① a) $A+A^t=\begin{pmatrix}1&2&1\\0&1&0\\2&0&3\end{pmatrix}+\begin{pmatrix}1&0&2\\2&1&0\\1&0&3\end{pmatrix}=\boxed{\begin{pmatrix}2&2&3\\2&2&0\\3&0&6\end{pmatrix}}=C \quad |C|=-18$

$\mathrm{Adj}\,C=\begin{pmatrix}12&-12&-6\\-12&3&6\\-6&6&0\end{pmatrix}=(\mathrm{Adj}\,C)^t=\begin{pmatrix}12&-12&-6\\-12&3&6\\-6&6&0\end{pmatrix}$ $\boxed{(A+A^t)^{-1}=\begin{pmatrix}-\frac{2}{3}&\frac{2}{3}&\frac{1}{3}\\\frac{2}{3}&-\frac{1}{6}&-\frac{1}{3}\\\frac{1}{3}&-\frac{1}{3}&0\end{pmatrix}}$

b) $ABX-I_3=B \Rightarrow ABX=B+I_3 \Rightarrow \boxed{X=B^{-1}A^{-1}(B+I_3)}$ $|AB|=2$

$AB=\begin{pmatrix}3&2&3\\1&0&1\\3&5&2\end{pmatrix}$ $\mathrm{Adj}\,AB=\begin{pmatrix}-5&1&5\\+11&-3&-9\\2&0&-2\end{pmatrix}$ $(\mathrm{Adj}\,AB)^t=\begin{pmatrix}-5&11&2\\1&-3&0\\5&-9&-2\end{pmatrix}$

$(AB)^{-1}=\begin{pmatrix}-\frac{5}{2}&\frac{11}{2}&1\\\frac{1}{2}&-\frac{3}{2}&0\\\frac{5}{2}&-\frac{9}{2}&-1\end{pmatrix}$ $\begin{pmatrix}1&1&1\\1&1&1\\1&1&1\end{pmatrix}=B+I_3$ $\boxed{X=\begin{pmatrix}4&4&4\\-1&-1&-1\\-3&-3&-3\end{pmatrix}}$

② a) $A^2=\begin{pmatrix}4&5&-1\\-3&-4&1\\-3&-4&0\end{pmatrix}\begin{pmatrix}4&5&-1\\-3&-4&1\\-3&-4&0\end{pmatrix}=\begin{pmatrix}4&4&1\\-3&-3&-1\\0&1&-1\end{pmatrix}$

$A^3=\begin{pmatrix}4&4&1\\-3&-3&-1\\0&1&-1\end{pmatrix}\begin{pmatrix}4&5&-1\\-3&-4&1\\-3&-4&0\end{pmatrix}=\begin{pmatrix}1&0&0\\0&1&0\\0&0&1\end{pmatrix}$

$A^{128}=A^{126}A^2=I_3\cdot A^2=A^2=\begin{pmatrix}4&4&1\\-3&-3&-1\\0&1&-1\end{pmatrix}$

c) $B^2+I_n=B \Rightarrow I_n=B-B^2 \Rightarrow I_n=B(I_n-B) \Rightarrow \boxed{B^{-1}=I_n-B}$

③ i) $\begin{vmatrix}1&1&1\\2&0&1\\3a&3b&3c\end{vmatrix}=3\begin{vmatrix}1&1&1\\2&0&1\\a&b&c\end{vmatrix}=\begin{vmatrix}1&1&1\\6&0&3\\a&b&c\end{vmatrix}=-2$

ii) $\begin{vmatrix}a&b&c\\2a+6&2b&2c+3\\a+1&b+1&c+1\end{vmatrix}\begin{matrix}F_2-2F_1\to F_2\\F_3-F_1\to F_3\end{matrix}\begin{vmatrix}a&b&c\\6&0&3\\1&1&1\end{vmatrix}=2$

b) $$\begin{vmatrix} -1 & a & a & a \\ a & -1 & a & a \\ a & a & -1 & a \\ a & a & a & -1\end{vmatrix} = 0 = \begin{vmatrix} -1 & a & a & a \\ 0 & a^2-1 & a^2+a & a^2+a \\ 0 & a^2+a & a^2-1 & a^2+a \\ 0 & a^2+a & a^2+a & a^2-1\end{vmatrix} = (a+1)^3 \begin{vmatrix} -1 & a & a & a \\ 0 & a-1 & a & a \\ 0 & a & a-1 & a \\ 0 & a & a & a-1\end{vmatrix} =$$

