

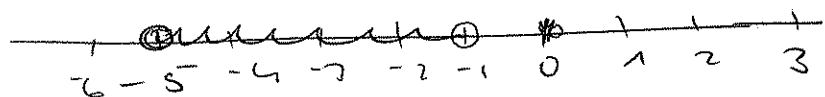
1) a) TEÓRICO

b) TEÓRICO

c) $A = (-5, -1)$

$B = (-5, 1)$

$A \cap B = (-5, -1)$



2) a) $(27 + \cancel{2} - 6\sqrt{6}) - (8 + 3 - 4\sqrt{6}) + 2\sqrt{6} - 3\sqrt{6} =$
 $= \overbrace{27} + \overbrace{\cancel{2}} - \underbrace{6\sqrt{6}} - \overbrace{8} - \overbrace{3} + \underbrace{4\sqrt{6}} + \underbrace{2\sqrt{6}} - \underbrace{3\sqrt{6}} = \underline{\underline{18 - 3\sqrt{6}}}$

b) $\frac{r}{r-2} = \frac{r(r+2)}{(r-2)(r+2)} = \frac{5+2r}{1}$

$\frac{3}{r+2} = \frac{3}{2r+3} = \frac{3}{5r} = \frac{3r}{2r}$

$\frac{5+2r}{1} + \frac{3r}{2r} = \frac{12r+50r+3r}{2r} = \frac{12r+53r}{2r} //$

3) a)

	2	3	-m	3
$\sqrt{3}$		$2\sqrt{3}$	$3\sqrt{3}+6$	$9+6\sqrt{3}-m\sqrt{3}$
	2	$3+2\sqrt{3}$	$3\sqrt{3}+6-m$	$12+6\sqrt{3}-m\sqrt{3}$

$12+6\sqrt{3}-m\sqrt{3} = 0 \Rightarrow \boxed{m = \frac{12+6\sqrt{3}}{\sqrt{3}}}$ ~~$\frac{12\sqrt{3}+18}{3}$~~ $\boxed{4\sqrt{3}+6}$

	2	3	-m	3
-1/2		-1	-1	$\frac{m}{2} + \frac{1}{2}$
	2	2	$-m-1$	$\frac{m}{2} + \frac{7}{2}$

$\frac{m}{2} + \frac{7}{2} = 0 \Rightarrow \boxed{m = -7}$

$$\begin{array}{r|rrrrrr}
 4) & 3 & -20 & 42 & -27 & 4 & -12 \\
 & & 6 & -28 & 28 & 2 & 12 \\
 \hline
 & 3 & -14 & 14 & 1 & 6 & \underline{0} \\
 & & 6 & -16 & -4 & -6 & \\
 \hline
 & 3 & -8 & -2 & -3 & \underline{0} & \\
 & & 9 & 3 & 3 & & \\
 \hline
 & 3 & 1 & 1 & \underline{0} & &
 \end{array}$$

$$3x^2 + x + 1 = 0 \rightarrow x = \frac{-1 \pm \sqrt{1-12}}{6} = \frac{-1 \pm \sqrt{-11}}{6}$$

$$P(x) = (x-2)^2 (x-3) (3x^2 + x + 1)$$

$$5) \frac{2(x+2)(x-2)}{(x-2)(x-4)} : \frac{x(x-1)(x-4)}{x^2(x-2)} =$$

$$= \frac{2(x+2)\cancel{(x-2)} \cancel{x^2} (x-2)}{(x-\cancel{2})(x-4) \cancel{x} (x-1)\cancel{(x-4)}} =$$

$$= \boxed{\frac{2x(x+2)(x-2)}{(x-4)^2(x-1)}}$$

①

$$1) a) \frac{4\sqrt{2} - \sqrt{2}}{2\sqrt{3} - 3} - \frac{2\sqrt{2}}{2\sqrt{3}} = \frac{3\sqrt{2}}{2\sqrt{3} - 3} - \frac{\sqrt{2}}{\sqrt{3}} =$$

$$= \frac{3\sqrt{2}(2\sqrt{3} + 3)}{(2\sqrt{3} - 3)(2\sqrt{3} + 3)} - \frac{\sqrt{2}\sqrt{3}}{\sqrt{3}\sqrt{3}} = \frac{3\sqrt{2}(2\sqrt{3} + 3)}{3} - \frac{\sqrt{2}\sqrt{3}}{3} =$$

$$= \frac{6\sqrt{6} + 9\sqrt{2} - \sqrt{6}}{3} = \frac{5\sqrt{6} + 9\sqrt{2}}{3} //$$

b)

	1	-1	0	-4
2		2	2	4
	1	1	2	0

 $(x-2)(x^2+x+2)$

	1	2	-3	-4	-12
2		2	8	10	12
	1	4	5	6	0
-3		-3	-3	-6	
	1	1	2	0	

 $(x-2)(x+3)(x^2+x+2)$

$$\frac{(x-2)(x^2+x+2)}{(x-2)(x+3)(x^2+x+2)} = \frac{1}{x+3} //$$

2) a) i) TEÓRICO

ii) $P(-1) = (-1)^{40} - 3(-1)^{17} - 2 = 1 + 3 - 2 = 2 //$

b) i)

	4	24	31	m	-8
-1/2		-2	-11	-10	$-\frac{m}{2} + 5$
	4	22	20	m-10	$-\frac{m}{2} - 3$

$$-\frac{m}{2} - 3 = 0, \quad -m - 6 = 0, \quad \boxed{m = -6}$$

$$\begin{array}{r|rrrrr}
 & 4 & 24 & 31 & -6 & -8 \\
 -1/2 & & -2 & -11 & -10 & 8 \\
 \hline
 & 4 & 22 & 20 & -16 & 0 \\
 -4 & & -16 & -24 & 16 & \\
 \hline
 & 4 & 6 & -4 & 0 &
 \end{array}$$

$$4x^2 + 6x - 4 = 0; \quad x = \frac{-6 \pm \sqrt{36 + 64}}{8} = \frac{-6 \pm 10}{8} = \begin{cases} 1/2 \\ -2 \end{cases}$$

3) a) $\overbrace{x^4 - 3x^2 + 2x^2 - 6} + \overbrace{x^4 + x^2} = \overbrace{4x^2 + 16 - 6}$

$$2x^4 - 4x^2 - 16 = 0; \quad x^4 - 2x^2 - 8 = 0$$

$$x^2 = t; \quad t^2 - 2t - 8 = 0; \quad t = \frac{2 \pm \sqrt{4 + 32}}{2} = \frac{2 \pm 6}{2} = \begin{cases} 4 \\ -2 \end{cases}$$

$$x = \pm \sqrt{4} \rightarrow \begin{cases} 2 \\ -2 \end{cases} \quad x = \pm \sqrt{-2} \rightarrow \text{no sol}$$

b) $\sqrt{5 - 2x} = 2x - 3;$ $\overline{5 - 2x} = \overline{4x^2 + 9 - 12x}$

$$0 = 4x^2 - 10x + 4; \quad 2x^2 - 5x + 2 = 0$$

$$x = \frac{5 \pm \sqrt{25 - 16}}{4} = \frac{5 \pm 3}{4} = \begin{cases} 2 \rightarrow \text{si} \\ 1/2 \rightarrow \text{no} \end{cases}$$

4) Películas $\rightarrow x$ (45€)
 Discos $\rightarrow y$ (72€)
 Libros $\rightarrow z$ (36€)

$$\begin{cases} 45x + 72y + 36z = 6300 \\ x = \frac{x+y+z}{2} \\ 72y = 45x + 180 \end{cases}$$

(2)

$$\begin{cases} 45x + 72y + 36z = 6300 \\ x - y - z = 0 \\ -45x + 72y = 180 \end{cases} \rightarrow \begin{pmatrix} 45 & 72 & 36 & | & 6300 \\ 1 & -1 & -1 & | & 0 \\ -45 & 72 & 0 & | & 180 \end{pmatrix}$$

$$\sim \begin{pmatrix} 45 & 72 & 36 & | & 6300 \\ 0 & 117 & 81 & | & 6300 \\ 0 & 144 & 36 & | & 6480 \end{pmatrix} \sim \begin{pmatrix} 45 & 72 & 36 & | & 6300 \\ 0 & 117 & 81 & | & 6300 \\ 0 & 0 & 7452 & | & 149040 \end{pmatrix}$$

$$\begin{cases} 45x + 72y + 36z = 6300 \\ 117y + 81z = 6300 \\ 7452z = 149040 \end{cases} \rightarrow \begin{cases} 45x + 2880 + 720 = 6300; x = 60 \\ 117y = 6300 - 1620 \Rightarrow y = \frac{4680}{117} = 40 \\ z = \frac{149040}{7452} = 20 \end{cases}$$

(60, 40, 20)

5) $\frac{x-6}{x-4} - 3 \leq 0; \frac{x-6}{x-4} - \frac{3x-12}{x-4} \leq 0$

$$\frac{x-6-3x+12}{x-4} \leq 0; \boxed{\frac{-2x+6}{x-4} \leq 0}$$

$-2x+6=0 \rightarrow x=3 //$

$x-4=0 \rightarrow x=4 //$

$$\begin{array}{c} \text{Sí} \quad \quad \quad \text{No} \quad \quad \quad \text{Sí} \\ \hline \quad \quad \quad 3 \quad \quad \quad 4 \\ S_1: (-\infty, 3] \cup (4, +\infty) \end{array}$$

$x^2+4-4x+2x-2-x > 0; x^2-3x+2 > 0$

$x^2-3x+2=0 \rightarrow \begin{matrix} 1 \\ 2 \end{matrix}$

$$\begin{array}{c} \text{Sí} \quad \quad \quad \text{No} \quad \quad \quad \text{Sí} \\ \hline \quad \quad \quad 1 \quad \quad \quad 2 \\ S_2 = (-\infty, 1) \cup (2, +\infty) \end{array}$$

