & a \\ a & a & 1 \end{vmatrix} = \begin{array}{l} -2(a+1) + a + a^2(a+2) \\ + 2a(a+1) - a^2 - (a+2) = \end{array}$$

$$= -2a - 2 + a + a^3 + 2a^2 + 2a^2 + 2a - a^2 - a - 2 =$$

$$= a^3 + 3a^2 - 4$$

$$\begin{array}{c|cccc} & 1 & 3 & 0 & -4 \\ 1 & & 1 & 4 & 4 \\ \hline & 1 & 4 & 4 & \llcorner 0 \end{array}$$

$$a^2 + 4a + 4 = 0 \begin{array}{l} \nearrow -2 \\ \searrow -2 \end{array}$$

$$y = \frac{\cancel{(a-1)}(a+2)^{\cancel{2}}}{(a-1)^{\cancel{2}}\cancel{(a+2)}} = \frac{a+2}{a-1} \quad //$$

$$\begin{vmatrix} 1 & a & a+2 \\ 1 & 1 & -2(a+1) \\ a & 1 & a \end{vmatrix} = \begin{array}{l} a + a + 2 - 2a^2(a+1) \\ - a(a+2) + 2(a+1) - a^2 = \end{array}$$

$$= a + a + 2 - 2a^3 - 2a^2 - a^2 - 2a + 2a + 2 - a^2 =$$

$$= -2a^3 - 4a^2 + 2a + 4$$

$$\begin{array}{c|cccc} & -2 & -4 & 2 & 4 \\ 1 & & -2 & -6 & -4 \\ \hline & -2 & -6 & -4 & \llcorner 0 \end{array}$$

$$2a^2 + 6a + 4 = 0$$

$$a^2 + 3a + 2 = 0 \begin{array}{l} \nearrow -1 \\ \searrow -2 \end{array}$$

$$z = \frac{-2(a+1)\cancel{(a-1)}\cancel{(a+2)}}{(a-1)^{\cancel{2}}\cancel{(a+2)}} = \frac{-2(a+1)}{(a-1)} \quad //$$

EXAMEN GEOMETRÍA

1

1) $A(3,1,0)$ $B(4,m,2)$ $C(1,2,n)$

$\overrightarrow{AB} = (1, m-1, 2)$ $\overrightarrow{AC}\,(-2, 1, n)$

$|\overrightarrow{AB}| = |\overrightarrow{AC}|$; $\sqrt{1^2+(m-1)^2+2^2} = \sqrt{(-2)^2+1^2+n^2}$

$\sqrt{1+m^2+1-2m+4} = \sqrt{4+1+n^2}$; $\sqrt{m^2-2m+6} = \sqrt{n^2+5}$

$\overrightarrow{AB}\cdot\overrightarrow{AC} = 0$; $-2+m-1+2n = 0$

$$\left.\begin{array}{l} m^2-2m+6 = n^2+5 \\ -2+m-1+2n = 0 \end{array}\right\} \quad \left.\begin{array}{l} m^2 - n^2 - 2m = -1 \\ m + 2n = 3 \end{array}\right\}$$

$\boxed{m = 3-2n}$ $(3-2n)^2 - n^2 - 2(3-2n) = -1$

$9 + 4n^2 - 12n - n^2 - 6 + 4n = -1$

$3n^2 - 8n + 4 = 0$

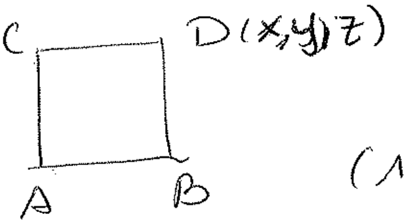
$$n = \frac{8 \pm \sqrt{64-48}}{6} = \frac{8 \pm 4}{6} = \left\langle \begin{array}{l} 2 \\ \frac{2}{3} \end{array}\right.$$

$n = 2 \Rightarrow m = 3-4 = -1$

$n = \frac{2}{3} \Rightarrow m = 3 - \frac{4}{3} = \frac{5}{3}$

$$\boxed{\begin{array}{l} m = -1;\ n = 2 \\ m = \frac{5}{3};\ n = \frac{2}{3} \end{array}}$$

$\boxed{m = -1;\ n = 2}$ $\overrightarrow{AB} = (1, -2, 2)$ $\overrightarrow{AC}\,(-2, 1, 2)$



$\overrightarrow{AB} = \overrightarrow{CD}$

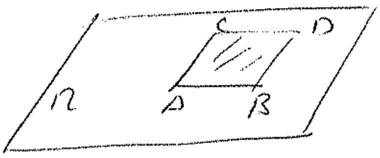
$(1, -2, 2) = (x-1, y-2, z-2)$

$$\left.\begin{array}{l} x-1 = 1 \\ y-2 = -2 \\ z-2 = 2 \end{array}\right\} \begin{array}{l} \to x = 2 \\ \to y = 0 \\ \to z = 4 \end{array}$$

$D(2, 0, 4)$

$l = |\overrightarrow{AB}| = \sqrt{1^2+(-2)^2+(-2)^2} = \sqrt{1+4+4} = \sqrt{9} = 3$

$P = 12u$ // $A = 9u^2$ //



$$\vec{n} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & -2 & 2 \\ -2 & 1 & 2 \end{vmatrix} = (-6,-6,-3)$$

$\vec{n}\,(2, 2, 1)$ $\qquad 2x + 2y + z + D = 0$

$2\cdot 3 + 2\cdot 1 + 0 + D = 0;\quad 6 + 2 + D = 0;\quad D = -8$

$$\boxed{\pi:\ 2x + 2y + z - 8 = 0}$$

2) $\vec{u}_r\,(1,-2,-1)$ $\qquad \vec{u}_s\,(2, 1, 3)$

$P_s\,(1,-2,0)$ $\qquad \overrightarrow{P_rP_s}\,(1-a,-3,-4)$

$P_r\,(a, 1, 4)$

$\dfrac{2}{1} \neq \dfrac{1}{-2} \neq \dfrac{3}{-1}$ $\quad$ No son paralelas

$$\begin{vmatrix} 1 & -2 & -1 \\ 2 & 1 & 3 \\ 1-a & -3 & -4 \end{vmatrix} = -4 + 6 - 6(1-a) + (1-a) + a - 16 =$$

$= -4 + 6 - 6 + 6a + 1 - a + a - 16 =$

$= 5a - 10$

$5a - 10 = 0 \Rightarrow a = 2$

si $a = 2 \rightarrow$ secantes

si $a \neq 2 \rightarrow$ se cruzan

$\boxed{a = 2}$

$$r: \begin{cases} x = 2 + t_1 \\ y = 1 - 2t_1 \\ z = 4 - t_1 \end{cases} \qquad s: \begin{cases} x = 1 + 2t_2 \\ y = -2 + t_2 \\ z = 3t_2 \end{cases}$$

$$\left.\begin{aligned} 2 + t_1 &= 1 + 2t_2 \\ 1 - 2t_1 &= -2 + t_2 \\ 4 - t_1 &= 3t_2 \end{aligned}\right\} \quad \left.\begin{aligned} t_1 - 2t_2 &= -1 \\ -2t_1 - t_2 &= -3 \\ -t_1 - 3t_2 &= -4 \end{aligned}\right\} \quad \begin{aligned} t_1 &= 1 \\ t_2 &= 1 \end{aligned}$$

$$I = (3, -1, 3)$$

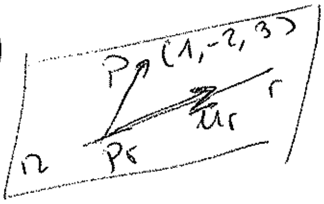
$$\vec{n} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & -2 & -1 \\ 2 & 1 & 3 \end{vmatrix} = (-5, -5, 5) \to (1, 1, -1)$$

$x + y - z + D = 0$; $1 - 2 - 0 + D = 0$; $D = 1$

$$\boxed{\pi : x + y - z + 1 = 0}$$

$$\cos\alpha = \frac{|\vec{u_r} \cdot \vec{u_s}|}{|\vec{u_r}||\vec{u_s}|} = \frac{|2 - 2 - 3|}{\sqrt{6}\sqrt{14}} = \frac{3}{\sqrt{84}} \; ; \; \underline{\underline{\alpha = 70'89^\circ}}$$

3) i)



$$\vec{u_r} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & 1 & -2 \\ 3 & -1 & 0 \end{vmatrix} = (-2, -6, -4) \to (1, 3, 2)$$

$Pr(0, -2, 2)$ $\qquad \vec{PrP} = (1, 0, 1)$

$$\vec{n} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & 3 & 2 \\ 1 & 0 & 1 \end{vmatrix} = (3, 1, -3)$$

$3x + y - 3z + D = 0$; $0 - 2 - 6 + D = 0$; $D = 8$

$$\boxed{\pi : 3x + y - 3z + 8 = 0}$$

ii) $$d(P, r) = \frac{|\vec{PrP} \wedge \vec{u_r}|}{|\vec{u_r}|} = \frac{\sqrt{3^2 + 1^2 + (-3)^2}}{\sqrt{1^2 + 3^2 + 2^2}} = \frac{\sqrt{19}}{\sqrt{14}} = \underline{\underline{\sqrt{\frac{19}{14}}\ u}}$$

iii) $\vec{n} = \vec{u_r} = (1, 3, 2)$ $\qquad x + 3y + 2z + D = 0$

$1 - 6 + 6 + D = 0$; $D = -1$; $\qquad \pi : x + 3y + 2z - 1 = 0$

$$r : \begin{cases} x = t \\ y = -2 + 3t \\ z = 2 + 2t \end{cases}$$

$t + 3(-2 + 3t) + 2(2 + 2t) - 1 = 0$

$t - 6 + 9t + 4 + 4t - 1 = 0$; $14t = 3$

$t = \frac{3}{14}$

$M = \left(\frac{3}{14}, \frac{-19}{14}, \frac{34}{14}\right)$

$$\left.\begin{array}{l} \frac{1+x}{2} = \frac{3}{14} \\ \frac{-2+y}{2} = -\frac{19}{14} \\ \frac{3+z}{2} = \frac{34}{14} \end{array}\right\}$$