$$= -(a+1)^3 \cdot \begin{vmatrix} a-1 & a & a \\ a & a-1 & a \\ a & a & a-1\end{vmatrix} = -(a+1)^3\left[(a-1)^3 + a^3 + a^3 - 3a^2(a-1)\right] =$$

$$= -(a+1)^3\left[\cancel{a^3} - \cancel{3a^2} + 3a - 1 \cancel{+a^3} \cancel{+a^3} - \cancel{3a^3} + \cancel{3a^2}\right] = (1-3a)(a+1)^3 = 0 \Leftrightarrow \boxed{\begin{matrix} a=-1 \\ a=\frac{1}{3}\end{matrix}}$$

4) $$\underset{A}{\begin{pmatrix} 1 & m & 1 \\ 1 & 1 & m \\ m & 1 & 1\end{pmatrix}} \underset{X}{\begin{pmatrix} x \\ y \\ z\end{pmatrix}} = \underset{B}{\begin{pmatrix} m+2 \\ -2m-2 \\ m\end{pmatrix}}$$

$|A| = 1+1+m^3-m-m-m = m^3-3m+2 =$
$= (m-1)^2(m+2) = 0 \Leftrightarrow \boxed{\begin{matrix} m=1 \\ m=-2\end{matrix}}$

Si $m \neq 1$, $m \neq -2$ S.C.D.

$$x = \frac{\begin{vmatrix} m+2 & m & 1 \\ -2m-2 & 1 & m \\ m & 1 & 1\end{vmatrix}}{(m+1)^2(m+2)} = \frac{m+2-2m-2+m^3-m-m^2-2m+2m^2+2m}{(m-1)^2(m+2)} = \frac{m\cancel{(m-1)}\cancel{(m+2)}}{(m-1)^{\cancel{2}}\cancel{(m+2)}} = \boxed{\frac{m}{m-1}}$$

m^3+m^2-2m

$$y = \frac{\begin{vmatrix} 1 & m+2 & 1 \\ 1 & -2m-2 & m \\ m & m & 1\end{vmatrix}}{(m-1)^2(m+2)} = \frac{-2m-2+m+m^3+2m^2+2m^2+2m-m^2-m-2}{(m-1)^2(m+2)} = \frac{\cancel{(m-1)}(m+2)^{\cancel{2}}}{(m-1)^{\cancel{2}}\cancel{(m+2)}} = \boxed{\frac{m+2}{m-1}}$$

m^3+3m^2-4

$$z = \frac{\begin{vmatrix} 1 & m & m+2 \\ 1 & 1 & -2m-2 \\ m & 1 & m\end{vmatrix}}{(m-1)^2(m+2)} = \frac{m+m+2-2m^3-2m^2-m^2-2m+2m+2-m^2}{(m-1)^2(m+2)} = \frac{-2\cancel{(m-1)}(m+1)\cancel{(m+2)}}{(m-1)^{\cancel{2}}\cancel{(m+2)}} = \boxed{\frac{-2(m+1)}{m-1}}$$

$-2m^3-4m^2+2m+4$

Si $m=1$

$A = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1\end{pmatrix}$ $B = \begin{pmatrix} 3 \\ -4 \\ 1\end{pmatrix}$ Rango $A = 1 \neq 2 =$ Rango(A/B) S.I (No hay solución

$$\left(\begin{array}{ccc|c} 1 & 1 & 1 & 3 \\ 1 & 1 & 1 & -4 \\ 1 & 1 & 1 & 1\end{array}\right) \underset{F_3-F_1}{\overset{F_2-F_1}{\sim}} \left(\begin{array}{ccc|c} 1 & 1 & 1 & 3 \\ 0 & 0 & 0 & -7 \\ 0 & 0 & 0 & -2\end{array}\right)$$

Si $m=-2$

$A = \begin{pmatrix} 1 & -2 & 1 \\ 1 & 1 & -2 \\ -2 & 1 & 1\end{pmatrix}$ $B = \begin{pmatrix} 0 \\ 2 \\ -2\end{pmatrix}$ $$\left(\begin{array}{ccc|c} 1 & -2 & 1 & 0 \\ 1 & 1 & -2 & 2 \\ -2 & 1 & 1 & -2\end{array}\right) \underset{2F_1+F_3}{\overset{F_1-F_2}{\sim}} \left(\begin{array}{ccc|c} 1 & -2 & 1 & 0 \\ 0 & -3 & 3 & -2 \\ 0 & -3 & 3 & -2\end{array}\right)$$