$S = (-\infty, 1) \cup (2, 3] \cup (4, +\infty)$

$$1) \quad a) \quad \frac{(2-\sqrt{3})(\sqrt{3}-1)}{(\sqrt{3}+1)(\sqrt{3}-1)} - \frac{1\sqrt{3}}{2\sqrt{3}\sqrt{3}} = \frac{2\sqrt{3}-2-3+\sqrt{3}}{2} - \frac{\sqrt{3}}{6} =$$

$$= \frac{3\sqrt{3}-5}{2} - \frac{\sqrt{3}}{6} = \frac{9\sqrt{3}-15-\sqrt{3}}{6} = \frac{8\sqrt{3}-15}{6} //$$

$$b) \quad \frac{2(x^2+2x-3)}{(x-2)} \cdot \frac{x^2-4x+4}{x^2-9} = \frac{2(x-1)(x+3)}{(x-2)} \cdot \frac{(x-2)^2}{(x+3)(x-3)}$$

$$= \frac{2(x-1)(x+3)(x-2)^2}{(x-2)(x+3)(x-3)} = \frac{2(x-1)(x-2)}{(x-3)} //$$

$$2) \quad \begin{array}{r|rrrrrr} & -1 & 0 & m & 0 & 1 & \\ -2 & & 2 & -4 & -2m+8 & 4m-16 & \\ \hline & -1 & 2 & m-4 & -2m+8 & 4m-16 & \end{array}$$

$$4m-15=1$$

$$4m=16$$

$$m=\frac{16}{4}=4 //$$

$$-x^4 + 4x^2 + 1 = 0, \quad x^4 - 4x^2 - 1 = 0 \quad x^2 = t$$

$$t^2 - 4t - 1 = 0; \quad t = \frac{4 \pm \sqrt{16+4}}{2} = \frac{4 \pm \sqrt{20}}{2} = \frac{4 \pm 2\sqrt{5}}{2} = 2 \pm \sqrt{5}$$

No se tiene $t = \pm \sqrt{2+\sqrt{5}} \quad t = \pm \sqrt{2-\sqrt{5}} \rightarrow \text{No se tiene}$

$$3) \quad a) \quad \widehat{x^3 - 5x^2} = x^2 + 1 + 2x - x^2 - 11$$

$$x^3 - 5x^2 + 2x + 10 = 0$$

$$\begin{array}{r|rrrr} 5 & 1 & -5 & -2 & 10 \\ & & 5 & 0 & -10 \\ \hline & 1 & 0 & -2 & 0 \end{array}$$

$$x^2 - 2 = 0$$

$$x^2 = 2$$

$$x = \pm \sqrt{2}$$

Sol: 5, \sqrt{2}, -\sqrt{2}

b) $\sqrt{2x-1} = x-2$; $2x-1 = x^2+4-4x$

$$0 = x^2 - 6x + 5 \quad x = \frac{6 \pm \sqrt{36-20}}{2} = \frac{6 \pm 4}{2} = \begin{matrix} 1 \rightarrow \underline{\underline{81}} \\ 1 \rightarrow \underline{\underline{100}} \end{matrix}$$

$$x: \Gamma \rightarrow \sqrt{9} + 4 = 5 + 2 \Rightarrow 8$$

$$x = 1 \rightarrow \sqrt{1+4} = 1 + 2 = 3$$

4) $H - x$
 $M - y$
 $N - z$

$$\begin{aligned} x + y + z &= 30 \\ x + y &= 2z \\ x + 3y &= 2z + 10 \end{aligned}$$

$$\begin{aligned}x + y + z &= 30 \\x + y - 2z &= 0 \\x + 3y - 2z &= 20\end{aligned}$$

$$\left(\begin{array}{ccc|c} 1 & 1 & 1 & 30 \\ 1 & 1 & -2 & 0 \\ 1 & 3 & -2 & 20 \end{array} \right)$$

$$\sim \begin{pmatrix} 1 & 1 & 1 \\ 0 & 0 & 3 \\ 0 & 2 & -3 \end{pmatrix} \begin{matrix} 30 \\ 30 \\ -10 \end{matrix}$$

$$\sim \left(\begin{array}{ccc|c} 1 & 2 & 1 & 30 \\ 0 & 2 & -3 & -6 \\ 0 & 0 & 3 & 30 \end{array} \right)$$

$$x + y + z = 30$$

$$27 - 37 = -10$$

$$37 = 10$$

4210

$$x = 10 \quad y = -10 + 20 = 10; \quad z = 10 \quad (\underline{10, 10, 10})$$

$$Z = \frac{30}{3} = 10$$

$$5) \quad \frac{5x+5}{10} - \frac{4-6x}{10} \leq \frac{1+4x}{10}; \quad \sqrt{5x+5} - \sqrt{4-6x} \leq \sqrt{1+4x}$$

$$5x + 6x - 4x \leq 1 - 5 + 4 \quad ; \quad 7x \leq 0 \quad \underline{\underline{x \leq 0}}$$

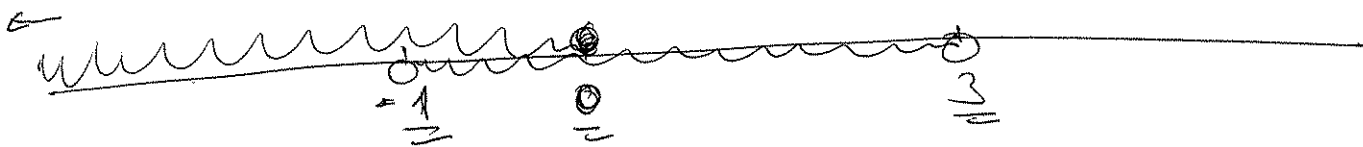
$$S_1 = (-\infty, 0]$$

$$x^2 - 7x - 3 \leq 0;$$

$$x^2 - 2x - 3 = 0 \Rightarrow \begin{matrix} -1 \\ 3 \end{matrix}$$

$\text{no} \quad | \quad \text{si} \quad | \quad \text{no}$

$$S_2 = (-1, 3)$$



$$S = S_1 \cap S_2 = (-1, 0] //$$

1) a) $x^2 + x = 0 \rightarrow x(x+1) = 0 \begin{cases} x=0 \\ x=-1 \end{cases}$

$D_f = \mathbb{R} \setminus \{0, -1\} //$

$D_g = \mathbb{R} //$

$\frac{x^3}{x^2-4} \geq 0$

NO	SÍ	NO	SÍ
	-2	0	2

$D_h = \text{~~unión~~} (-2, 0] \cup (2, +\infty) //$

b) $\frac{3x-6}{x^2+x} = 3$; $3x-6 = 3x^2+3x$

$0 = 3x^2+6 \rightarrow \text{No pertenece}$

$\frac{3x^2+1}{x^2+2} = 3$; $3x^2+1 = 3x^2+6 \rightarrow \text{No pertenece}$

c) $f(x) \rightarrow$ Corte en $0x \rightarrow (2, 0)$

AV: $x = -1$; $x = 0$

-	+	-	+
	-1	0	2

$+$: $(-1, 0) \cup (2, +\infty)$

$-$: $(-\infty, -1) \cup (0, 2)$

$g(x) \rightarrow + : (-\infty, +\infty)$

d) $f(-x) = \frac{3(-x)-6}{(-x)^2+(-x)} = \frac{-3x-6}{x^2-x} \rightarrow \text{NO TIENE}$

$g(-x) = \frac{3(-x)^2+1}{(-x)^2+2} = \frac{3x^2+1}{x^2+2} \rightarrow \text{PAR (Resalta en 04)}$

e) $f(x) \begin{cases} 0x: 3x-6=0 \rightarrow x=2 \text{ (2.º)} \\ 0y: \text{No Corte.} \end{cases}$

2) i) $f(x) = \begin{cases} -2x+1 & \text{si } x \leq -1 \\ -x^2+4 & \text{si } -1 < x \leq 2 \\ x^2-6x+8 & \text{si } x > 2 \end{cases}$

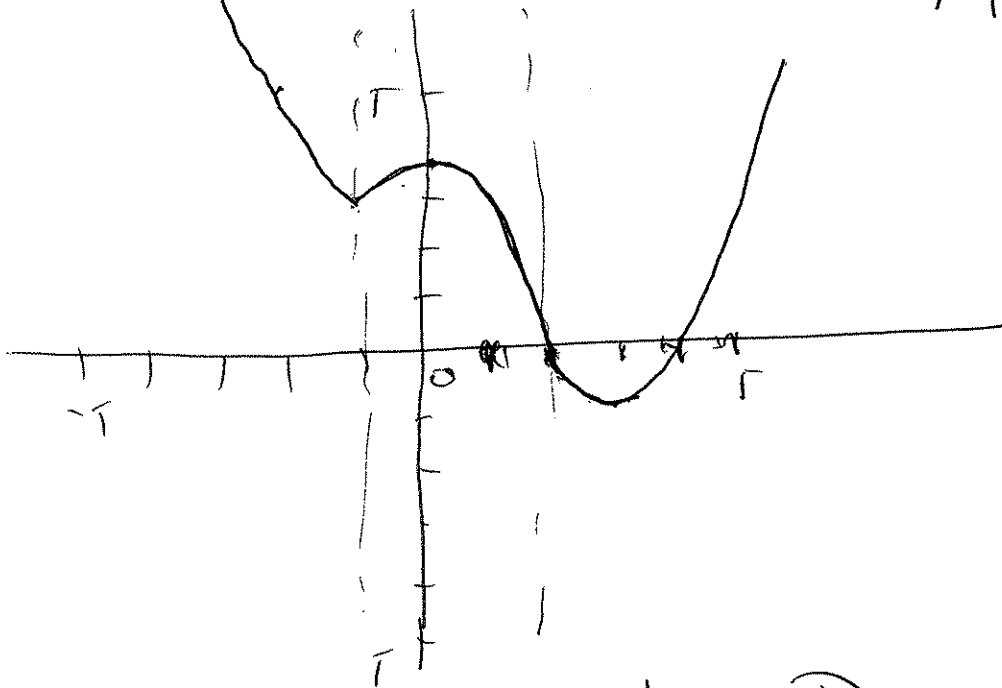
x	-2	-1
y	5	3

x	-1	2
y	3	0

 $V(0,4)$

x	2	3
y	0	-1

 $V(3,-1)$



ii) Continua en todo $\mathbb{R}$

iii) Creciente: $(-1, 0) \cup (3, +\infty)$

Decrecante: $(-\infty, -1) \cup (0, 3)$

iv) He 0X: $x^2-6x+8=0 \rightarrow x=2 \rightarrow (2,0)$
 $\rightarrow x=4 \rightarrow (4,0) //$