$7+7x=3;\ x=\frac{-4}{7}$

$-14+7y=-19;\ y=\frac{-5}{7}$

$21+7z=34;\ y=\frac{13}{7}$

$$\boxed{P'\left(-\frac{4}{7}, -\frac{5}{7}, \frac{13}{7}\right)}$$

4) $\overline{u_r}(2,0,1)$ $\overline{u_s}(1,-1,0)$ $r: \begin{cases} x=-3+2t \\ y=-1 \\ z=t \end{cases}$

$P_r(-3+2t_1, -1, t_1)$ $P_s(3+t_2, -1-t_2, 0)$

$\overrightarrow{P_rP_s} = (6-2t_1+t_2, -t_2, -t_1)$

$$\left.\begin{array}{l} 2(6-2t_1+t_2)+0(-t_2)+1(-t_1)=0 \\ 1(6-2t_1+t_2)-1(-t_2)+0\cdot(-t_1)=0 \end{array}\right\}$$

$$\left.\begin{array}{l} 12-4t_1+2t_2-t_1=0 \\ 6-2t_1+t_2+t_2=0 \end{array}\right\} \left.\begin{array}{l} -5t_1+2t_2=-12 \\ -2t_1+2t_2=-6 \end{array}\right\}$$

$\underline{\underline{t_1=2}}$ $\underline{\underline{t_2=-1}}$

$$\boxed{\begin{array}{l} P_r(1,-1,2) \\ P_s(2,0,0) \end{array}}$$

$\overline{u_t} = \overrightarrow{P_rP_s} = (1, 1, -2)$

$$\boxed{t: \begin{cases} x-5y-2z-2=0 \\ x+y+z-2=0 \end{cases}}$$

$$\boxed{t: \begin{cases} x=1+t \\ y=-1+t \\ z=2-2t \end{cases}}$$

iii) $[\overline{u_r}, \overline{u_s}, \overrightarrow{P_rP_s}] = \begin{vmatrix} 2 & 0 & 1 \\ 1 & -1 & 0 \\ 1 & 1 & -2 \end{vmatrix} = 4+1+1=6$

$$\vec{u_r} \wedge \vec{u_s} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 2 & 0 & 1 \\ 1 & -1 & 0 \end{vmatrix} = (1, 1, -2)$$

$$d(r, s) = \frac{|6|}{\sqrt{1^2+1^2+(-2)^2}} = \frac{6}{\sqrt{6}} = \underline{\sqrt{6}\ u}$$

iv) $d(P_r, P_s) = |\overrightarrow{P_r P_s}| = \sqrt{1^2+1^2+(-2)^2} = \sqrt{6}\ u$

<u>es lo mismo</u>

1) EXAMEN GEOMETRÍA (RECUPERACIÓN) (1)

b)

$$\begin{vmatrix} 1 & 1 & m \\ 0 & m & -1 \\ 1 & 2m & 0 \end{vmatrix} = -1-m^2+2m =$$

$$= -m^2+2m-1 = 0 \to 1, \; 1$$

c) $(1,2,0) = \alpha(1,1,+1) + \beta(0,1,-1)$

$$\left.\begin{aligned} 1 &= \alpha \\ 2 &= \alpha + \beta \\ 0 &= +\alpha - \beta \end{aligned}\right\} \quad \boxed{\begin{aligned} \alpha &= +1 \\ \beta &= +1 \end{aligned}}$$

2) a) $\overline{u_r} = (3,4,5)$ $\quad \overline{u_s} = (-1,2,3)$

$P_r(2,k,0)$; $P_s(-2,1,3)$. $\overline{P_rP_s}(-4,1-k,3)$

$$\begin{vmatrix} 3 & 4 & 5 \\ -1 & 2 & 3 \\ -4 & 1-k & 3 \end{vmatrix} = 18-5(1-k)-48+40-9(1-k)+12 =$$

$$= 18-5+5k-48+40-9+9k+12 =$$

$$= 14k+8 = 0 \; ; \quad k = \frac{-8}{14} = \frac{-4}{7}$$

b) $$\overline{n_\pi} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 3 & 4 & 5 \\ -1 & 2 & 3 \end{vmatrix} = (2,-14,10) \approx (1,-7,5)$$

$x-7y+5z+D=0$ $\quad -2-7+15+D=0$

$D=-6$; $\boxed{\pi : x-7y+5z-6=0}$

c) $$r: \begin{cases} x = 2+3t_1 \\ y = -\frac{4}{7}+4t_1 \\ z = 5t_1 \end{cases} \qquad s: \begin{cases} x = -2 - t_2 \\ y = 1+2t_2 \\ z = 3+3t_2 \end{cases}$$

$$\left.\begin{aligned} 2+3t_1 &= -2-t_2 \\ -\frac{4}{7}+4t_1 &= 1+2t_2 \\ 5t_1 &= 3+3t_2 \end{aligned}\right\} \quad \left.\begin{aligned} 3t_1+t_2 &= -4 \\ 28t_1-14t_2 &= 11 \\ 5t_1-3t_2 &= 3 \end{aligned}\right\}$$

$$\left.\begin{aligned} 9t_1+3t_2 &= -12 \\ 5t_1-3t_2 &= 3 \end{aligned}\right\} \quad t_1 = -\frac{9}{14}$$

$$\overline{14t_1 = -9} \quad t_2 = -4+\frac{27}{14} = \frac{-29}{14}$$

$$I = \begin{cases} x = 2-\frac{27}{14} \\ y = -\frac{4}{7} - \frac{36}{14} \\ z = \frac{-45}{14} \end{cases} \qquad \underline{I = \left(\frac{1}{14}, \frac{-44}{14}, \frac{-45}{14}\right)}$$

3) a) $\vec{u_r} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 3 & -1 & -1 \\ 1 & 1 & -1 \end{vmatrix} = (2,2,4) \approx (1,1,2)$

$P_r(0,-4,2)$ $\quad r: \begin{cases} x = t \\ y = -4+t \\ z = 2+2t \end{cases}$

$x+y+2z+D=0$; $\quad 1-3+14+D=0$; $\quad D=-12$

$\pi_\perp: x+y+2z-12=0$

$t+(-4+t)+2(2+2t)-12=0$;

$t-4+t+4+4t-12=0$; $\quad 6t=12$; $\quad \underline{t=2}$

$P_M = (2,-2,6)$ $\quad P'(x,y,z)$

$\frac{x+1}{2} = 2 \rightarrow x=3$

$\frac{y-3}{2} = -2 \rightarrow y=-1$ $\quad \underline{P'(3,-1,5)}$

$\frac{z+7}{2} = 6 \rightarrow z=5$

b) $d(P, P') = \overrightarrow{PP'} = \sqrt{(3-1)^2+(-1+3)^2+(5-7)^2} =$

$= \sqrt{4+4+4} = \sqrt{12} = \underline{2\sqrt{3}\ u}$

c) $\vec{u}_r(1, 1, 2)$; $P_r(0, -4, 2)$ $\quad P(1, -3, 7)$

$\overrightarrow{P_rP}(+1, +1, +5)$

$$\begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & 1 & 5 \\ 1 & 1 & 2 \end{vmatrix} = (-3, 3, 0)$$

$$d(P, r) = \frac{\sqrt{(-3)^2+3^2+0^2}}{\sqrt{1^2+1^2+2^2}} = \frac{\sqrt{18}}{\sqrt{6}} = \underline{\sqrt{3}\ u}$$

<u>Es la mitad</u>

4) a) $$\vec{u}_r = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & -1 & 1 \\ 1 & 1 & -1 \end{vmatrix} = (0, 2, 2) \approx (0, 1, 1)$$

$$r: \begin{cases} x = 1 \\ y = 2+t \\ z = 3+t \end{cases} \qquad r: \frac{x-1}{0} = \frac{y-2}{1} = \frac{z-3}{1}$$

b) $$d(P, \pi_1) = \frac{|1-2+3|}{\sqrt{1^2+(-1)^2+1^2}} = \frac{2}{\sqrt{3}} = \frac{2\sqrt{3}}{3}\ u \ //$$

$$d(P, \pi_2) = \frac{|1+2-3-2|}{\sqrt{1^2+1^2+(-1)^2}} = \frac{2}{\sqrt{3}} = \frac{2\sqrt{3}}{3}\ u \ //$$

c) $\overrightarrow{PQ} = (1,-1,-3)$; $M_{PQ} = \left(\frac{3}{2}, \frac{3}{2}, \frac{3}{2}\right)$

$\pi_{med}: x - y - 3z + D = 0$

$\frac{3}{2} - \frac{3}{2} - \frac{9}{2} + D = 0$, $D = \frac{9}{2}$

$\pi_{med}: x - y - 3z + \frac{9}{2} = 0$

$\pi_{med}: 2x - 2y - 6z + 9 = 0$

$2 \cdot 1 - 2(2+t) - 6(3+t) + 9 = 0$

$2 - 4 - 2t - 18 - 6t + 9 = 0$

$-8t = 11$; $t = -\frac{11}{8}$

$I: \begin{cases} x = 1 \\ y = 2 - \frac{11}{8} \\ z = 3 - \frac{11}{8} \end{cases}$ $\quad \underline{\underline{I\left(1, \frac{5}{8}, \frac{13}{8}\right)}}$

EXAMEN ANÁLISIS (1º) ①

1) a)

i) $\lim_{x \to -1} \left(\frac{1}{2(x+1)} - \frac{x}{(x+1)(x-1)} \right) =$

$= \lim_{x \to -1} \frac{(x-1) - 2x}{2(x+1)(x-1)} = \lim_{x \to -1} \frac{-\cancel{(x+1)}}{2\cancel{(x+1)}(x-1)} = \frac{1}{4}$ //

ii) $\lim_{x \to 3} \left(\frac{(4x+1)+(x^2-3x)}{4x+1} \right)^{\frac{x^2}{x-3}} = \lim_{x \to 3} \left(1 + \frac{1}{\frac{4x+1}{x^2-3x}} \right)^{\frac{x^2}{x-3}} =$