Rango $A = 2 =$ Rango(A/B)
S.C.I

$z = \lambda$ $y = \frac{2+3\lambda}{3}$ $x = \frac{4+3\lambda}{3}$

$$\boxed{\left(\frac{4+3\lambda}{3}, \frac{2+3\lambda}{3}, \lambda\right) \forall \lambda \in \mathbb{R}}$$

1 a) $x \to$ nº suizos $y \to$ nº zuavos $z \to$ nº sajones

$$\left.\begin{aligned} x+\frac{1}{2}y+\frac{1}{2}z&=901\\ \frac{1}{3}x+y+\frac{1}{3}z&=901\\ \frac{1}{4}x+\frac{1}{4}y+z&=901\end{aligned}\right\} \quad \left.\begin{aligned} 2x+y+z&=1802\\ x+3y+z&=2703\\ x+y+4z&=3604\end{aligned}\right\} \quad \begin{pmatrix} 1 & 1 & 4 & 3604\\ 2 & 1 & 1 & 1802\\ 1 & 3 & 1 & 2703\end{pmatrix} \underset{F_1-F_3\to F_3}{\overset{2F_1-F_2\to F_2}{\sim}} \begin{pmatrix} 1 & 1 & 4 & 3604\\ 0 & 1 & 7 & 5406\\ 0 & -2 & 3 & 901\end{pmatrix}$$

$$\overset{2F_2+F_3\to F_3}{\sim} \begin{pmatrix} 1 & 1 & 4 & 3604\\ 0 & 1 & 7 & 5406\\ 0 & 0 & 17 & 11713\end{pmatrix} \quad \left.\begin{aligned} x+y+4z&=3604\\ y+7z&=5406\\ 17z&=11713\end{aligned}\right\} \begin{aligned} x&=3604-583-4\cdot 689=\underline{\underline{265}} \text{ suizos}\\ y&=5406-7\cdot 689=\underline{\underline{583}} \text{ zuavos}\\ z&=\underline{\underline{689}} \text{ sajones}\end{aligned}$$

b)
$$\begin{vmatrix} x & -x & -1 & -1\\ 1 & x & 1 & 1\\ 1 & 1 & x & 0\\ 0 & -1 & -1 & x\end{vmatrix} \overset{F_1+F_2\to F_1}{=} \begin{vmatrix} x+1 & 0 & 0 & 0\\ 1 & x & 1 & 1\\ 1 & 1 & x & 0\\ 0 & -1 & -1 & x\end{vmatrix} = (x+1)\left[x^3-1+x-x\right]=(x+1)(x^3-1)=$$

$$=(x+1)(x-1)(x^2+x+1)=0 \iff \begin{cases} \boxed{x=-1}\\ \boxed{x=1}\end{cases}$$

2- a) $3A^2=A \Rightarrow |3A^2|=|A| \Rightarrow 9|A|^2=|A| \Rightarrow 9|A|^2-|A|=0 \Rightarrow$

$$\Rightarrow |A|(9|A|-1)=0 \Rightarrow \begin{cases} \boxed{|A|=0}\\ 9|A|-1=0 \Rightarrow \boxed{|A|=\frac{1}{9}}\end{cases}$$

b) $A^2 X = A + CB \overset{|A|\neq 0}{\Rightarrow} X = (A^2)^{-1}(A+CB)$

$A^2 = \begin{pmatrix} 2 & 0 & 3 \\ 4 & 1 & 2 \\ 3 & 0 & 5 \end{pmatrix} \qquad |A| = 1 \Rightarrow |A^2| = 1 \qquad (A^2)^{-1} = \begin{pmatrix} 5 & 0 & -3 \\ -14 & 1 & 8 \\ -3 & 0 & 2 \end{pmatrix}$