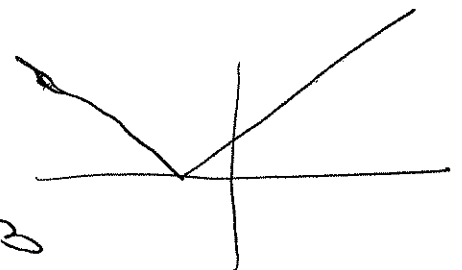
He 0Y: $-x^2+4=4 \rightarrow (0,4) //$

3) a) $-3x-5=0, -3x=5, x=-\frac{5}{3}$

+ -
 $\frac{-5/3}{-}$

~~He~~

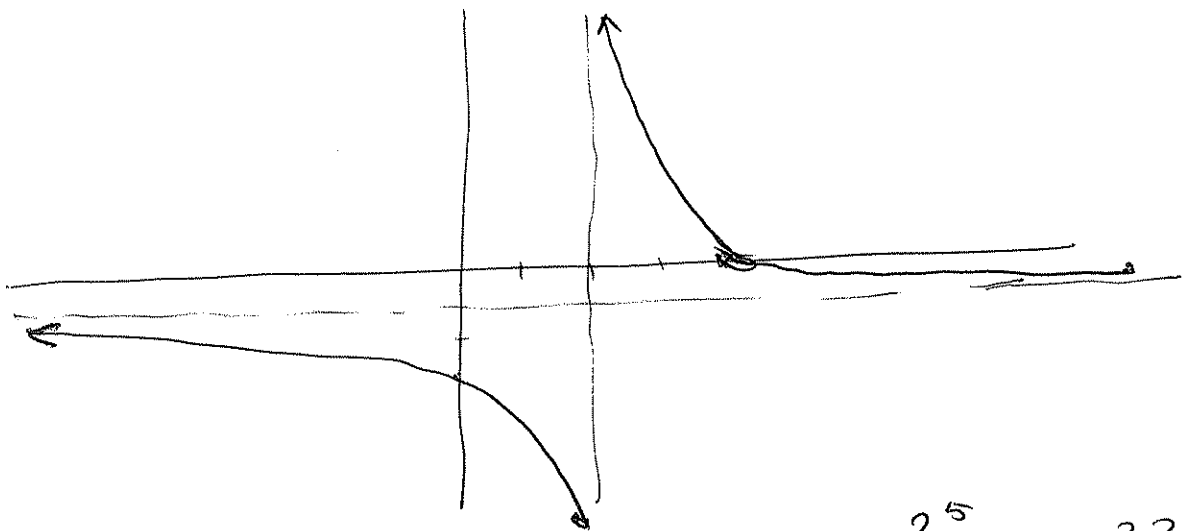
$f(x) = \begin{cases} -3x-5 & \text{si } x < -5/3 \\ 3x+5 & \text{si } x \geq -5/3 \end{cases}$



(2)

b) Corte eja $\begin{cases} OX: -2x+8=0 \rightarrow x=4 \rightarrow (4, 0) \\ OY: (0, -\frac{4}{3}) \end{cases}$

Asintotas $\begin{cases} H: y = -2/3 \\ V: 3x-6=0 \rightarrow x=2 \end{cases}$



c)

x	-2	-1
y	-32/3	-8

$$g(-2) = \frac{2^5}{-2-1} = -\frac{32}{3}$$

$$g(-1) = \frac{2^4}{-1-1} = -8$$

$$m = \frac{-8 + 32/3}{-1 + 2} = \frac{8}{3}$$

$$y + 8 = \frac{8}{3}(x + 1) ; y = \frac{8}{3}x + \frac{8}{3} - 8$$

$$y = \frac{8}{3}x - \frac{16}{3}$$

$$g(-1/5) = \frac{8(-1/5)}{3} - \frac{16}{3} = \frac{-12-16}{3} = -\frac{28}{3} //$$

1

1) a) $x^2 + x = 0 \rightarrow \begin{matrix} x=0 \\ x=-1 \end{matrix}$ $D_f = \mathbb{R} - \{0, -1\}$

$D_g = \mathbb{R} - \{0\}$

$x-3=0 \rightarrow x=3$ $x=0$ $\begin{matrix} + & - & + \\ | & | & | \\ 0 & & 3 \end{matrix}$ $D_h = (-\infty, 0) \cup [3, +\infty)$

b) $2x^2 - 8 = 0; 2x^2 = 8; x^2 = 4; x = \pm 2$

$x^2 + x = 0 \rightarrow x = -1, 0$

$\begin{matrix} + & - & + & - & + \\ | & | & | & | & | \\ -2 & -1 & 0 & 2 & \end{matrix}$

f(x)

$+: (-\infty, -2) \cup (-1, 0) \cup (2, +\infty)$

$-: (-2, -1) \cup (0, 2)$

g(x): $+: (0, +\infty)$
 $-: (-\infty, 0)$

c) $f(x) \begin{cases} 0x: (-2, 0), (2, 0) \\ 0y: \text{no curve} \end{cases}$

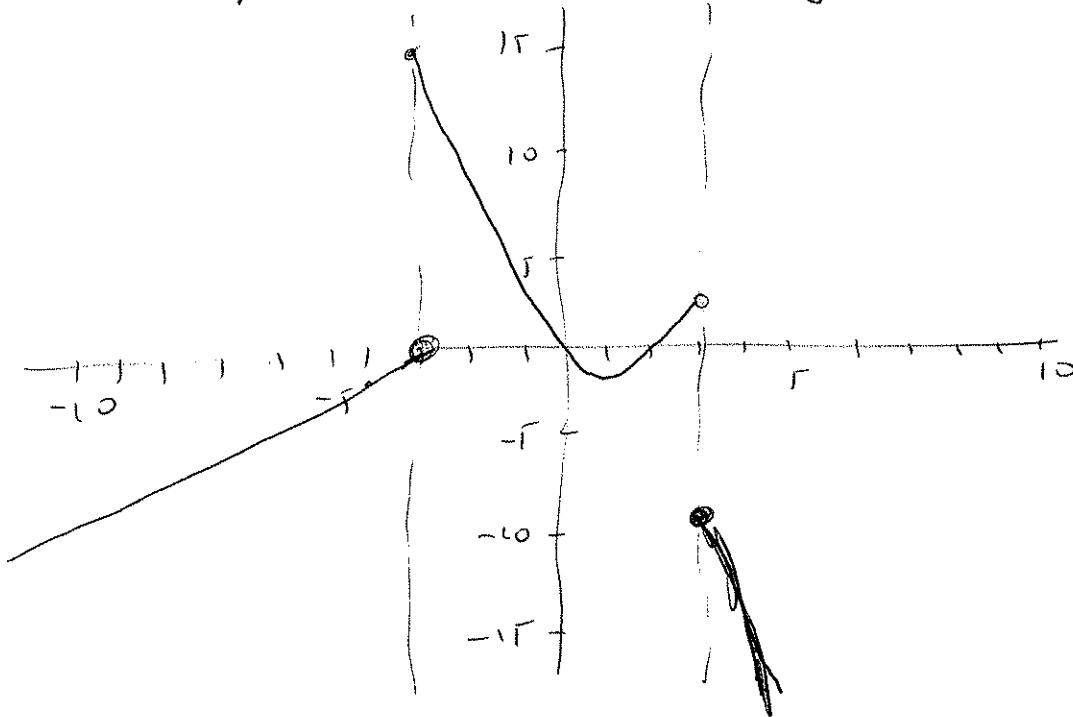
$g(x) \begin{cases} 0x: \text{no curve} \\ 0y: \text{no curve} \end{cases}$

$h(x) \begin{cases} 0x: (3, 0) \\ 0y: \text{no curve} \end{cases}$

d) $\frac{2x^2 - 8}{x^2 + x} = 2$ $2x^2 - 8 = 2x^2 + 2x; x = -4 \rightarrow \underline{\underline{(-4, 2)}}$
 $\sqrt[3]{\frac{1}{x}} = 2; \frac{1}{x} = 8; x = \frac{1}{8}; \underline{\underline{(\frac{1}{8}, 2)}}$

2) i) $f(x) = \begin{cases} 2x+6 & \text{si } x < -3 \\ x^2-2x & \text{si } -3 \leq x < 3 \\ -x^2 & \text{si } x \geq 3 \end{cases}$

$\begin{array}{r|rr} 1 & -4 & -3 \\ \hline 7 & -2 & 0 \end{array}$
 $\begin{array}{r|rr} x & -3 & 3 \\ \hline 7 & 15 & 3 \end{array}$ $N(1, -1)$
 $\begin{array}{r|rr} x & 3 & 4 \\ \hline 8 & -9 & 16 \end{array}$ $N(0, 0)$



ii) Creciente: $(-\infty, -3) \cup (1, 3)$

Decreciente: $(-3, 1) \cup (3, +\infty)$

iii) Eje Ox: $x^2-2x=0 \Rightarrow \frac{0}{2} \rightarrow (0,0)$
 $\frac{2}{2} \rightarrow (2,0)$
 Eje Oy: $(0,0)$

iv) Discontinua en $x = -3$ y $x = 3$

3) a) $2^x \cdot 2^3 - 3 \cdot \frac{2^x}{2^3} - 5 \cdot 2^x = 21$; $\boxed{2^x = t}$