$= \lim_{x \to 3} \left(1 + \frac{1}{\frac{4x+1}{x^2-3x}} \right)^{\frac{x^2}{x-3} \cdot \frac{4x+1}{x^2-3x} \cdot \frac{x^2-3x}{4x+1}} =$

$= e^{\lim_{x \to 3} \frac{x^2 \cdot (x^2-3x)}{(x-3)(4x+1)}} = e^{\lim_{x \to 3} \frac{x^3 \cancel{(x-3)}}{\cancel{(x-3)}(4x+1)}} = e^{\frac{27}{13}}$

b) $f(x) = \begin{cases} \frac{3x}{2-x} & \text{si } x < 0 \\ \frac{x}{2+x} & \text{si } x \geq 0 \end{cases}$

CONTINUIDAD Continua en todo $\mathbb{R}$

DERIVABILIDAD

$f'(x) = \begin{cases} \frac{6}{(2-x)^2} & \text{si } x < 0 \\ \frac{2}{(2+x)^2} & \text{si } x > 0 \end{cases}$ En $x=0$ no derivable

$\lim_{x \to 0^-} f(x) = \frac{3}{2}$ $\lim_{x \to 0^+} f(x) = \frac{1}{2}$

2) $f(x) = \begin{cases} e^{x-2} - x + 1 & \text{si } x \leq 2 \\ ax + \sqrt{x^2+b^2-4} & \text{si } x > 2 \end{cases}$

CONTINUIDAD

$\lim_{x \to 2^-} f(x) = e^0 - 2 + 1 = 0$

$\lim_{x \to 2^+} f(x) = 2a + \sqrt{4+b^2-4} = 2a + b$ $\quad 2a+b=0$

$f(2) = -1$

DERIVABILIDAD

$f'(x) = \begin{cases} e^{x-2} - 1 & \text{si } x < 2 \\ a + \dfrac{x}{\sqrt{x^2+b^2-4}} & \text{si } x > 2 \end{cases}$

$\lim_{x \to 2^-} f(x) = e^0 - 1 = 1 - 1 = 0$

$\lim_{x \to 2^+} f(x) = a + \dfrac{2}{b}$

$a + \dfrac{2}{b} = 0$

$ab = -2$

$\begin{cases} 2a+b = 0 \\ ab = -2 \end{cases}$ $\quad b = -2a$

$-2a^2 = -2 \quad a^2 = 1 \Rightarrow a = \pm 1$

$b = \mp 2$

$\boxed{a = 1;\ b = -2}$ $\quad \boxed{a = -1;\ b = 2}$

2) $f(x) = \begin{cases} e^{x-2} + bx & si\ x \leq 2 \\ ax^2 + \sqrt{x^2-3} & si\ x > 2 \end{cases}$

$f'(x) = \begin{cases} e^{x-2} + b & si\ x < 2 \\ 2ax + \dfrac{x}{\sqrt{x^2-3}} & si\ x > 2 \end{cases}$

$\lim_{x \to 2^-} f(x) = 1 + 2b$
$\lim_{x \to 2^+} f(x) = 4a + 1$
$f(2) = 1 + 2b$

$1 + 2b = 4a + 1$
$\underline{\underline{b = 2a}}$

$\lim_{x \to 2^-} f'(x) = 1 + b$
$\lim_{x \to 2^+} f'(x) = 4a + 2$

$1 + b = 4a + 2$
$\left.\begin{array}{l} b - 4a = 1 \\ b = 2a \end{array}\right\}$ $\boxed{\begin{array}{c} a = -1/2 \\ b = -1 \end{array}}$

$P_1(2, -1)$ $\quad m = 0$

$\boxed{y = -1}$

$-\frac{7}{2} + 2 = -\frac{3}{2}$

$P_2\left(\sqrt{7}, -\frac{3}{2}\right)$ $\quad m = -\sqrt{7} + \frac{\sqrt{7}}{2} = -\frac{\sqrt{7}}{2}$

$y + \frac{3}{2} = -\frac{\sqrt{7}}{2}(x - \sqrt{7})$ $\quad \boxed{y = -\frac{\sqrt{7}}{2}x + 2}$

3) i) $y = \ln\left(\sqrt{1+e^x} - 1\right) - \ln\left(\sqrt{1+e^x} + 1\right)$

$y' = \dfrac{1}{\sqrt{1+e^x} - 1} \cdot \dfrac{e^x}{2\sqrt{1+e^x}} - \dfrac{1}{\sqrt{1+e^x} + 1} \cdot \dfrac{e^x}{2\sqrt{1+e^x}} =$

$= \dfrac{e^x\left(\sqrt{1+e^x} + 1\right) - e^x\left(\sqrt{1+e^x} - 1\right)}{2\sqrt{1+e^x}\left(\sqrt{1+e^x} - 1\right)\left(\sqrt{1+e^x} + 1\right)} =$

$$= \frac{\cancel{2}\, e^{x}}{2\sqrt{1+e^{x}}\cdot e^{x}} = \boxed{\frac{1}{\sqrt{1+e^{x}}}}$$

ii)

$$y = \left(1+\frac{1}{x}\right)^{x}; \qquad \ln y = x \cdot \ln\left(1+\frac{1}{x}\right)$$

$$\frac{y'}{y} = \ln\left(1+\frac{1}{x}\right) + x \cdot \frac{1}{1+\frac{1}{x}} \cdot -\frac{1}{x^{2}} =$$

$$= \ln\left(1+\frac{1}{x}\right) - \frac{x^{2}}{x^{2}(x+1)} = \ln\left(1+\frac{1}{x}\right) - \frac{1}{x+1}$$

$$\boxed{y' = \left(1+\frac{1}{x}\right)^{x} \cdot \left(\ln\left(1+\frac{1}{x}\right) - \frac{1}{x+1}\right)}$$

iii)

$$y' = \frac{1}{1+\left(\frac{1+x^{2}}{1-x^{2}}\right)^{2}} \cdot \frac{2x(1-x^{2})+2x(1+x^{2})}{(1-x^{2})^{2}} - 2\left[\operatorname{arctg} x + \frac{x}{1+x^{2}}\right] =$$

$$= \frac{1}{\frac{(1-x^{2})^{2}+(1+x^{2})^{2}}{(1-x^{2})^{2}}} \cdot \frac{4x}{(1-x^{2})^{2}} - 2\left[\operatorname{arctg} x + \frac{x}{1+x^{2}}\right] =$$

$$= \frac{4x}{2+2x^{4}} - 2\operatorname{arctg} x + \frac{x}{1+x^{2}} =$$

$$= \frac{2x}{1+x^{4}} - 2\operatorname{arctg} x + \frac{x}{1+x^{2}}$$

iii) $$y' = \frac{1}{1+\left(\frac{1+x}{1-x}\right)^2} \cdot \frac{(1-x)+(1+x)}{(1-x)^2}$$

$$-2\operatorname{arctg} x - 2x \cdot \frac{1}{1+x^2} =$$

$$= \frac{2}{(1-x)^2+(1+x)^2} - 2\operatorname{arctg} x - 2x \cdot \frac{1}{1+x^2} =$$

$$= \frac{2}{2+2x^2} - 2\operatorname{arctg} x - \frac{2x}{1+x^2} =$$

$$= \frac{1}{1+x^2} - 2\operatorname{arctg} x - \frac{2x}{1+x^2} = \underline{\underline{\frac{1-2x}{1+x^2} - 2\operatorname{arctg} x}}$$

4) a) $f(x) = \frac{x}{x-a}$ $\quad g(x) = \ln(x+b)$

$f(0) = g(0) = 0$; $\quad 0 = \ln b = 0$; $\underline{\underline{b=1}}$

$g'(b) = \frac{1}{x+b}$

$f'(x) = \frac{-a}{(x-a)^2}$

$f'(0) = g'(0)$; $\quad \frac{-a}{a^2} = \frac{1}{b}$; $\quad -\frac{1}{a} = 1$; $\underline{\underline{a=-1}}$

b) $(f \circ g)(x) = \sqrt{(x^2+2)+1} = \sqrt{x^2+3}$

$(g \circ f)(x) = \left(\sqrt{x+1}\right)^2 + 2 = x+3$

$\sqrt{x^2+3} = z$; $x^2+3=z^2$; $x^2 = z^2 - 3$; $x = \sqrt{z^2-3}$

$(f \circ g)^{-1}(x) = \sqrt{x^2-3}$

$x+3 = z$; $x = z-3$

$(g \circ f)^{-1}(x) = x-3$

$\sqrt{x^2-3} = x-3$; $x^2-3 = x^2+9-6x$

$6x = 12$; $\underline{\underline{x=2}}$

EXAMEN ANÁLISIS (2º) 1

1) a) i) $L = \lim_{x \to \frac{\pi}{2}^-} (tg\, x)^{\cos x}$

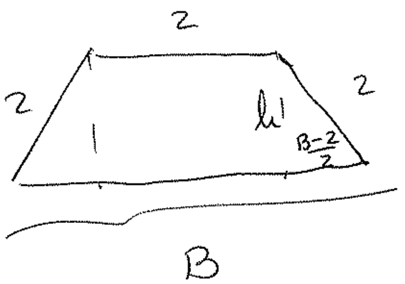
$$\ln L = \lim_{x \to \frac{\pi}{2}^-} \cos x \cdot \ln tg\, x = \lim_{x \to \frac{\pi}{2}^-} \frac{\ln tg\, x}{\frac{1}{\cos x}} =$$

$$= \lim_{x \to \frac{\pi}{2}^-} \frac{\frac{1}{tg\, x} \cdot \frac{1}{\cos^2 x}}{\frac{\operatorname{sen} x}{\cos^2 x}} = \lim_{x \to \frac{\pi}{2}^-} \frac{\operatorname{sen} x}{tg\, x} =$$

$$= \lim_{x \to \frac{\pi}{2}^-} \cos x = 0 \Rightarrow L = e^0 = \underline{\underline{1}}$$

ii) $$\lim_{x \to 0} \frac{\frac{1}{1+x^2} \cdot \cancel{2x}}{\cancel{2x}} = \lim_{x \to 0} \frac{1}{1+x^2} = 1$$

b)



$$h^2 + \left(\frac{B}{2} - 1\right)^2 = 4. \quad h^2 + \frac{B^2}{4} + 1 - B = 4$$

$$h^2 = 3 + B - \frac{B^2}{4} = \frac{12 + 4B - B^2}{4}$$

$$h = \frac{\sqrt{-B^2 + 4B + 12}}{2}$$

$$A = \frac{(B+2) \cdot \sqrt{-B^2+4B+12}}{4}$$

$$A' = \frac{\sqrt{-B^2+4B+12} + \frac{(B+2)(-2B+4)}{2\sqrt{-B^2+4B+12}}}{4} =$$

$$= \frac{2(-B^2+4B+12) + (B+2)(-2B+4)}{8\sqrt{-B^2+4B+12}} =$$

$$= \frac{-2B^2 + 8B + 24 - 2B^2 + \cancel{4B} - \cancel{4B} + 8}{8\sqrt{-B^2+4B+12}} = \frac{-4B^2 + 8B + 32}{8\sqrt{-B^2+4B+12}}$$