$X = \begin{pmatrix} 5 & 0 & -3 \\ -14 & 1 & 8 \\ -3 & 0 & 2 \end{pmatrix} \cdot \left[\begin{pmatrix} 1 & 0 & 1 \\ 2 & 1 & 0 \\ 1 & 0 & 2 \end{pmatrix} + \begin{pmatrix} -1 & 3 \\ 1 & 1 \\ 6 & 2 \end{pmatrix} \begin{pmatrix} 1 & 1 & 2 \\ -1 & 3 & 1 \end{pmatrix} \right] = \begin{pmatrix} 5 & 0 & -3 \\ -14 & 1 & 8 \\ -3 & 0 & 2 \end{pmatrix} \left[\begin{pmatrix} 1 & 0 & 1 \\ 2 & 1 & 0 \\ 1 & 0 & 2 \end{pmatrix} + \begin{pmatrix} -4 & 8 & 1 \\ 0 & 4 & 3 \\ 4 & 12 & 14 \end{pmatrix} \right] =$

$= \begin{pmatrix} 5 & 0 & -3 \\ -14 & 1 & 8 \\ -3 & 0 & 2 \end{pmatrix} \begin{pmatrix} -3 & 8 & 2 \\ 2 & 5 & 3 \\ 5 & 12 & 16 \end{pmatrix} = \begin{pmatrix} -30 & 4 & -38 \\ 84 & -11 & 103 \\ 19 & 0 & 26 \end{pmatrix}$

3 a) i) $M^2 = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix} \begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$

$M^3 = M$

$M^4 = I_3$

ii) $|2M^{2018}| = 2^3 |M^{2018}| = 2^3 \cdot |I_3| = 8$

b) $N^2 - 2N - 3I_2 = 0_2 \Leftrightarrow N^2 - 2N + I_2 = 4I_2 \Leftrightarrow (N - I_2)^2 = 4I_2$

$N - I_2 = \begin{pmatrix} a-1 & b \\ b & a-1 \end{pmatrix} \qquad (N - I_2)^2 = \begin{pmatrix} (a-1)^2 + b^2 & 2b(a-1) \\ 2b(a-1) & (a-1)^2 + b^2 \end{pmatrix} = 4I_2 \Leftrightarrow$

$\Leftrightarrow \left.\begin{matrix} (a-1)^2 + b^2 = 4 \\ 2b(a-1) = 0 \end{matrix}\right\} \longrightarrow \begin{cases} b = 0 \Rightarrow (a-1)^2 = 4 \Leftrightarrow a-1 = \begin{cases} 2 \\ -2 \end{cases} \Leftrightarrow a = \begin{cases} 3 \\ -1 \end{cases} \\ a-1 = 0 \Rightarrow a = 1 \Leftrightarrow b^2 = 4 \Leftrightarrow b = \pm 2 \end{cases}$

$\begin{pmatrix} 3 & 0 \\ 0 & 3 \end{pmatrix}, \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}, \begin{pmatrix} 1 & 2 \\ 2 & 1 \end{pmatrix}, \begin{pmatrix} 1 & -2 \\ -2 & 1 \end{pmatrix}$

④ $(m+2)x+(m-1)y-z=3$
$mx-y+z=2$
$x+my-z=0$

$$\left.\begin{array}{l}\\ \\ \end{array}\right\}\left(\begin{array}{ccc|c}m+2 & m-1 & -1 & 3\\ m & -1 & 1 & 2\\ 1 & m & -1 & 0\end{array}\right) \quad \text{columnas } 1\text{–}3 = A$$

$$|A| = m+2-m^2+m-1-1-m^2-2m+m^2-1 = -m^2-m = 0 \Leftrightarrow -m(m+1)=0 \Leftrightarrow \begin{cases} m=0\\ m=-1\end{cases}$$

Si $m\neq 0,-1$ Rango $A=3=$ Rango $A/B=$ nº incógnitas (SCD)

$$x=\frac{\begin{vmatrix}3 & m-1 & -1\\ 2 & -1 & 1\\ 0 & m & -1\end{vmatrix}}{-m(m+1)}=\frac{3\cancel{-2m}-3m+\cancel{2m}-2}{-m(m+1)}=\frac{3m-1}{m(m+1)}$$

$$y=\frac{\begin{vmatrix}m+2 & 3 & -1\\ m & 2 & 1\\ 1 & 0 & -1\end{vmatrix}}{-m(m+1)}=\frac{\overset{m+1}{-2m-4+3+2+3m}}{-m(m+1)}=\frac{-1}{m}$$

$$z=\frac{\begin{vmatrix}m+2 & m-1 & 3\\ m & -1 & 2\\ 1 & m & 0\end{vmatrix}}{-m(m+1)}=\frac{\overset{m^2-2m+1}{3m^2+2m-2+3-2m^2-4m}}{-m(m+1)}=\frac{-(m-1)^2}{m(m+1)}$$