$8t - \frac{3t}{8} - 5t = 21$; $64t - 3t - 40t = 168$

$21t = 168$; $t = \frac{168}{21} = 8$

$2^x = 8 \Rightarrow \underline{\underline{x = 3}}$

$$b) \frac{3^x}{3} - 2 \cdot \frac{3^{2x}}{3} - (3^2)^x = 30 \quad (2)$$

$$\boxed{3^x = t} \quad \frac{t}{3} - \frac{2t^2}{3} - t^2 = 30$$

$$t - 2t^2 - 3t^2 = 90; \quad 0 = 5t^2 - t + 90$$

$$t = \frac{1 \pm \sqrt{1 - 1800}}{2} \rightarrow \text{no sol}$$

$$c) \quad 10 - 2x > 0; \quad -2x > -10; \quad x < 5$$

$$D = (-\infty, 5)$$

$$\text{Cble eqs} \begin{cases} 0x: 10 - 2x = 1; x = \frac{9}{2} \\ 0y: (0, 1) \end{cases}$$



$$4) a) \quad i) (x^2 - 1)^{-1} = \frac{1}{8}; \quad x^2 - 1 = 8; \quad x^2 = 9; \quad \underline{\underline{x = \pm 3}}$$

$$ii) \quad 5^{\frac{x+1}{2}} = 10; \quad \frac{x+1}{2} = \frac{\log 10}{\log 5} = 1.43$$

$$x+1 = 2.86; \quad \underline{\underline{x = 1.86}}$$

$$iii) \quad 64^{1/3} = x^2 - 3x; \quad 4 = x^2 - 3x$$

$$0 = x^2 - 3x - 4; \quad x = \frac{3 \pm \sqrt{9+16}}{2} = \frac{3 \pm 5}{2}; \quad \begin{cases} 4 // \\ -1 // \end{cases}$$

$$b) \quad 2400 = 800(1+r)^{15}, \quad 3 = (1+r)^{15}$$

$$1+r = 1.075; \quad r = 0.075 \rightarrow \underline{\underline{r = 7.5\%}}$$

$$c) \quad 3 \log x - \frac{1}{2} \log x + \log x = 7$$

$$6 \log x - \log x + 2 \log x = 14$$

$$7 \log x = 14; \quad \log x = 2; \quad \underline{\underline{x = 100}}$$

①

1) a) ~~xxxxxxxxxx~~ $D_f = \mathbb{R} - \{0\}$
~~xxxxxxxxxx~~ $D_g = \mathbb{R} - \{5, -2\}$

$x^2 + 1 = 0 \rightarrow \text{No sol}$

$x^2 - 1 = 0 \Rightarrow x = \begin{matrix} -1 \\ 1 \end{matrix}$

$\begin{array}{c} + & - & + \\ | & | & | \\ -1 & & 1 \end{array} \quad D_h = (-\infty, -1) \cup (1, +\infty)$

b) $x - 1 = 0 \rightarrow x = 1 //$
 $x^2 = 0 \rightarrow x = 0 //$

$\begin{array}{c} - & - & + \\ | & | & | \\ 0 & & 1 \end{array}$

$x^2 = 0 \rightarrow x = 0$

$\begin{array}{c} - & + & + \\ | & | & | \\ -2 & & 0 \end{array}$

$x^3 + 8 = 0 \rightarrow x = -2$

$g(x) \begin{cases} 0x: (0, 0) \\ 0y: (0, 0) \end{cases}$

c) $f(x) \begin{cases} 0x: \rightarrow (1, 0) \\ 0y: \rightarrow \text{no curve} \end{cases}$

$h(x) \begin{cases} 0x: \text{no curve} \\ 0y: \text{no curve} \end{cases}$

d) $\frac{x-1}{x^2} = \frac{x^2}{x^3+8} \quad ; \quad (x-1)(x^3+8) = x^4$

$x^4 + 8x - x^3 - 8 = x^4 \quad ; \quad 0 = x^3 - 8x + 8$

$\begin{array}{r|rrrr} 2 & 1 & 0 & -8 & 8 \\ & & 2 & 4 & -8 \\ \hline & 1 & 2 & -4 & 0 \end{array}$

$x^2 + 2x - 4 = 0 \quad ; \quad x = \frac{-2 \pm \sqrt{4 + 16}}{2} = \frac{-2 \pm \sqrt{20}}{2} = \begin{cases} 1'23 \\ -3'23 \end{cases}$

$(2, \frac{1}{4})$ $(1'23, 0'15)$ $(-3'23, 0'40)$

2) a)

$$f(x) = \begin{cases} x^2 + 2x + 1 & \text{si } x \leq 0 \\ -x^2 + 1 & \text{si } 0 < x < 2 \\ -x - 1 & \text{si } x \geq 2 \end{cases}$$

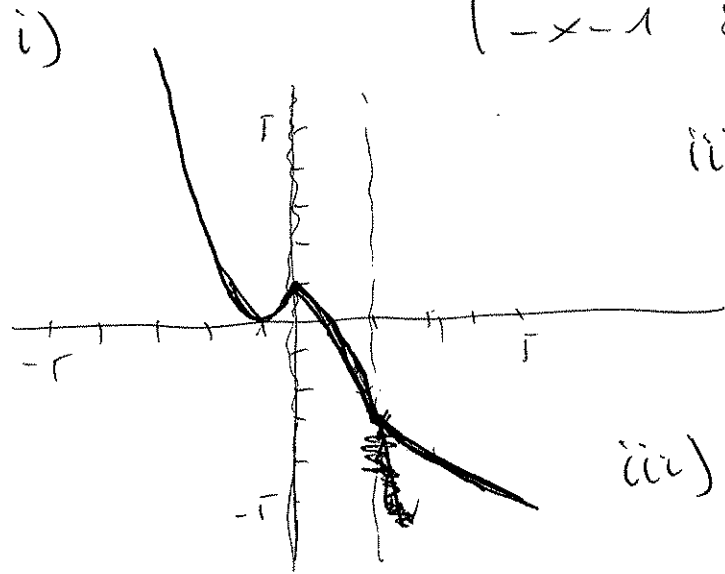
x	-1	0
y	0	1

 $M(-1, 0)$

x	0	2
y	1	-3

 $M(0, 1)$

x	2	3
y	-3	-4



(i) Creciente: $(-1, 0)$

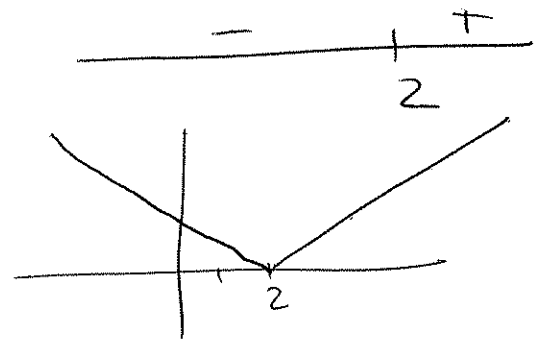
Decreciente: $(-\infty, -1) \cup (0, +\infty)$

(ii) OX: $(-1, 0), (1, 0)$

OY: $(0, 1)$

b) $3x - 6 = 0; 3x = 6; x = \frac{6}{3} = 2$

$$g(x) = \begin{cases} -3x + 6 & \text{si } x \leq 2 \\ 3x - 6 & \text{si } x \geq 2 \end{cases}$$



$$|3x - 6| = 9; \quad \begin{aligned} 3x - 6 = 9 & \Rightarrow 3x = 15; x = 5 // \\ 3x - 6 = -9 & \Rightarrow 3x = -3; x = -1 // \end{aligned}$$

3) a) $25 \cdot 5^x + \frac{3 \cdot 5^x}{5} - 4 \cdot 5^x = 108; \quad \boxed{5^x = t}$

$$25t + \frac{3t}{5} - 4t = 108$$

$$125t + 3t - 20t = 540$$

$$108t = 540; \quad t = \frac{540}{108} = 5 //$$

$$5^x = 5 \Rightarrow \boxed{x = 1}$$

$$b) \frac{2^{2x}}{8} - 2 \cdot 2^x + 2 \cdot 2^{2x} = 26 \quad | 2^x = t \quad (2)$$

$$\frac{t^2}{8} - 2t + 2t^2 = 26 \quad | \quad t^2 - 16t + 16t^2 = 208$$

$$17t^2 - 16t - 208 = 0 \quad ; \quad t = \frac{16 \pm \sqrt{256 + 14144}}{34} =$$

$$= \frac{16 \pm 120}{34} = \begin{cases} 4 \\ -3.05 \end{cases} \quad \begin{array}{l} 2^x = 4 \rightarrow |x = 2| \\ 2^x = -3.05 \rightarrow \text{no sol} \end{array}$$

$$c) 5.00 = 25.00 (0.88)^t \quad ; \quad 0.2 = 0.88^t$$

$$t = \frac{\log 0.2}{\log 0.88} = \underline{\underline{12.59 \text{ años}}}$$

$$4) a) i) \sqrt{x} + 2 = 4; \quad \sqrt{x} = 2; \quad \underline{\underline{x = 4}}$$

$$ii) (x^2 + 1)^2 = 100; \quad x^4 + 1 + 2x^2 = 100$$

$$x^4 + 2x^2 - 99 = 0 \quad | x^2 = t$$

$$t^2 + 2t - 99 = 0; \quad t = \frac{-2 \pm \sqrt{4 + 396}}{2} = \frac{-2 \pm 20}{2} = \begin{cases} 9 \\ -11 \end{cases}$$

$$t = \pm \sqrt{9} = \begin{cases} 3 \\ -3 \end{cases}$$

$$t = \pm \sqrt{-11} \rightarrow \text{no sol}$$

$$iii) 6 \log x - 8 \log x + 5 \log x = 3$$

$$3 \log x = 3; \quad \log x = 1; \quad \underline{\underline{x = 10}}$$

$$\begin{aligned}
 \text{b) } \log \sqrt[3]{0'225} &= \log \sqrt[3]{\frac{225}{1000}} = \log \sqrt[3]{\frac{3^2 \cdot 5^2}{10^3}} = \\
 &= \log \frac{3^{2/3} \cdot 5^{2/3}}{10^{3/3}} = \frac{2}{3} \log 3 + \frac{2}{3} \log 5 - \frac{3}{3} \log 10 = \\
 &= 0'313 + 0'46 - 1 = \underline{\underline{-0'23}}
 \end{aligned}$$

$$\begin{aligned}
 \text{c) } f(1) &= 3^1 - 1^2 = 3 - 1 = 2 \rightarrow (1, 2) \\
 f(2) &= 3^2 - 2^2 = 9 - 4 = 5 \rightarrow (2, 5)
 \end{aligned}$$

$$m = \frac{5-2}{2-1} = \frac{3}{1} = 3$$

$$y - 2 = 3(x - 1); \quad \underline{\underline{y = 3x - 1}}$$

$$y(1'8) = 3 \cdot 1'8 - 1 = 5'4 - 1 = \underline{\underline{4'4}}$$

1) i) $x^3 - 8 = 0$

$$\begin{array}{r|rrrr} 2 & 1 & 0 & 0 & -8 \\ & & 2 & 4 & 8 \\ \hline & 1 & 2 & 4 & 0 \end{array}$$

$$x^2 - 3x + 2 = 0 \rightarrow x = 1$$

$$\rightarrow x = 2$$

$$\lim_{x \rightarrow 2} \frac{(x-2)(x^2+2x+4)}{(x-2)(x-1)} = \frac{4+4+4}{2-1} = 12 //$$