$-4B^2 + 8B + 32 = 0$, $B^2 - 2B - 8 = 0$ $\begin{cases} B = 4 \\ \cancel{B = -2} \end{cases}$

$B \in [2, 6]$

$A(2) = 4$

$A(6) = 0$

$A(4) = \frac{6 \cdot \sqrt{12}}{4} = 3\sqrt{3} \rightarrow$ Máximo

2) 1º) Dominio: $\mathbb{R}$

2º) Corte ejes $\begin{cases} OX: (-2, 0) \\ OY: (0, 2) \end{cases}$

3º) Asíntotas:

H: $\lim_{x \to \pm\infty} f(x) = \begin{cases} 0 \\ \infty \end{cases}$ $\boxed{y=0}$ Dcha

Verticales: No tiene

Oblicuas: $\lim_{x \to \pm\infty} \frac{x+2}{x \cdot e^x} = \begin{cases} 0 \\ \infty \end{cases}$ No tiene

4º) $y' = \frac{e^x - (x+2)e^x}{e^{2x}} = \frac{e^x(-x-1)}{e^{2x}} = \frac{-x-1}{e^x}$

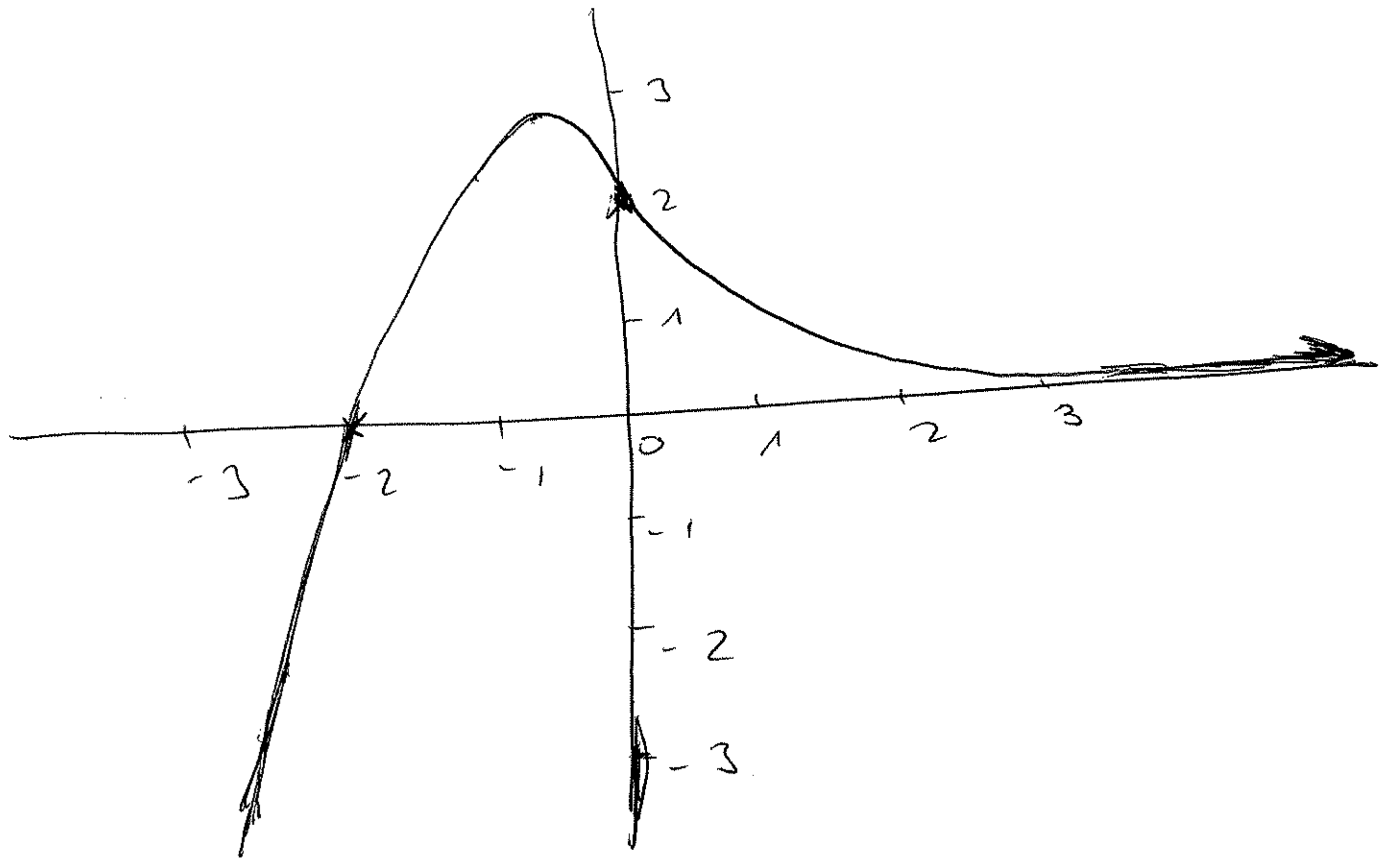
$y' = 0 \Rightarrow x = -1$

$y' > 0$ Crece | -1 | $y' < 0$ Decrece

$\downarrow$

$(-1, e) \to$ Máximo

5º) Simetrías $f(-x) = \frac{-x+2}{e^{-x}} \to$ No tiene



3) i) $\sqrt{1+x^2} = t$ $\frac{x\,dx}{\sqrt{1+x^2}}\,dt$

$$\int \cancel{A}\, t\, dt = -\ln\cos t = -\ln\sqrt{1+x^2} + cte$$

ii)
$$\begin{array}{l|l}
x^4 + 0x^3 - 9x^2 + 5x + 14 & x^3 - x^2 - 8x + 12 \\ \hline
-x^4 \quad x^3 + 8x^2 - 12x & x + 1 \\
\hline
/ \quad x^3 - x^2 - 7x + 14 & \\
\quad -x^3 + x^2 \quad 8x \quad -12 & \\
\hline
/ \quad / \quad x + 2 &
\end{array}$$

$$x^4 - 9x^2 + 5x + 14 = (x+1)(x^3 - x^2 - 8x + 12) + (x+2)$$

$$\int \frac{x^4 - 9x^2 + 5x + 14}{x^3 - x^2 - 8x + 12}\,dx = \int (x+1)\,dx + \int \frac{x+2}{x^3 - x^2 - 8x + 12}\,dx$$

$x^3 - x^2 - 8x + 12 = 0$

$$\begin{array}{r|rrrr}
 & 1 & -1 & -8 & 12 \\
+2 & & +2 & 2 & -12 \\
\hline
 & 1 & +1 & -6 & \boxed{0}
\end{array}$$

$x^2 + x - 6 = 0 \to 2, -3$

$(x-2)^2(x+3)$

$$\frac{x+2}{x^3 - x^2 - 8x + 12} = \frac{A}{(x-2)} + \frac{B}{(x-2)^2} + \frac{C}{(x+3)} =$$

$$= \frac{A(x-2)(x+3) + B(x+3) + C(x-2)^2}{(x-2)^2(x+3)}$$

$x = 2 \to 4 = 5B \Rightarrow B = \frac{4}{5}$ // $C = -1/25$

$x = -3 \to -1 = 25C$

$x = 0 \to 2 = -6A + 3B + 4C$; $2 = -6A + \frac{12}{5} - \frac{4}{25}$; $A = \frac{1}{25}$ //

$$I = \frac{x^2}{2} + x + \frac{1}{25}\ln|x-2| - \frac{4}{5(x-2)} - \frac{1}{25}\ln|x+3| + cte$$

---

4) a) $$\int \frac{\ln x \sqrt{1+(\ln x)^2}}{x}\,dx =$$

$$1+(\ln x)^2 = t$$
$$\frac{2\ln x}{x}\,dx = dt$$

$$= \int \frac{\sqrt{t}}{2}\,dt = \frac{t^{3/2}}{2 \cdot 3/2} = \frac{t^{3/2}}{3} = \frac{\sqrt{t^3}}{3}$$

$$I = \left.\frac{\sqrt{(1+(\ln x)^2)^3}}{3}\right]_1^e = \frac{\sqrt{8}}{3} - \frac{1}{3} = \frac{\sqrt{8}-1}{3}$$ //

b) $$\frac{x+2}{e^x} = 0 \Rightarrow x = -2$$

$f<0$ $f>0$

-3 -2 1

$$A = -\int_{-3}^{-2} \frac{x+2}{e^x}\,dx + \int_{-2}^{1} \frac{x+2}{e^x}\,dx = -\left[(-x-3)e^{-x}\right]_{-3}^{-2} +$$

$$+ \left[(-x-3)e^{-x}\right]_{-2}^{1} = -\left[(-e^{2}-0)\right] + \left[-4e^{-1} + e^{2}\right] =$$

$$\boxed{= 2e^2 - \frac{4}{e} = \frac{2e^3-4}{e}}$$ //

$$\int (x+2)e^{-x}\,dx =$$

$u = x+2$ $v = -e^{-x}$

$du = dx$ $dv = e^{-x}dx$

$$= -(x+2)e^{-x} + \int e^{-x}\,dx =$$

$$= -(x+2)e^{-x} - e^{-x} = e^{-x}(-x-3)$$

EXAMEN ANÁLISIS (RECUPERACIÓN) 1

1) a) $\lim_{x \to 0^-} f(x) = e^b$

$\lim_{x \to 0^+} f(x) = \ln a$

$f(0) = 1$

$e^b = 1 \Rightarrow \underline{\underline{b=0}}$

$\ln a = 1 \Rightarrow \underline{\underline{a=e}}$

$$f(x) = \begin{cases} e^{ex} & \text{si } x < 0 \\ 1 & \text{si } x = 0 \\ \ln(x^2+e) & \text{si } x > 0 \end{cases}$$