Si $m=0$

$$\begin{pmatrix}2 & -1 & -1 & 3\\ 0 & -1 & 1 & 2\\ 1 & 0 & -1 & 0\end{pmatrix}\underset{F_1-2F_3\to F_3}{\sim}\begin{pmatrix}2 & -1 & -1 & 3\\ 0 & -1 & 1 & 2\\ 0 & -1 & 1 & 3\end{pmatrix}\underset{F_2-F_3\to F_3}{\sim}\begin{pmatrix}2 & -1 & -1 & 3\\ 0 & -1 & 1 & 2\\ 0 & 0 & 0 & -1\end{pmatrix}$$

Rango $A=2\neq 3=$ Rango $A/B \Rightarrow$ SI

Si $m=-1$

$$\begin{pmatrix}1 & -2 & -1 & 3\\ -1 & -1 & 1 & 2\\ 1 & -1 & -1 & 0\end{pmatrix}\underset{\substack{F_1+F_2\to F_2\\ F_1-F_3\to F_3}}{\sim}\begin{pmatrix}1 & -2 & -1 & 3\\ 0 & -3 & 0 & 5\\ 0 & -1 & 0 & 3\end{pmatrix}\underset{F_2-3F_3\to F_3}{\sim}\begin{pmatrix}1 & -2 & -1 & 3\\ 0 & -3 & 0 & 5\\ 0 & 0 & 0 & -4\end{pmatrix}$$

Rango $A=2\neq 3=$ Rango $A/B \Rightarrow$ SI

① $\vec{u}(-1,2,3)$ $\vec{v}(2,0,-1)$ $\vec{w}(1,-1,-1)$

a) $\begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ -1 & 2 & 3 \\ 2 & 0 & -1 \end{vmatrix} = -2\vec{i}+5\vec{j}-4\vec{k}$ $|(2,-5,4)| = \sqrt{4+25+16} = \sqrt{45} = 3\sqrt{5}$

$\boxed{\vec{a}\left(\frac{2}{3\sqrt{5}}, \frac{-5}{3\sqrt{5}}, \frac{4}{3\sqrt{5}}\right)}$

b) $\vec{a} = (-t+2s,\ 2t,\ 3t-s) \perp \vec{v} \Leftrightarrow$

$\Leftrightarrow 2(-t+2s)-(3t-s)=0 \Leftrightarrow -2t+4s-3t+s=0 \Leftrightarrow -5t+5s=0 \Leftrightarrow \boxed{t=s}$

$\vec{a}(t,2t,2t)$ $\forall t\in\mathbb{R}\setminus\{0\}$ Por ejemplo $t=1 \Rightarrow \boxed{\vec{a}(1,2,2)}$

c) $\begin{vmatrix} -1 & 2 & 3 \\ 2 & 0 & -1 \\ 1 & -1 & -1 \end{vmatrix} = 0-6-2-0+1+4 = -8+5 = -3$

$V_{tetraedro} = \frac{1}{6}|-3| = \frac{1}{2}\,u^3$

② $P(1,0,5)$ $r\begin{cases} x=3+2t \\ y=-1-t \\ z=1 \end{cases}$ $a(3,-1,1)$ $\vec{u}_r(2,-1,0)$

a) $2x-y+D=0 \xrightarrow{P} 2+D=0 \Rightarrow D=-2$ $\boxed{2x-y-2=0 \rightarrow \text{Plano} \perp r \text{ por } P}$

Sea M pto de corte de r y π

$2(3+2t)-(-1-t)-2=0 \Leftrightarrow 6+4t+1+t-2=0 \Leftrightarrow 5t+5=0 \Leftrightarrow \boxed{t=-1}$

$M(1,0,1)$ pto de intersección de r y π.