ii) ~~$\lim_{x \rightarrow \infty} \left(\frac{2}{3}\right)^x = 0 //$~~

$$(ii) \lim_{x \rightarrow +\infty} \frac{(\sqrt{9x^4 - x^2 + x} - \sqrt{9x^4 - 2x^2 - x})(\sqrt{9x^4 - x^2 + x} + \sqrt{9x^4 - 2x^2 - x})}{\sqrt{9x^4 - x^2 + x} + \sqrt{9x^4 - 2x^2 - x}} =$$

$$= \lim_{x \rightarrow +\infty} \frac{(9x^4 - x^2 + x) - (9x^4 - 2x^2 - x)}{\sqrt{9x^4 - x^2 + x} + \sqrt{9x^4 - 2x^2 - x}} = \lim_{x \rightarrow +\infty} \frac{x^2 + 2x}{3x^2 + 3x^2} = \frac{1}{6}$$

$$(iv) \lim_{x \rightarrow 1} \left(\frac{(x^2 - 2x)(x^2 - 1) - x^3(x-1)}{(x-1)(x^2 - 1)} \right) =$$

$$= \lim_{x \rightarrow 1} \left(\frac{x^4 - x^2 - 2x^3 + 2x - x^4 + x^3}{(x-1)(x^2 - 1)} \right) =$$

$$= \lim_{x \rightarrow 1} \frac{-x^3 - x^2 + 2x}{(x-1)(x^2 - 1)} = \lim_{x \rightarrow 1} \frac{-x(x^2 + x - 2)}{(x-1)(x^2 - 1)} =$$

$$= \lim_{x \rightarrow 1} \frac{-x(x-1)(x+2)}{(x-1)^2(x+1)} = \frac{-3}{0} = \infty$$

$$v) \lim_{x \rightarrow -2} \frac{(\sqrt{5-2x}-3)(\sqrt{5-2x}+3)}{(2x^2+5x+2)(\sqrt{5-2x}+3)} =$$

$$= \lim_{x \rightarrow -2} \frac{5-2x-9}{(2x^2+5x+2)(\sqrt{5-2x}+3)} = \lim_{x \rightarrow -2} \frac{-2(x+2)}{2(x+1)(x+1/2)(\sqrt{5-2x}+3)} =$$

$$2x^2+5x+2=0; \quad x = \frac{-5 \pm \sqrt{25-16}}{4} = \frac{-5 \pm 3}{4} = \begin{cases} -1/2 \\ -2 \end{cases}$$

$$= \lim_{x \rightarrow -2} \frac{-1}{(x+1/2)(\sqrt{5-2x}+3)} = \frac{-1}{-\frac{3}{2} \cdot 6} = \frac{-1}{-9} = \frac{1}{9} //$$

2) FUNCIONES: Nada

CONEXIONES: $|x=4|$

$$\lim_{x \rightarrow 4^-} \frac{x^2-16}{x-4} = \lim_{x \rightarrow 4^-} \frac{(x+4)(x-4)}{(x-4)} = 8$$

$$\lim_{x \rightarrow 4^+} \frac{x^2-16}{x-4} = \lim_{x \rightarrow 4^+} \frac{(x+4)(x-4)}{(x-4)} = 8$$

$$f(4) = 6$$

D.E

$$3) a) \lim_{x \rightarrow -2^-} f(x) = (-2)^2 - 4 = 4 - 4 = 0 //$$

$$\lim_{x \rightarrow -2^+} f(x) = \frac{(-2)^2 + k}{(-2)^2 - 1} = \frac{4+k}{3}$$

$$f(-2) = 0$$

$$\frac{4+k}{3} = 0 \Rightarrow |k = -4|$$

b) Funciones: $|x = -1|$

~~8~~

Conexiones: $|x = -2|$ $|x = 1|$

$$\lim_{x \rightarrow -2^-} f(x) = 0$$

$$\lim_{x \rightarrow -2^+} f(x) = 0 \Rightarrow \text{CONTINUA}$$

$$f(-2) = 0$$

$$\lim_{x \rightarrow -1^-} f(x) = -\infty$$

$$\lim_{x \rightarrow -1^+} f(x) = +\infty \Rightarrow \text{D.I. S.I.}$$

$$f(-1) = \text{A}$$

$$\lim_{x \rightarrow 1^-} f(x) = +\infty$$

$$\lim_{x \rightarrow 1^+} f(x) = 3 \Rightarrow \text{D.I. S.I.}$$

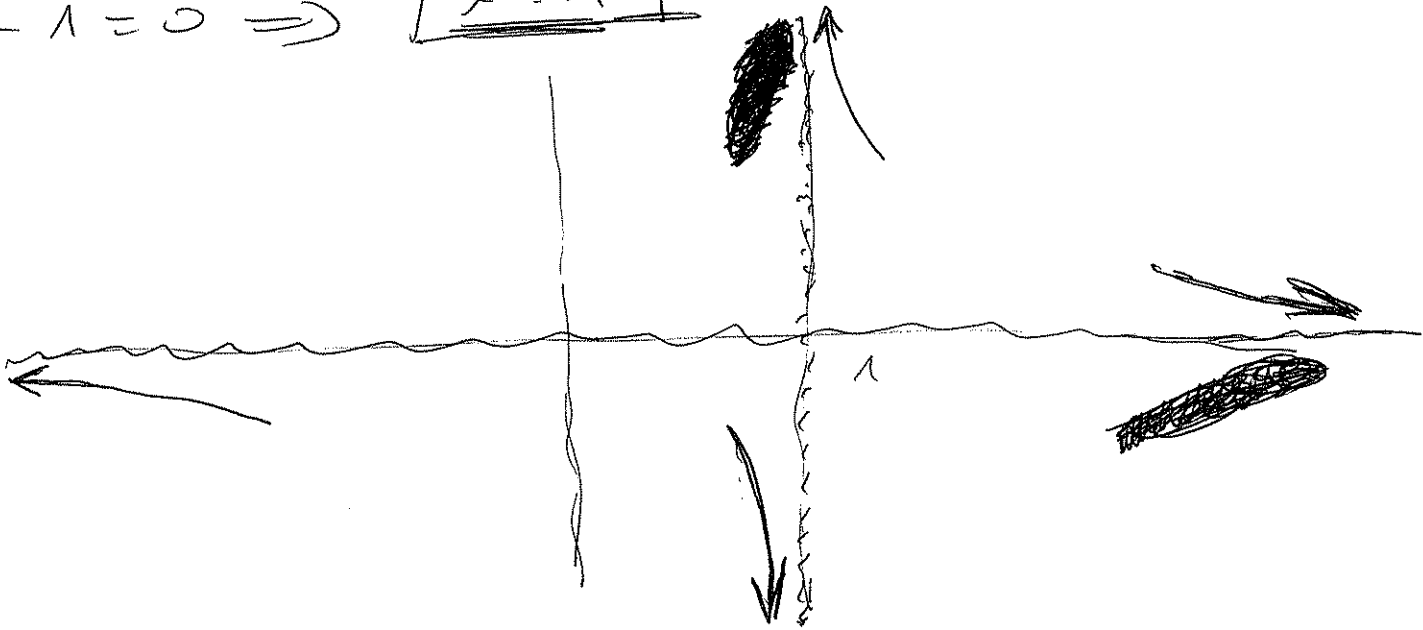
$$f(1) = 3$$

4) a) TEÓRICO

b) $\lim_{x \rightarrow +\infty} f(x) = 0$; $\lim_{x \rightarrow -\infty} f(x) = 0$

$y=0$ A.H (Dos lados)

$x^3 - 1 = 0 \Rightarrow x=1$ A.V



1) $\lim_{x \rightarrow 0^-} f(x) = a$ $a=0$

$\lim_{x \rightarrow 0^+} f(x) = 0$

$f(0) = a$

FUNCIONES

$x^2 - 3x + 2 = 0 \rightarrow x=1 //$
 $\rightarrow x=2 //$

$x-2=0 \rightarrow x=2 //$

CONEXIONES

$x=0 //$ $x=2 //$

$x=0$

$\lim_{x \rightarrow 0^-} f(x) = 0$

$\lim_{x \rightarrow 0^+} f(x) = 0$

$f(0) = 0$

CONTINUA

$x=1$

$\lim_{x \rightarrow 1^-} f(x) = -\infty$

$\lim_{x \rightarrow 1^+} f(x) = +\infty$

$f(1) = \cancel{0}$

D.I.S.I

$x=2$

$\lim_{x \rightarrow 2^-} f(x) = \lim_{x \rightarrow 2^-} \frac{x(x-1)}{(x-1)(x-2)} = \frac{2}{1} = 2$

$\lim_{x \rightarrow 2^+} f(x) = \frac{4}{2-3} = \frac{4}{-1} = -4$

$f(2) = \frac{4}{2-3} = -4$

D.I.S.F

$x=3$

$\lim_{x \rightarrow 3^-} f(x) = -\infty$

$\lim_{x \rightarrow 3^+} f(x) = +\infty$

$f(3) = \cancel{0}$

D.I.S.I

2)

i) $\lim_{x \rightarrow 1} \frac{(\sqrt{x^2+3}-2)(\sqrt{x^2+3}+2)}{(x^2+x-2)(\sqrt{x^2+3}+2)} =$

$= \lim_{x \rightarrow 1} \frac{x^2+3-4}{(x^2+x-2)(\sqrt{x^2+3}+2)} = \lim_{x \rightarrow 1} \frac{(x+1)(x-1)}{(x-1)(x+2)(\sqrt{x^2+3}+2)} =$