Continua en $\mathbb{R}$

$$f'(x) = \begin{cases} e \cdot e^{x} & \text{si } x < 0 \\ \dfrac{2x}{x^2+e} & \text{si } x > 0 \end{cases}$$

$\lim_{x \to 0^-} f'(x) = e$

$\lim_{x \to 0^+} f'(x) = 0$

Derivable en $\mathbb{R} - \{0\}$

b) $f'(x) = \dfrac{1}{\sqrt{ax+2}} \cdot \dfrac{a}{2\sqrt{ax+2}} = \dfrac{a}{2(ax+2)}$

$f'(1) = \dfrac{a}{2(a+2)} = \dfrac{1}{4}$; $4a = 2(a+2)$; $4a = 2a+4$

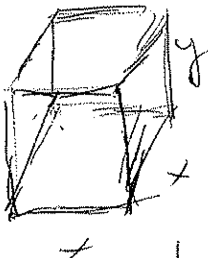
$2a = 4$; $\boxed{a = 2}$

$P_0(1, \ln 2)$ $m = \dfrac{1}{4}$

$y - \ln 2 = \dfrac{1}{4}(x-1)$ $\boxed{y = \dfrac{1}{4}(x-1) + \ln 2}$

2) a) $\lim_{x \to 0} \dfrac{e^x - \cos x\, e^{\operatorname{sen} x} + 4x}{2x} =$

$= \lim_{x \to 0} \dfrac{e^x + \operatorname{sen} x \cdot e^{\operatorname{sen} x} - \cos^2 x\, e^{\operatorname{sen} x} + 4}{2} = \dfrac{4}{2} = 2$



$V = x^2 y = 16$; ~~[illegible]~~ $\rightarrow y = \frac{16}{x^2}$

$C_1 = 2 \cdot 2x^2 + 4xy = 4x^2 + 4xy$

$C_1(x) = 4x^2 + 4x\frac{16}{x^2} = \frac{4x^4 + 64x}{x^2} \rightarrow$ mínimo

$\cancel{C_1'(x) = \frac{16x^3 \cdot x^2 - 2x(4x^4+64)}{x^4} = \frac{8x^5 - 128x}{x^4} = \frac{8x^4 - 128}{x^3}}$; $\cancel{x^4 = \frac{128}{8} = 16}$; $\cancel{x = \sqrt[4]{16} = 2}$; $\cancel{y = 4}$

$\cancel{x \in (0, \infty)}$; $\cancel{C_1(0) = \infty}$; $\cancel{C_1(\infty) = \infty}$

$\cancel{C_1(2) = \frac{128}{4} = 32 €}$

$C_1'(x) = \frac{(16x^3 + 64)x^2 - 2x(4x^4 + 64x)}{x^4} =$

$= \frac{16x^5 + 64x^2 - 8x^5 - 128x^2}{x^4} = \frac{8x^5 - 64x^2}{x^4}$

$8x^5 - 64x^2 = 0$; $\quad 8x^2(x^3 - 8) = 0 \rightarrow x^3 = 8 \rightarrow \underline{x = 2}$

$\underline{y = 4}$

$C_1(2) = \frac{4 \times 16 + 64 \times 2}{2^2} = \frac{64 + 128}{4} = \frac{192}{4} = \underline{\underline{48 €}}$

3) i) $x^2 - 6x + 9 = 0$; $x = 3$; $D = \mathbb{R} \setminus \{3\}$

ii) Corte ejes $\begin{cases} OX: (0,0) \\ OY: (0,0) \end{cases}$

iii) A.H: $\lim_{x \to \pm\infty} f(x) = \begin{cases} 2 \\ 2 \end{cases}$ $\quad \boxed{y = 2}$

A.V: $\boxed{x = 3}$ $\quad \lim_{x \to 3^-} f(x) = +\infty$; $\lim_{x \to 3^+} f(x) = +\infty$

A.O: $\lim_{x \to \pm\infty} \frac{2x^2}{x^3 - 6x^2 + 9x} = 0 \rightarrow$ no tiene

2

iv) $f'(x) = \dfrac{4x(x^2-6x+9) - 2x^2(2x-6)}{(x^2-6x+9)^2} =$

$= \dfrac{\cancel{4x^3} - 24x^2 + 36x - \cancel{4x^3} + 12x^2}{(x^2-6x+9)^2} = \dfrac{-12x^2+36x}{(x^2-6x+9)^2}$

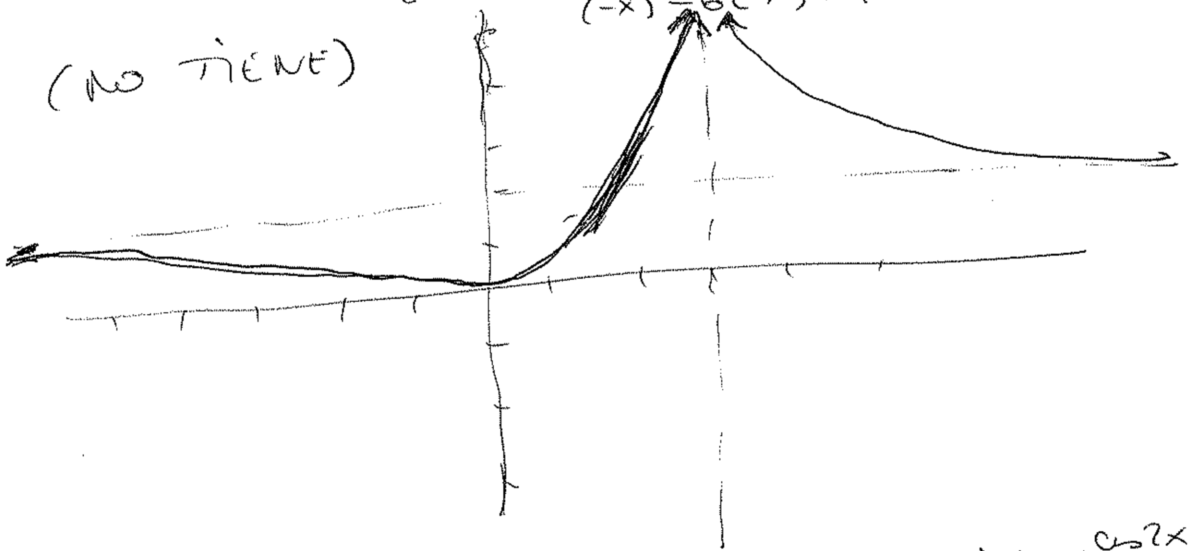
$-12x^2+36x = 0;\quad -12x(x-3) = 0 \quad \begin{cases} x = 0 \\ x = 3 \end{cases}$

$f'<0$	$f'>0$	$f'<0$
Decrec	Crec	Decrec

0 → (0,0) → (mínimo) 3 → $\nexists$

v) Simetrías $f(-x) = \dfrac{2(-x)^2}{(-x)^2-6(-x)+9} = \dfrac{2x^2}{x^2+6x+9}$

(NO TIENE)



4) a)

$u = x^2-1 \quad v = \dfrac{-\cos 2x}{2}$

$du = 2x\,dx \quad dv = \operatorname{sen} 2x$

$I = \dfrac{(x^2-1)(-\cos 2x)}{2} + \underbrace{\int x \cos 2x\, dx}_{I_1}$

$u = x \quad v = \dfrac{\operatorname{sen} 2x}{2}$

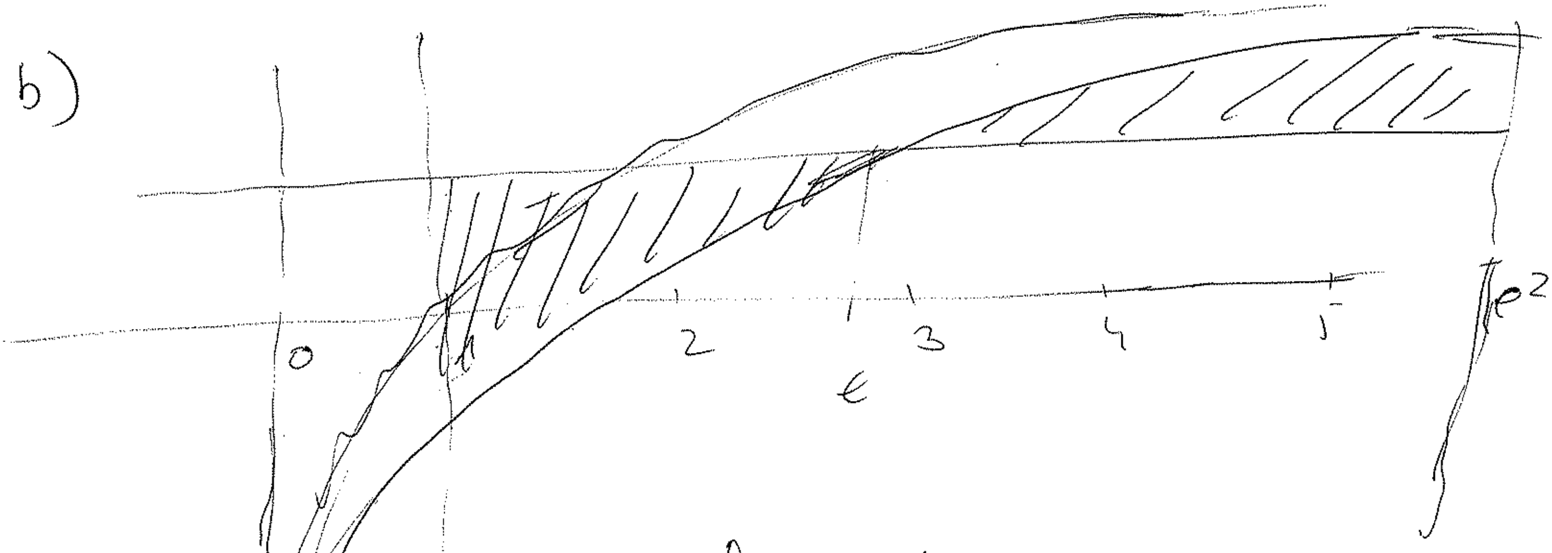
$du = dx \quad dv = \cos 2x\, dx$

$I_1 = \dfrac{x \operatorname{sen} 2x}{2} - \int \dfrac{\operatorname{sen} 2x\, dx}{2} =$