$= \lim_{x \rightarrow 1} \frac{x+1}{(x+2)(\sqrt{x^2+3}+2)} = \frac{1+1}{(1+2)(2+2)} = \frac{2}{12} = \frac{1}{6} //$

$$(i) \lim_{x \rightarrow -1} \frac{x^2(x+1)}{2(x+1)(x-\frac{3}{2})} = \frac{(-1)^2}{2(-1-\frac{3}{2})} = -\frac{1}{5} //$$

$$(ii) \lim_{x \rightarrow \infty} \left(\frac{(x^2-4)(x^2-2) - x^3 \cdot x}{x(x^2-2)} \right) =$$

$$= \lim_{x \rightarrow \infty} \left(\frac{x^4 - 2x^2 - 4x^2 + 8 - x^4}{x^3 - 2x} \right) = \lim_{x \rightarrow \infty} \frac{-6x^2 + 8}{x^3 - 2x} = 0 //$$

$$3) a) f(4) = \sqrt{\frac{4}{4+5}} = \sqrt{\frac{4}{9}} = \frac{2}{3} \quad P_0(4, 2/3)$$

$$f'(x) = \frac{1}{2\sqrt{\frac{x}{x+5}}} \cdot \frac{1(x+5) - 1 \cdot x}{(x+5)^2} = \frac{1}{2\sqrt{\frac{x}{x+5}}} \cdot \frac{5}{(x+5)^2} =$$

$$= \frac{5}{2\sqrt{\frac{x}{x+5}}(x+5)^2} = \frac{5}{2 \cdot \frac{2}{3} \cdot 81} = \frac{5}{108}$$

$$\boxed{y - \frac{2}{3} = \frac{5}{108}(x-4)}$$

$$b) i) f(1) = 2, \quad f'(-1) = 0, \quad f'(x) = 3ax^2 - 8x + 4$$

$$\begin{cases} a(+1)^3 - 4(+1)^2 + 4(+1) + b = 2 \\ 3a(-1)^2 - 8(-1) + 4 = 0 \end{cases} \Rightarrow \begin{cases} a - 4 + 4 + b = 2 \\ 3a + 8 + 4 = 0 \end{cases}$$

$$a + b = 2$$

$$3a = -12$$

$$a = -\frac{12}{3} = -4$$

$$\boxed{\begin{matrix} a = -4 \\ b = 6 \end{matrix}}$$

$$-4 + b = 2 \Rightarrow b = 2 + 4 = 6$$

(ii) $f(x) = -4x^3 - 4x^2 + 4x + 6$

$$f'(x) = -12x^2 - 8x + 4$$

$$f''(x) = -24x - 8$$

$$-12x^2 - 8x + 4 = 0 \quad ; \quad 3x^2 + 2x - 1 = 0$$

$$x = \frac{-2 \pm \sqrt{4+12}}{6} = \frac{-2 \pm 4}{6} = \begin{matrix} 1/3 \\ -1 \end{matrix}$$

$$f(-1) = -4(-1)^3 - 4(-1)^2 + 4(-1) + 6 = 4 - 4 - 4 + 6 = 2$$

$$\begin{aligned} f(1/3) &= -4 \cdot \frac{1}{27} - 4 \cdot \frac{1}{9} + 4 \cdot \frac{1}{3} + 6 = \frac{-4}{27} - \frac{4}{9} + \frac{4}{3} + 6 = \\ &= \frac{-4 - 12 + 36 + 162}{27} = \frac{182}{27} \end{aligned}$$

$E_1(-1, 2) \rightarrow$ Min rel

$E_2(\frac{1}{3}, \frac{182}{27}) \rightarrow$ Max rel

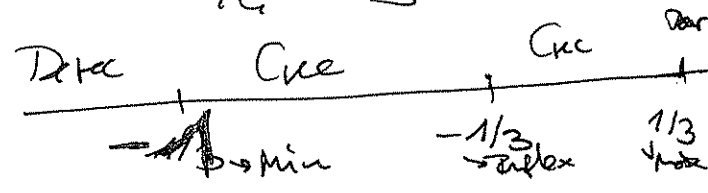
~~$f'(x) = -12x^2 - 8x + 4 = 0 \Rightarrow 12x^2 + 8x - 4 = 0$~~

$$f''(-1) = -24(-1) - 8 = 24 - 8 = 16 > 0$$

$$f''(1/3) = -24 \cdot 1/3 - 8 = -8 - 8 = -16 < 0$$

$$-24x - 8 = 0 \quad ; \quad -24x = 8, \quad x = -\frac{8}{24} = -\frac{1}{3}$$

$E_3(-\frac{1}{3}, \frac{118}{27}) \rightarrow$ Inflex



$$\begin{aligned} f(-1/3) &= -4 \cdot \left(-\frac{1}{27}\right) - 4 \cdot \frac{1}{9} + 4 \cdot \left(-\frac{1}{3}\right) + 6 = \frac{4}{27} - \frac{4}{9} - \frac{4}{3} + 6 = \\ &= \frac{4 - 12 - 36 + 162}{27} = \frac{118}{27} \end{aligned}$$

4) $D = \mathbb{R} - 3, 3$

Cepts $\begin{cases} OX: (0, 0) \\ OY: (0, 0) \end{cases}$

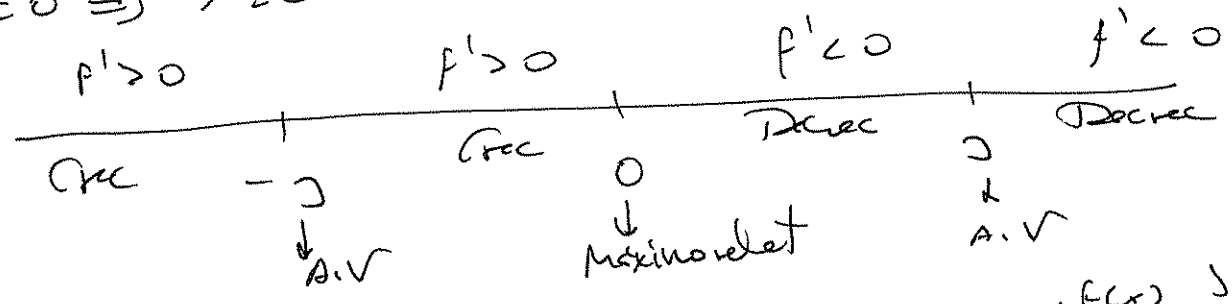
* Asintots $\begin{cases} H: \boxed{y=2} \text{ Do Cad} \\ V: \boxed{x=-3} \end{cases}$

$\lim_{x \rightarrow -3^-} f(x) = +\infty$
 $\lim_{x \rightarrow -3^+} f(x) = -\infty$

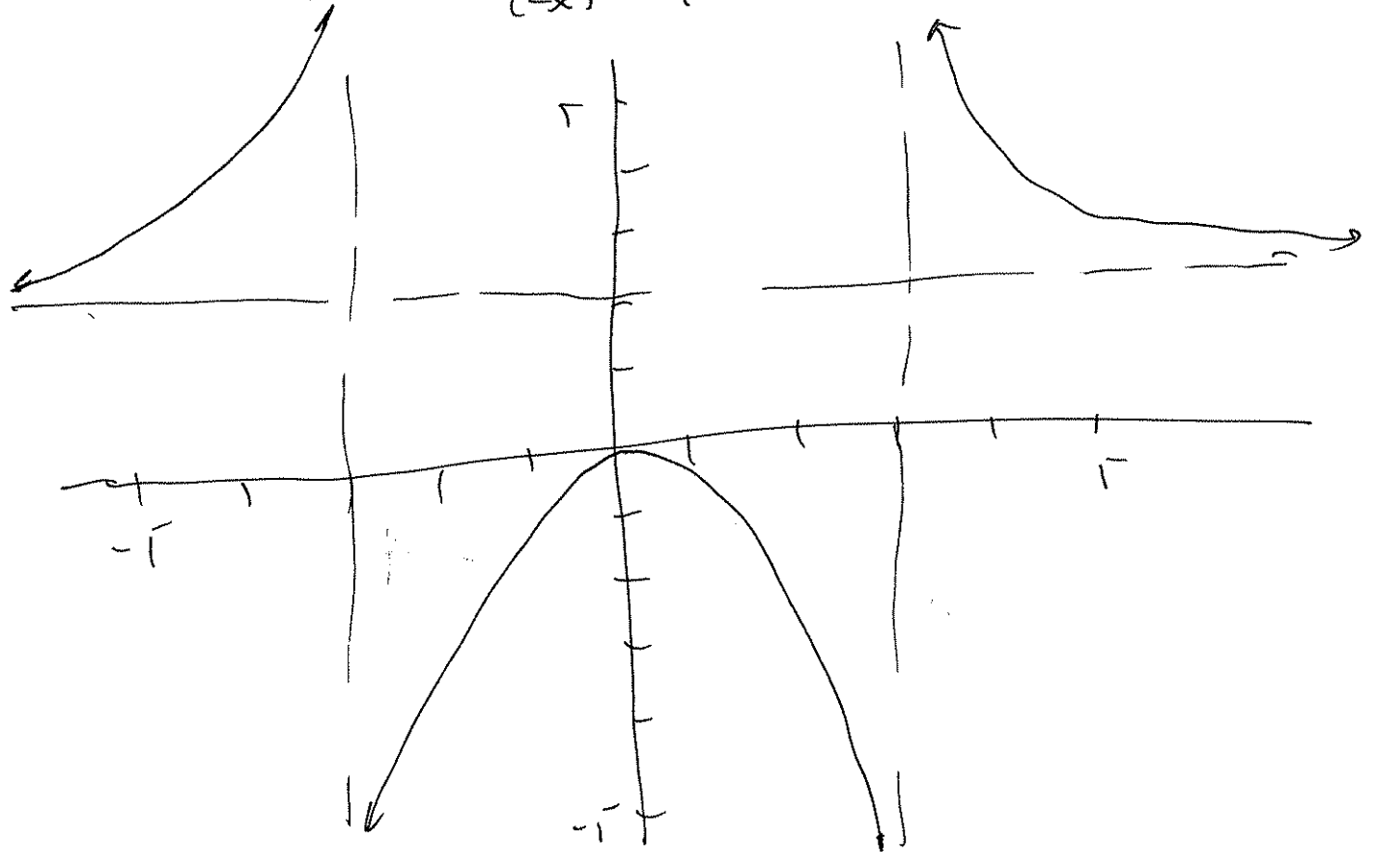
$\lim_{x \rightarrow 3^-} f(x) = -\infty$
 $\lim_{x \rightarrow 3^+} f(x) = +\infty$

* 2^a der: $f'(x) = \frac{4x(x^2-9) - 2x \cdot 2x^2}{(x^2-9)^2} = \frac{-36x}{(x^2-9)^2}$

$-36x = 0 \Rightarrow x = 0$



* Simetria: $f(-x) = \frac{2(-x)^2}{(-x)^2-9} = \frac{2x^2}{x^2-9} = f(x)$ Si $\rightarrow$ par



1) i) $\lim_{x \rightarrow -1^-} f(x) = \frac{-a+2}{-1+2} = -a+2$
 $\lim_{x \rightarrow -1^+} f(x) = (-1)^2 + (-1) = 0$
 $f(-1) = -a+2$

$$\left\{ \begin{array}{l} -a+2=0 \\ \Rightarrow \boxed{a=2} \end{array} \right.$$

ii) FUNCIONES $x = -2$; $x = 3$

CONEXIONES $x = -1$; $x = 3$

$x = -2$
 $\lim_{x \rightarrow -2^-} f(x) = \frac{2(-2)+2}{-2+2} \rightarrow +\infty$
 $\lim_{x \rightarrow -2^+} f(x) = -\infty$
 $f(-2) \rightarrow \nexists$

$$\left\{ \begin{array}{l} \text{D.I.S.I} \end{array} \right.$$

$x = -1$ CONTINUA

$x = 3$
 $\lim_{x \rightarrow 3^-} f(x) = 12$
 $\lim_{x \rightarrow 3^+} f(x) = +\infty$
 $f(3) \Rightarrow \nexists$

$$\left\{ \begin{array}{l} \text{D.I.S.I} \end{array} \right.$$

2) i) $\lim_{x \rightarrow 2} \frac{(\sqrt{3x-2}-x)(\sqrt{3x-2}+x)}{(x^2-5x+6)(\sqrt{3x-2}+x)} =$

$$= \lim_{x \rightarrow 2} \frac{(3x-2)-x^2}{(x-3)(x-2)(\sqrt{3x-2}+x)} = \lim_{x \rightarrow 2} \frac{-x^2+3x-2}{(x-3)(x-2)(\sqrt{3x-2}+x)} =$$

$$= \lim_{x \rightarrow 2} \frac{-(x-1)(x-2)}{(x-3)(x-2)(\sqrt{3x-2}+x)} = \lim_{x \rightarrow 2} \frac{-(x-1)}{(x-3)(\sqrt{3x-2}+x)} = \frac{-1}{-1 \cdot 4} = \frac{1}{4}$$

ii)

	1	0	0	1
-1		-1	1	-1
	1	-1	1	0

$$ii) \lim_{x \rightarrow -1} \frac{(x+1)(x^2-x+1)}{x(x+1)(x-5)} = \frac{(-1)^2(-1)+1}{(-1)(-1-5)} = \frac{3}{6} = \frac{1}{2} //$$

$$iii) \lim_{x \rightarrow \infty} \frac{(\sqrt{4x^2+2x+1} - \sqrt{4x^2-x-2})(\sqrt{4x^2+2x+1} + \sqrt{4x^2-x-2})}{(\sqrt{4x^2+2x+1} + \sqrt{4x^2-x-2})} =$$

$$= \lim_{x \rightarrow \infty} \frac{(4x^2+2x+1) - (4x^2-x-2)}{\sqrt{4x^2+2x+1} + \sqrt{4x^2-x-2}} =$$

$$= \lim_{x \rightarrow \infty} \frac{4x^2+2x+1-4x^2+x+2}{\sqrt{4x^2} + \sqrt{4x^2}} = \lim_{x \rightarrow \infty} \frac{3x+3}{4x} = \frac{3}{4} //$$

$$3) b) f(3) = -27, \quad f'(3) = 0$$

$$f'(x) = 3ax^2 + 2bx - 9$$

$$f(3) = 27a + 9b - 27 = -27$$

$$f'(3) = 27a + 6b - 9 = 0$$

$$\begin{cases} 27a + 9b = 0 \\ 27a + 6b = 9 \end{cases} \Rightarrow \begin{cases} 3b = -9; \quad b = -3 // \\ 27a - 27 = 9; \quad a = +1 // \end{cases}$$

$$f(x) = x^3 - 3x^2 - 9x$$

$$f'(x) = 3x^2 - 6x - 9 = 0$$

$$x^2 - 2x - 3 = 0; \quad x = \frac{2 \pm \sqrt{4+12}}{2} = \frac{2 \pm 4}{2} = \begin{cases} 3 \\ -1 \end{cases}$$

$$f''(x) = 6x - 6$$

$$f''(-1) = -6 - 6 = -12 < 0$$

$$f''(3) = 18 - 6 = 12 > 0$$

$$f''(x) = 0 \Rightarrow x = 1$$

$$\underline{E_1(3, -27)}$$

Minimum relat

$$\underline{E_2(-1, 5)}$$

Maximum relat

$$I_1(1, -11) \rightarrow \text{Inflection}$$

3) a)

$$i) f'(x) = 2x\sqrt{x^2-1} + x^2 \cdot \frac{2x}{2\sqrt{x^2-1}} =$$

$$= \frac{2x(x^2-1) + x^3}{\sqrt{x^2-1}} = \frac{3x^3 - 2x}{\sqrt{x^2-1}} //$$

$$ii) g'(x) = 3 \left(\frac{x}{x-2} \right)^2 \cdot \left(\frac{1(x-2) - 1 \cdot x}{(x-2)^2} \right) =$$

$$= 3 \left(\frac{x}{x-2} \right)^2 \cdot \frac{-2}{(x-2)^2} = \frac{-6x^2}{(x-2)^4} //$$

4) Dominio: $x^2 - 4x = 0$; $x(x-4) = 0 \rightarrow \begin{matrix} x=0 \\ x=4 \end{matrix}$

$D = \mathbb{R} \setminus \{0, 4\}$

Check $\rightarrow$ $\begin{matrix} < 0 & x: & \text{no crite} \\ < 0 & y: & \text{no crite} \end{matrix}$

A.H.: $\lim_{x \rightarrow -\infty} f(x) = 0$; $\lim_{x \rightarrow -\infty} f(x) = 0 \mid \boxed{5=0} \text{ bed}$

$\lim_{x \rightarrow 0^-} f(x) = +\infty$ $\lim_{x \rightarrow 0^+} f(x) = -\infty$

A.V.: $x=0, x=4$

$\lim_{x \rightarrow 4^-} f(x) = +\infty$ $\lim_{x \rightarrow 4^+} f(x) = -\infty$

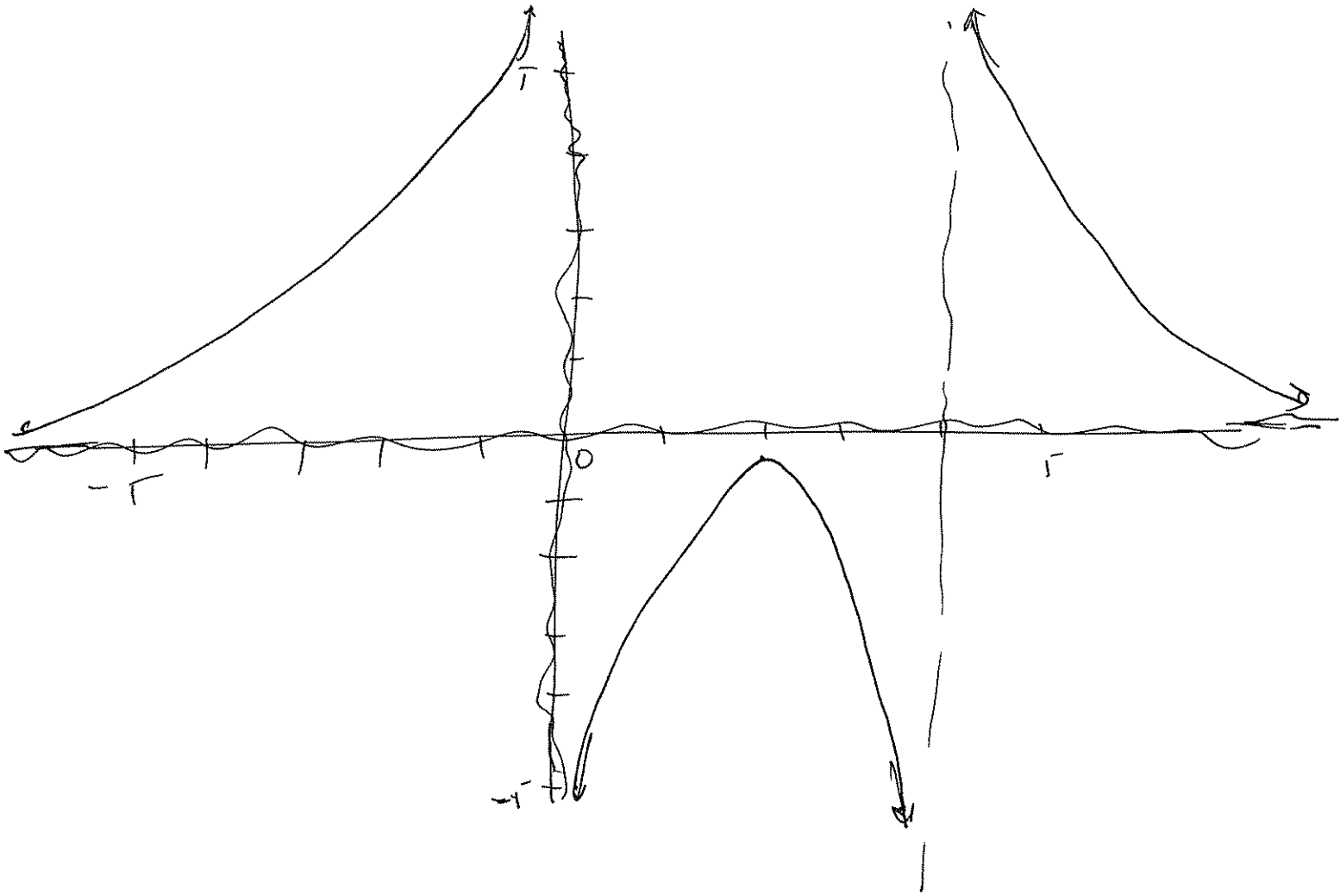
Extremal $f'(x) = \frac{0(x^2-4x) - (2x-4) \cdot 2}{(x^2-4x)^2} = \frac{-4x+8}{(x^2-4x)^2} = 0$

$-4x+8=0 \Rightarrow x=2 \rightarrow f(2) = \frac{2}{2^2-4 \cdot 2} = \frac{2}{4-8} = \frac{2}{-4} = -\frac{1}{2}$