$= \dfrac{x \operatorname{sen} 2x}{2} + \dfrac{\cos 2x}{4}$

$$\boxed{I = \dfrac{-(x^2-1)\cos 2x}{2} + \dfrac{x \operatorname{sen} 2x}{2} + \dfrac{\cos 2x}{4}}$$

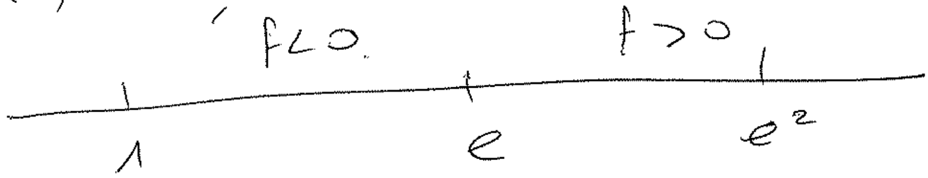
b)



$D(x) = \ln x - 1$

$\ln x - 1 = 0 \rightarrow x = e$

$D(x) = 0$



$$A = -\int_1^e (\ln x - 1)\,dx + \int_e^{e^2} (\ln x - 1)\,dx =$$

$$= -\Big[x(\ln x - 2)\Big]_1^e + \Big[x(\ln x - 2)\Big]_e^{e^2} = -\Big[\big(e(1-2) - (-2)\big)\Big] +$$

$$+ \Big[(e^2 \cdot 0) - \big(e(1-2)\big)\Big] = -[-e+2] + e = 2e - 2 =$$

$$= \underline{\underline{2(e-1)\,u^2}}$$

I $\int (\ln x - 1)\,dx = x(\ln x - 1) - \int dx =$ $\quad u = \ln x - 1 \quad v = x$

$= x\ln x - x - x = \underline{\underline{x(\ln x - 2)}}$ $\quad du = \frac{1}{x}dx \quad dv = dx$

EXAMEN JUNIO

1) a) $P_r(2,1,0)$ $P_s(-1,-2,1)$ $\overrightarrow{P_rP_s}(-3,-3,1)$

$\overline{u_r}=(3,-2,1)$, $\overline{u_s}(2,-1,2)$

$$\begin{vmatrix} -3 & -3 & 1 \\ 3 & -2 & 1 \\ 2 & -1 & 2 \end{vmatrix} = 12-3-6+4-3+18=22\neq 0$$

Se cruzan

$$\overline{u_r}\wedge\overline{u_s}=\begin{vmatrix} \bar{i} & \bar{j} & \vec{k} \\ 3 & -2 & 1 \\ 2 & -1 & 2 \end{vmatrix} = (-3,-4,1)$$

b)

$$d(r,s)=\frac{22}{\sqrt{9+16+1}}=\frac{22}{\sqrt{26}}\ u$$

c)

$$\begin{vmatrix} \bar{i} & \bar{j} & \vec{k} \\ 3 & -2 & 1 \\ -3 & -4 & 1 \end{vmatrix} = (2,-6,-18)\simeq(1,-3,-9)$$

$x-3y-9z+D=0$ $\boxed{x-3y-9z+1=0}$

$$\begin{vmatrix} \bar{i} & \bar{j} & \vec{k} \\ 2 & -1 & 2 \\ -3 & -4 & 1 \end{vmatrix} = (7,-8,-11)$$

$7x-8y-11z+D=0$ $\boxed{7x-8y-11z+2=0}$

2) $M_{PQ}(0,3,3)$ $\overrightarrow{PQ}=(-2,2,0)\simeq(1,-1,0)$

$\pi_{med}=x-y+3=0$

$x-y+D \Rightarrow D=3$

$$\left.\begin{array}{l} x+y+z=2 \\ x-y+3=0 \end{array}\right\}$$

$$\begin{vmatrix} \bar{i} & \bar{j} & \vec{k} \\ 1 & 1 & 1 \\ 1 & -1 & 0 \end{vmatrix} = (1,1,-2)$$

$P_r=(0,3,-1)$

$$r:\begin{cases} x=t \\ y=3+t \\ z=-1-2t \end{cases} \rightarrow \underline{P(-1,2,1)}$$

$|\overline{PQ}|=\sqrt{4+4+0}=\sqrt{8}$

$t^2+1-2t+t^2+1+2t+4t^2+16+16t=8$

$\sqrt{(t-1)^2+(1+t)^2+(-2t-4)^2}=\sqrt{8}$

$6t^2+16t+10=0$

$3t^2+8t+5=0 \rightarrow t=-1$, $t=\frac{-5}{3}$

$A=\frac{\sqrt{8}\sqrt{8}\sqrt{3}}{4}=\frac{8\sqrt{3}}{4}=2\sqrt{3}$

3) $r: \begin{cases} x = -2 - t \\ y = -2t \\ z = 3t \end{cases}$ $(-2-t)+(-2t)-3t+2=0$

$-2-t-2t-3t+2=0; \ -6t=0, t=0$

$I(-2,0,0)$ $\overline{u_s} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & 1 & 0 \\ 0 & 1 & -1 \end{vmatrix} = (-1,1,1)$

$u_t = (1,2,-1)$ $\overline{u_s} \wedge \overline{u_t} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ -1 & 1 & 1 \\ 1 & 2 & -1 \end{vmatrix} = (-3,0,-3) \simeq (1,0,1)$

$\boxed{x + z + 2 = 0}$

4) $\overline{u_r} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 3 & -1 & 2 \\ 1 & 4 & 1 \end{vmatrix} = (-9,-1,13)$

$\overline{n_\pi} = (2,-5,a)$

$\overline{u_r} \cdot \overline{n_\pi} = -18+5+13a; \ -13+13a=0$

i) si $a = 1 \Rightarrow r \parallel \pi$
si $a \neq 1 \Rightarrow r \not\parallel \pi$
b cualquiera.

ii) $a = 1$

iii) $a=1$; $\pi: 2x-5y+z+2=0$

$Pr: \ x=0 \ \begin{cases} -y+2z=1 \\ 4y+z=b \end{cases}$

$9z = 4+b$
$z = \frac{4+b}{9}$

$y = \frac{8+2b}{9} - 1 = \frac{2b-1}{9}$

$(0, \frac{b+4}{9}, \frac{2b-1}{9})$

$\frac{-5(b+4)}{9} + \frac{2b-1}{9} + 2 = 0; \ -5b-20+2b-1+18=0$

$-3b-3=0$; $\boxed{\begin{matrix} b=-1 \\ a=1 \end{matrix}}$

1) a) $f(x) = \begin{cases} e^x(x^2+ax) & si\ x \leq 0 \\ \dfrac{bx^2+c}{x+1} & si\ x > 0 \end{cases}$

$$\left.\begin{array}{l} \lim\limits_{x\to 0^-} f(x) = 0 \\ \lim\limits_{x\to 0^+} f(x) = c \\ f(0) = 0 \end{array}\right\} \boxed{c=0}$$

$$f'(x) = \begin{cases} \underbrace{e^x(x^2+ax)+(2x+a)e^x}_{} \\ \boxed{e^x(x^2+(a+2)x+a)} & si\ x \leq 0 \\ \dfrac{2bx(x+1)-(bx^2)}{(x+1)^2} = \boxed{\dfrac{bx^2+2bx}{(x+1)^2}} & si\ x > 0 \end{cases}$$

$$\left.\begin{array}{l} \lim\limits_{x\to 0^-} f'(x) = a \\ \lim\limits_{x\to 0^+} f'(x) = 0 \end{array}\right\} \boxed{a=0}$$

$$m = \frac{b.1^2+2b.1}{(1+1)^2} = \frac{3b}{4} = 3; \quad 3b = 12; \quad \boxed{b=4}$$

b)

$$f'(x) = 2\cdot\left(\frac{\cancel{2}(\sqrt{x}+2)\cdot\frac{1}{\cancel{2}\sqrt{x}}(\sqrt{x}+1)-(\sqrt{x}+2)^2\cdot\frac{1}{2\sqrt{x}}}{(\sqrt{x}+1)^2}\right) =$$

$$= \frac{\cancel{2}}{\sqrt{x}}\cdot\left(\frac{(\sqrt{x}+2)(2(\sqrt{x}+1)-(\sqrt{x}+2))}{\cancel{2}(\sqrt{x}+1)^2}\right) =$$

$$= \frac{1}{\cancel{\sqrt{x}}}\left(\frac{(\sqrt{x}+2)(\cancel{\sqrt{x}})}{(\sqrt{x}+1)^2}\right) = \frac{\sqrt{x}+2}{(\sqrt{x}+1)^2}$$

2) a) i) $L = \lim_{x \to +\infty} \left(\cos\left(\frac{1}{x} \right) \right)^x$

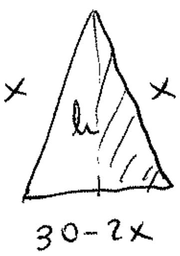
$\ln L = \lim_{x \to +\infty} x \cdot \ln \cos\left(\frac{1}{x} \right) =$

$= \lim_{x \to +\infty} \frac{\ln \cos\left(\frac{1}{x} \right)}{\frac{1}{x}} = \lim_{x \to \infty} \frac{\frac{1}{\cos\left(\frac{1}{x}\right)} \left(-\operatorname{sen} \frac{1}{x} \right)\left(-\frac{1}{x^2} \right)}{\left(-1/x^2 \right)} =$

$= \lim_{x \to \infty} \frac{-\operatorname{sen} \frac{1}{x}}{\cos \frac{1}{x}} = \frac{0}{1} = 0 ; \quad \underline{\underline{L = e^0 = 1}}$

ii) $\lim_{x \to 1} \frac{x \ln x - x + 1}{(x-1) \ln x} = \lim_{x \to 1} \frac{\ln x + \cancel{1} - \cancel{1}}{\ln x + \frac{x-1}{x}} =$

$= \lim_{x \to 1} \frac{x \ln x}{x \ln x + x - 1} = \lim_{x \to 1} \frac{\ln x + 1}{\ln x + 1 + 1} = \frac{1}{2}$ //

b) 

$A = b \cdot h = ((30-2x) \cdot h)/2 = (15-x) \cdot h$

$h^2 + (15-x)^2 = x^2 ; \quad h^2 + 225 + \cancel{x^2} - 30x = \cancel{x^2}$

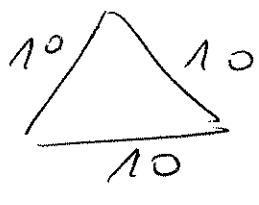
$h^2 = 30x - 225 ; \quad h = \sqrt{30x - 225}$

$A = (15-x) \cdot \sqrt{30x - 225} \longrightarrow$ máximo

$A' = -\sqrt{30x-225} + \frac{(15-x) \cdot 30}{2\sqrt{30x-225}} = \frac{-2(30x-225) + (15-x) \cdot 30}{2\sqrt{30x-225}} =$

$= \frac{-60x + 450 + 450 - 30x}{2\sqrt{30x-225}} = \frac{-90x + 900}{2\sqrt{30x-225}} = 0$

$-90x + 900 = 0 ; \quad x = \frac{900}{90} = 10$ //



3) 1º) Dominio $\mathbb{R} - \{1\}$

2º) Corte ejes: $(0,0)$

3º) A.H: $\lim_{x \to \pm\infty} f(x) = \begin{cases} +\infty \\ -\infty \end{cases}$ No tiene

A.V: $\boxed{x = 1}$ $\lim_{x \to 1^-} f(x) = +\infty$; $\lim_{x \to 1^+} f(x) = +\infty$

A.O: $m = \lim_{x \to \infty} \frac{x^3}{x^3 - 2x^2 + x} = 1$

$$n = \lim_{x \to \infty} \frac{x^3}{x^2 - 2x + 1} - 1 \cdot x = \lim_{x \to \infty} \frac{x^3 - x^3 + 2x^2 - x}{x^2 - 2x + 1} =$$

$$= \lim_{x \to \infty} \frac{2x^2 - x}{x^2 - 2x + 1} = 2 \qquad \boxed{y = x + 2}$$

4º) Extr. relat.
$$y' = \frac{3x^2(x^2 - 2x + 1) - (2x - 2) \cdot x^3}{(x^2 - 2x + 1)^2} =$$

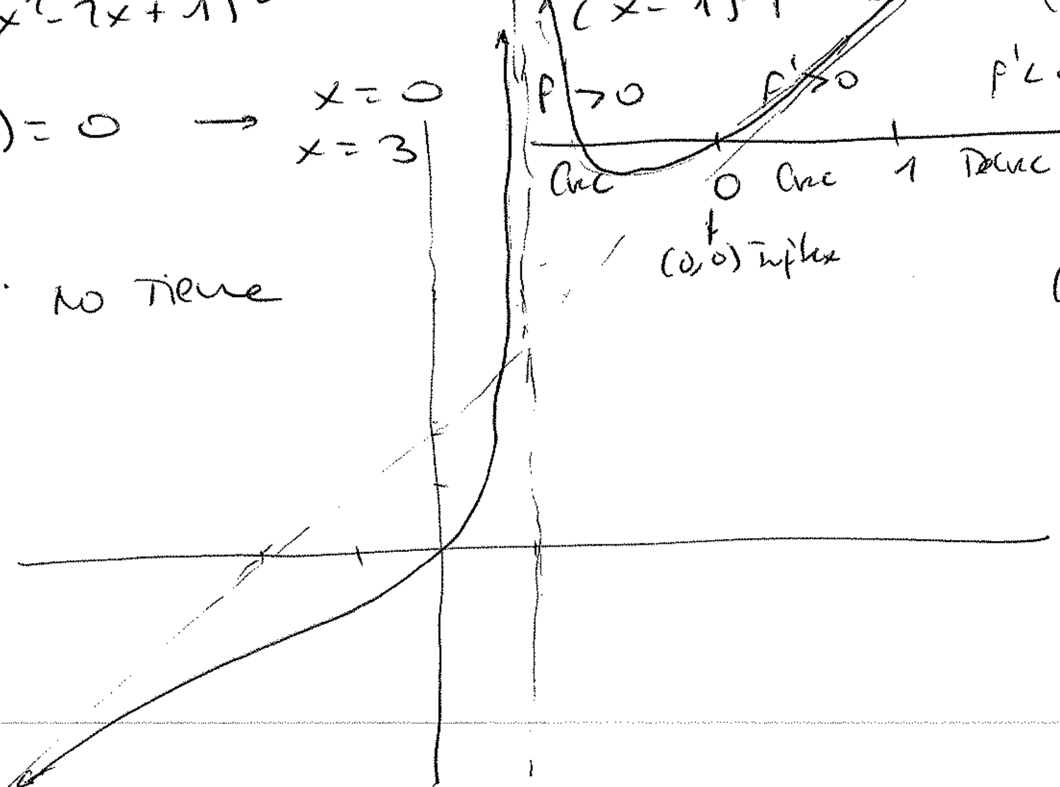
$$= \frac{3x^4 - 6x^3 + 3x^2 - 2x^4 + 2x^3}{(x^2 - 2x + 1)^2} = \frac{x^4 - 4x^3 + 3x^2}{(x^2 - 2x + 1)^2} =$$

$$= \frac{x^2(x^2 - 4x + 3)}{(x^2 - 2x + 1)^2} = \frac{x^2\cancel{(x-1)}(x-3)}{(x-1)^{\cancel{4}}} = \frac{x^2(x-3)}{(x-1)^3}$$

$x^2(x-3) = 0 \rightarrow x = 0$, $x = 3$

$f' > 0$		$f' > 0$		$f' < 0$		$f' > 0$
Crec	0	Crec	1	Decrec	3	Crec
	$(0,0)$ Inflex				$(3, \frac{27}{4})$ Mínimo	

5º) Inflex: No tiene



4) a) $I = \int e^{2x} \operatorname{sen} x \, dx$ $u = e^{2x}$ $v = -\cos x$

$du = 2e^{2x}dx$ $dv = \operatorname{sen} x \, dx$

$I = -e^{2x}\cos x + 2\underbrace{\int e^{2x}\cos x \, dx}_{I_1}$ $u = e^{2x}$ $v = \operatorname{sen} x$

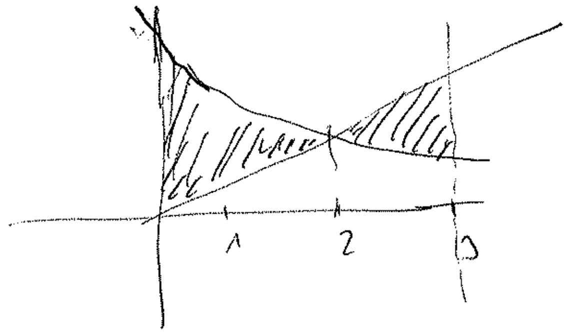
$du = 2e^{2x}dx$ $dv = \cos x \, dx$

$I_1 = e^{2x}\operatorname{sen} x - 2\underbrace{\int e^{2x}\operatorname{sen} x \, dx}_{I}$

$I = -e^{2x}\cos x + 2\left(e^{2x}\operatorname{sen} x - 2I\right) = -e^{2x}\cos x + 2e^{2x}\operatorname{sen} x - 4I$

$5I = e^{2x}(2\operatorname{sen} x - \cos x)$; $\boxed{I = \frac{e^{2x}(2\operatorname{sen} x - \cos x)}{5}}$

b)



$\frac{x+4}{x+1} = x$; $x+4 = x^2 + x$

$x^2 = 4$; $x = \pm 2$

$D(x) = \frac{x+4}{x+1} - x = \frac{x+4-x^2-x}{x+1} = \frac{-x^2+4}{x+1}$

D>0   D<0

0   2   3

$$\begin{array}{r|l} -x^2+0x+4 & x+1 \\ \hline x^2+x & -x+1 \\ \hline x+4 & \\ -x-1 & \\ \hline 3 & \end{array}$$

$-x^2+4 = (x+1)(-x+1)+3$

$\frac{-x^2+4}{x+1} = -x+1+\frac{3}{x+1}$

$\int \frac{-x^2+4}{x+1} = \int(-x+1)\,dx + \int \frac{3}{x+1}\,dx = -\frac{x^2}{2} + x + 3\ln(x+1)$

$A = \left[-\frac{x^2}{2} + x + 3\ln(x+1)\right]_0^2 - \left[-\frac{x^2}{2} + x + 3\ln(x+1)\right]_2^3 =$

$= \left[(-\cancel{2}+\cancel{2}+3\ln 3) - 0\right] - \left[\left(-\frac{9}{2} + 3 + 3\ln 4\right) - \left(-\cancel{2}+\cancel{2}+3\ln 3\right)\right] = \left(6\ln 3 - 3\ln 4 + \frac{3}{2}\right)$