$f' > 0$	$f' = 0$	$f' < 0$	$f' < 0$
Crc.	0 Reper	2 Dance	4 Dance
	$\downarrow$ A.V	$\downarrow$ Mäxndat (2, -1/2)	$\downarrow$ A.V

Plot

$$f(-x) = \frac{2}{(-x)^2 - 4(-x)} = \frac{2}{x^2 + 4x} \rightarrow \text{no trace}$$



1)

x	y	f	xf	yf	xxf	yyf	xyf
0	8	2	0	16	0	128	0
1	6	3	3	18	3	108	18
1	7	5	5	35	5	245	35
2	5	7	14	35	28	175	70
3	5	4	12	20	36	100	60
4	3	6	24	18	96	54	72
5	0	3	15	0	75	0	0
		30	73	142	243	810	255

Med x	2,43333	Cov	-3,0178
Med y	4,73333	r	-0,9537
Var x	2,17889	Fiab	90,9497
Var y	4,59556		
Dt x	1,47611	Y(x)	-1,385 8,10352
Dt y	2,14372	X(y)	-0,6567 5,54159

a) $r = -0,9537$. Rel inversa > fuerte

b) $y(3,5) = -1,385 \times 3,5 + 8,10352 = \underline{\underline{3,26}}$

2) $(0,0)^0 (0,1)^0 (0,2)^0 (0,3)^0$
 $(1,0)^0 (1,1)^1 (1,2)^2 (1,3)^3$
 $(2,0)^0 (2,1)^2 (2,2)^4 (2,3)^6$
 $(3,0)^0 (3,1)^3 (3,2)^6 (3,3)^9$

$E = \{0, 1, 2, 3, 4, 6, 9\}$

$P(0) = \frac{7}{16}$; $P(1) = \frac{1}{16}$; $P(2) = \frac{2}{16}$

$P(3) = \frac{2}{16}$; $P(4) = \frac{1}{16}$; $P(6) = \frac{2}{16}$; $P(9) = \frac{1}{16}$

b) TEÓRICO

c) $P(A \cup B) = \frac{9}{16}$

$P(A \cap B) = \frac{1}{16} + \frac{2}{16} + \frac{1}{16} = \frac{4}{16}$

$A \cup B = \{1, 2, 3, 4, 6\}$

$A \cap B = \{1, 3, 9\}$

3) A) $\begin{array}{|c|} \hline 4R \\ \hline 1B \\ \hline \end{array}$ 5 balls B) $\begin{array}{|c|} \hline 1R \\ \hline 3B \\ \hline \end{array}$ 4 balls

$(\frac{4}{5}) R$ $\swarrow$ $(\frac{1}{4}) R$ $\swarrow$ $(\frac{3}{3}) R \rightarrow 0$
 $(\frac{4}{5}) R$ $\swarrow$ $(\frac{3}{4}) B$ $\swarrow$ $(\frac{2}{3}) R$ $\rightarrow \frac{4}{5} \cdot \frac{3}{4} \cdot \frac{2}{3} = \frac{24}{60} = \frac{2}{5}$
 $(\frac{1}{5}) B$ $\swarrow$ $(\frac{1}{5}) R$ $\swarrow$ $(\frac{2}{4}) R \rightarrow 0$
 $(\frac{1}{5}) B$ $\swarrow$ $(\frac{4}{5}) B$ $\swarrow$ $(\frac{1}{4}) B$ $\rightarrow 0$
 $(\frac{1}{5}) B$ $\swarrow$ $(\frac{4}{5}) B$ $\swarrow$ $(\frac{3}{4}) B \rightarrow \frac{1}{5} \cdot \frac{4}{5} \cdot \frac{3}{4} = \frac{12}{100} = \frac{3}{25}$

a) $P(R, R) = 0 //$

b) $P(B, B) = \frac{2}{5} + \frac{3}{25} = \frac{10}{25} + \frac{3}{25} = \frac{13}{25} //$ 52%

c) $= 1 - 0 - \frac{13}{25} = \frac{25-13}{25} = \frac{12}{25} //$ 48%

4) $X \rightarrow B(6, 0.7)$
 $P = 0.7$ (vive < 80 ans) $q = 0.3$ (vive > 80 ans)

a) $P(4F) = P(2E) = P(X=2) = \binom{6}{2} (0.7)^2 (0.3)^4 = 0.01512$
 $+ P(5F) + P(6F) + P(1E) \rightarrow 0.01008$
 $+ P(0E) \rightarrow 0.00007$

b) $P(1E) + P(2E) + \dots + P(6E) = 1 - P(0E) = 1 - \binom{6}{0} (0.7)^0 (0.3)^6 = \frac{0.99}{0.00007}$

c) $P(X=3) + P(X=4) + P(X=5) = \binom{6}{3} (0.7)^3 (0.3)^3 + \binom{6}{4} (0.7)^4 (0.3)^2 + \binom{6}{5} (0.7)^5 (0.3)^1$
 $= 0.117 + 0.322 + 0.309 = 0.748 = 74.8\%$

5) a) $z_1 = \frac{160 - 166}{5} = -1.2$ $P(z \leq -1.2) = P(z \leq 1.2) = 0.8849$

b) $z_1 = \frac{158 - 166}{5} = -1.6$; $z_2 = \frac{167 - 166}{5} = 0.2$
 $P(-1.6 \leq z \leq 0.2) = P(z \leq 0.2) - P(z \leq -1.6) =$
 $= (1 - P(z \leq 0.2)) - (1 - P(z \leq 1.6)) = (1 - 0.5793) - (1 - 0.9498) =$
 $= 0.4207 - 0.0507 = 0.3699$ c) $(0.8849)^4 = 0.6131$

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x	y	f	xf	yf	xxf	yyf	xyf
0	0	8	0	0	0	0	0
1	1	4	4	4	4	4	4
1	2	5	5	10	5	20	10
1	0	4	4	0	4	0	0
2	1	1	2	1	4	1	2
2	2	2	4	4	8	8	8
3	2	6	18	12	54	24	36
		30	37	31	79	57	60

Med x 1,23333

Cov 0,72556

Med y 1,03333

r 0,75415

Var x 1,11222

Fiab 56,8736

Var y 0,83222

Y(x) 0,65235 0,22877

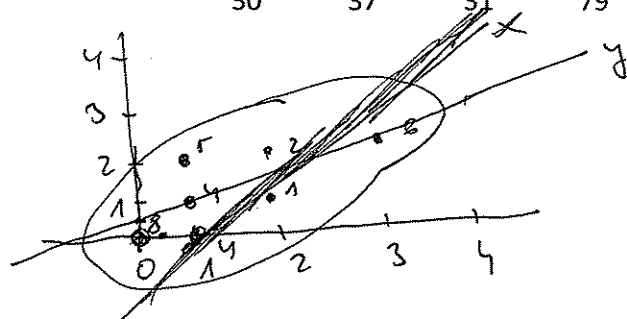
Dt x 1,05462

X(y) 0,87183 0,33244

Dt y 0,91226

$$\frac{x}{y} \begin{array}{|c|c|c|} \hline 0 & 4 & \\ \hline 0 & 2 & 3 \\ \hline \end{array}$$

$$\frac{x}{y} \begin{array}{|c|c|c|} \hline 0 & 3 & 7 \\ \hline 0 & 0 & 4 \\ \hline \end{array}$$



$$x(3) = 0,87 \times 3 + 0,33 = 2,61 + 0,33 = \underline{\underline{2,94}}$$

$$2) P(A) = \frac{7}{12} \text{ ; } P(B) = \frac{4}{12} \text{ ; } P(C) = \frac{8}{12}$$

$$P(A \cup B) = \frac{7}{12} \text{ ; } P(A \cap B) = \frac{4}{12} \text{ ; } P(B \cap C) = 0$$

$$5) Z = \frac{x-15}{4} \text{ ; } a) P(10 \leq x \leq 13) = P(-1,25 \leq z \leq -0,75) =$$

$$= P(z \leq -0,75) - P(z \leq -1,25) = (1 - P(z \leq 0,75)) - (1 - P(z \leq 1,25)) =$$

$$= (1 - 0,6915) - (1 - 0,8944) = 0,3085 - 0,1056 = \underline{\underline{0,2029}}$$

$$b) P(x \leq 17) = P(z \leq 0,75) = \underline{\underline{0,6915}}$$

$$180 \times 0,1056 = \underline{\underline{19,008}}$$

$$c) P(x \geq 20) = P(z \geq 1,25) = 1 - P(z \leq 1,25) = 1 - 0,8944 = 0,1056$$

(D) $\begin{array}{|c|} \hline 3B \\ \hline 1N \\ \hline \end{array}$ (B) $\begin{array}{|c|} \hline 3P \\ \hline 1N \\ \hline \end{array}$ (C) $\begin{array}{|c|} \hline 2R \\ \hline 2B \\ \hline \end{array}$

$\frac{3}{4}$	R	$\frac{2}{3}$	R
$\frac{1}{4}$	V	$\frac{1}{3}$	V
$\frac{3}{4}$	R	$\frac{2}{3}$	R
$\frac{1}{4}$	V	$\frac{2}{3}$	V
$\frac{3}{4}$	R	$\frac{2}{4}$	12
$\frac{1}{4}$	V	$\frac{2}{4}$	V
		$\frac{2}{4}$	R
		$\frac{2}{4}$	V

$$\rightarrow \frac{9}{24} + \frac{3}{32} = \frac{24}{96} + \frac{9}{96} = \frac{33}{96} = \frac{11}{32}$$

$$e) \frac{3}{4} \cdot \frac{3}{4} \cdot \frac{1}{3} + \frac{3}{4} \cdot \frac{1}{4} \cdot 0 + \frac{1}{4} \cdot \frac{3}{4} \cdot \frac{2}{4} + \frac{1}{4} \cdot \frac{1}{4} \cdot \frac{2}{4} =$$

4) $X \sim B(12, 0.35)$ $P = 0.35$ $q = 0.65$

b) $P(1E) + \dots + P(12E) = 1 - P(0E) = 1 - 0,005 = \underline{\underline{0,995 \rightarrow 99,5\%}}$

$$c) P(10F) + P(11F) + P(12F) = P(7E) + P(1E) + P(0E) =$$

$$= \underline{0'149} - \underline{14'9\%}$$