EXAMEN SEPTIEMBRE

1) $$|A| = \begin{vmatrix} m & -2 & 2 \\ -m & 2 & 1-m \\ 0 & 3 & m \end{vmatrix} = \cancel{2m^2} - 6m - 3m(1-m) \cancel{+ 2m^2} = 3m^2 - 9m = 3m(m-3)$$

$$3m(m-3) = 0 < \begin{matrix} m = 0 \\ m = 3 \end{matrix} \quad \exists A^{-1} \text{ si } m \neq 0, 3$$

m = 0

$$\left(\begin{array}{ccc|c} 0 & -2 & 2 & 0 \\ 0 & 2 & 1 & 0 \\ 0 & 3 & 0 & 2 \end{array}\right) \sim \left(\begin{array}{ccc|c} 0 & -2 & 2 & 0 \\ 0 & 0 & 3 & 0 \\ 0 & 0 & 6 & 4 \end{array}\right) \sim \left(\begin{array}{ccc|c} 0 & -2 & 2 & 0 \\ 0 & 0 & 3 & 0 \\ 0 & 0 & 0 & 4 \end{array}\right)$$

S.I

m = 3

$$\left(\begin{array}{ccc|c} 3 & -2 & 2 & 0 \\ -3 & 2 & -2 & 0 \\ 0 & 3 & 3 & 2 \end{array}\right) \sim \left(\begin{array}{ccc|c} 3 & -2 & 2 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 3 & 3 & 2 \end{array}\right)$$

$$\left.\begin{matrix} 3x - 2y + 2z = 0 \\ 3y + 3z = 2 \end{matrix}\right\} \rightarrow y = \frac{2-3z}{3}$$

$$3x - \frac{4-6z}{3} + 2z = 0 \quad 9x - 4 + 6z + 6z = 0$$

$$9x = 4 - 12z; \quad x = \frac{4-12z}{9} =$$

$$\text{S.C.I} \longrightarrow \left(\frac{4-12\lambda}{9}, \frac{2-3\lambda}{3}, \lambda\right) \lambda \in \mathbb{R}$$

Si $m \neq 0, 3 \longrightarrow$ S.C.D $\quad -4(1-m) - 8 = 4m - 12 = 4(m-3)$

$$x = \frac{\begin{vmatrix} 0 & -2 & 2 \\ 0 & 2 & 1-m \\ 2 & 3 & m \end{vmatrix}}{3m(m-3)} = \frac{4(m-\cancel{3})}{3m(m-\cancel{3})} = \frac{4}{3m}$$

$$y = \frac{\begin{vmatrix} m & 0 & 2 \\ -m & 0 & 1-m \\ 0 & 2 & m \end{vmatrix}}{3m(m-3)} = \frac{2\cancel{m}(m-3)}{3\cancel{m}(m-3)} = \frac{2}{3}$$

$-4m-2m(1-m) = 2m^2-6m$

$$z = \frac{\begin{vmatrix} m & -2 & 0 \\ -m & 2 & 0 \\ 0 & 3 & 2 \end{vmatrix}}{3m(m-3)} = \frac{0}{3m(m-3)} = 0$$

$4m-4m$

2) a) $P_r(1,-2,0)$ $\quad P_s(0,-1,2)$

$\overline{P_rP_s}(-1,1,2)$; $\overline{u_r}(k, 6+k, 3)$; $\overline{u_s}(3,1,-1)$

$$\begin{vmatrix} -1 & 1 & 2 \\ k & 6+k & 3 \\ 3 & 1 & -1 \end{vmatrix} = 6+k+2k+9-6(6+k)+3+k = 6+k+2k+9-36-6k+3+k =$$

$= -2k-18$; $\quad -2k-18=0$; $\boxed{k=-9}$

Si $k \neq -9 \Rightarrow$ Se cruzan

Si $k=-9 \Rightarrow u_r(-9,-3,3)$ $\quad u_s(3,1,-1)$

¿$\frac{0-1}{-9} = \frac{-1+2}{-3} = \frac{2}{3}$? $\rightarrow$ ¿$\frac{1}{9} = -\frac{1}{3} = \frac{2}{3}$? NO

Si $k=-9 \Rightarrow$ son paralelas no coincidentes

b) Si $k=-3 \Rightarrow \overline{u_r}(-3,3,3)$ $\quad \overline{u_s}(3,1,-1)$

$$\overline{u_r} \wedge \overline{u_s} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ -3 & 3 & 3 \\ 3 & 1 & -1 \end{vmatrix} = (-6, 6, -12)$$

$$d(r,s) = \frac{|-2(-3)-18|}{\sqrt{(-6)^2+6^2+(-12)^2}} = \frac{12}{\sqrt{216}} \ u$$

②

c) $P(1,-2,0)$ $P_s(0,-1,2)$ $\vec{u}_s(3,1,-1)$

$$\overrightarrow{PP_s} = (-1, 1, 2)$$

$$\vec{u}_s \wedge \overrightarrow{PP_s} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 3 & 1 & -1 \\ -1 & 1 & 2 \end{vmatrix} = (3, -5, 4)$$

$$d(P, s) = \frac{\sqrt{3^2+(-5)^2+4^2}}{\sqrt{3^2+1^2+(-1)^2}} = \frac{\sqrt{50}}{\sqrt{11}} = \underline{\underline{\sqrt{\frac{50}{11}}\ u}}$$

3) $f(x) = \dfrac{x^2-3}{e^x}$

a) $\underline{\underline{H:}}$ $\lim\limits_{x\to+\infty} f(x) = 0$; $\lim\limits_{x\to-\infty} f(x) = +\infty$

$\boxed{y=0}$ AH por la dcha

$\underline{\underline{V:}}$ $e^x = 0 \to \nexists$ sol $\underline{\underline{\text{No tiene}}}$

$\underline{\underline{O:}}$ $\lim\limits_{x\to\pm\infty} \dfrac{x^2-3}{x\,e^x} = \begin{cases} 0 \\ -\infty \end{cases}$ $\underline{\underline{\text{No tiene}}}$

b) $$f'(x) = \frac{2x\,e^x - e^x(x^2-3)}{e^{2x}} = \frac{e^x(-x^2+2x+3)}{e^{2x}} =$$

$$= \frac{-x^2+2x+3}{e^x} = 0$$

$-x^2+2x+3=0$; $x^2-2x-3=0 \begin{cases} x=-1 \\ x=3 \end{cases}$

$f'<0$ Decrec		$f'>0$ Crec		$f'<0$ Decrec
	-1		3	

$(-1, -2e) \to$ Mínimo $\left(3, \dfrac{6}{e^3}\right) \to$ Máximo

c) $f''(x) = \dfrac{(-2x+2)e^x - e^x(-x^2+2x+3)}{e^{2x}} =$

$= \dfrac{e^x(-2x+2+x^2-2x-3)}{e^{2x}} = \dfrac{x^2-4x-1}{e^x}$

$x^2-4x-1=0;\quad x = \dfrac{4 \pm \sqrt{16+4}}{2} = \dfrac{4 \pm \sqrt{20}}{2} \approx \begin{cases} 4'23 \\ -0'23 \end{cases}$

$f''>0$		$f''<0$		$f''>0$
Cóncava	$-0'23$ ↓ Inflex	Convexa	$4'23$ ↓ Inflex	Cóncava

d) $f(-x) = \dfrac{(-x)^2-3}{e^{-x}} = \dfrac{x^2-3}{e^{-x}} \begin{cases} \neq f(x) \\ \neq -f(x) \end{cases}$

No tiene simetrías

$OX:\ y=0;\quad x^2-3=0 \Rightarrow x=\pm\sqrt{3} \begin{cases} \rightarrow (\sqrt{3}, 0) \\ \rightarrow (-\sqrt{3}, 0) \end{cases}$

$OY:\ x=0;\quad y = \dfrac{-3}{1} = -3 \longrightarrow (0, -3)$

4) a) $\ln x = t \Rightarrow \frac{1}{x}dx = dt$

$$\int \frac{t-1}{t^2-3t+2}\,dt \qquad t^2-3t+2=0 \begin{cases}1\\2\end{cases}$$

$$\frac{t-1}{t^2-3t+2} = \frac{A}{t-1} + \frac{B}{t-2} = \frac{A(t-2)+B(t-1)}{(t-1)(t-2)}$$

$t=1 \Rightarrow 0 = -A \Rightarrow A=0$

$t=2 \Rightarrow 1 = B$

$$\int \frac{t-1}{t^2-3t+2}\,dt = \int \frac{1}{t-2} = \ln|t-2| =$$

$$= \Big[\ln|\ln x - 2|\Big]_1^4 = \ln|\ln 4 - 2| - \ln|\ln 1 - 2| =$$

$$= \ln|\ln 4 - 2| - \ln|-2| \;//$$

$$= -0'48 - 0'69 = -1'17 \;//$$

b) $L = \lim_{x\to0}\left(\frac{\operatorname{sen} x}{x}\right)^{1/x^2}$; $\ln L = \lim_{x\to0} \frac{\ln\left(\frac{\operatorname{sen} x}{x}\right)}{x^2}$

$$\ln L = \lim_{x\to0} \frac{\frac{x}{\operatorname{sen} x}\cdot\frac{x\cos x - \operatorname{sen} x}{x^2}}{2x} =$$

$$= \lim_{x\to0} \frac{x\cos x - \operatorname{sen} x}{2x^2 \operatorname{sen} x} = \lim_{x\to0} \frac{\cancel{\cos x} - x\operatorname{sen} x - \cancel{\cos x}}{2(2x\operatorname{sen} x + x^2\cos x)}$$

$$= \lim_{x\to0} \frac{-\operatorname{sen} x}{2(2\operatorname{sen} x + x\cos x)} = \lim_{x\to0} \frac{-\cos x}{2(2\cos x + \cos x - x\operatorname{sen} x)} = -\frac{1}{6}$$

$$\boxed{L = e^{-1/6}}$$