Recta s:

$\vec{PM}(0,0,-4)$ $\vec{u}_s(0,0,1)$ $\boxed{s\equiv\begin{cases} x=1 \\ y=0 \\ z=5+\lambda \end{cases}}$ $\boxed{\frac{x-1}{0}=\frac{y}{0}=\frac{z-5}{1}}$

b) $(1,0,1) = \left(\frac{x+1}{2}, \frac{y+0}{2}, \frac{z+5}{2}\right) \Leftrightarrow (2,0,2) = (x+1,\ y,\ z+5) \Leftrightarrow \boxed{P'(1,0,-3)}$

c) $\begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 2 & -1 & 0 \\ 0 & 0 & 1 \end{vmatrix} = -\vec{i}-2\vec{j}+0\vec{k}$ $\vec{n}(1,2,0)$

$x+2y+D=0 \xrightarrow{P} D=-1$ $\boxed{\pi \equiv x+2y-1=0}$

③ $\pi_1 \equiv 3x+4y-5z-7=0$ $\qquad \pi_2 \equiv x-2y+z-4=0$

a)

$$\frac{3}{1} \neq \frac{4}{-2} \neq \frac{-5}{1} \neq \frac{-7}{-4}$$

π_1 y π_2 se cortan.

b)
$$\left.\begin{array}{l} 3x+4y=7 \\ x-2y=4 \end{array}\right\} \quad z=0 \qquad \begin{array}{r} 3x+4y=7 \\ + \; 2x-4y=8 \\ \hline 5x \quad =15 \end{array} \qquad x=3 \quad y=\frac{-1}{2}$$

$Q(3, \frac{-1}{2}, 0)$

$P(3,-1,2)$ $\quad Q(3, \frac{-1}{2}, 0)$ $\quad \overrightarrow{PQ}(0, \frac{1}{2}, -2)$ $\quad \vec{u}(3,4,5)$

$$d(P,r) = \frac{|\overrightarrow{PQ} \wedge \vec{u}|}{|\vec{u}|} = \frac{|(\frac{-21}{2}, 6, \frac{3}{2})|}{|(3,4,5)|} = \frac{\frac{\sqrt{594}}{2}}{5\sqrt{2}} = \frac{3\sqrt{66}}{10\sqrt{2}} = \boxed{\frac{3\sqrt{33}}{10}}$$

c) $\vec{n_1}(3,4,-5)$ $\quad \vec{n_2}(1,-2,1)$

$$\left.\begin{array}{l} \vec{n_1} \cdot \vec{n_2} = 3-8-5=-10 \\ |\vec{n_1}| = 5\sqrt{2} \quad |\vec{n_2}| = \sqrt{6} \end{array}\right\} \cos\alpha = \frac{|-10|}{10\sqrt{3}} = \frac{|-1|}{\sqrt{3}} = \frac{\sqrt{3}}{3} \Rightarrow \boxed{\alpha = 54^\circ 44'}$$

④ $\pi_1 \equiv ax+y-z+1=0$ $\quad \pi_2 \equiv 2x+ay+z-2=0$ $\quad \vec{n_1}(a,1,-1)$ $\quad \vec{n_2}(2,a,1)$

a) $\pi_1 // \pi_2 \Leftrightarrow \vec{n_1} // \vec{n_2} \Leftrightarrow \frac{a}{2} = \frac{1}{a} = \frac{-1}{1} \neq \frac{1}{-2}$ $\quad \nexists a \Rightarrow \pi_1$ y π_2 no paralelos

b) $\pi_1 \perp \pi_2 \Leftrightarrow \vec{n_1} \perp \vec{n_2} \Leftrightarrow 2a+a-1=0 \Leftrightarrow 3a=1 \Leftrightarrow \boxed{a=\frac{1}{3}}$

c)
$$\begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ a & 1 & -1 \\ 2 & a & 1 \end{vmatrix} = (1+a)\vec{i} - (a+2)\vec{j} + (a^2-2)\vec{k} \Rightarrow \vec{u_r} = (1+a, -a-2, a^2-2)$$

Plano $2(x+y)=z$; $\vec{n}(2,2,-1)$

Recta $\perp$ plano $\Leftrightarrow \vec{u_r} // \vec{n} \Leftrightarrow \frac{1+a}{2} = \frac{-a-2}{2} = \frac{a^2-2}{-1} \Leftrightarrow \boxed{a=\frac{-3}{2}}$

① $A(1,3,-1)$ $B(a,2,0)$ $C(1,5,4)$ $D(2,0,2)$

a) $\vec{AC}(0,2,5)$ $\vec{CD}(1,-5,-2)$ $\begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 0 & 2 & 5 \\ 1 & -5 & -2 \end{vmatrix} = 21\vec{i}+5\vec{j}-2\vec{k}$ $\vec{n}(21,5,-2)$

Plano que contiene a A, C y D $\rightarrow 21x+5y-2z+D=0 \xrightarrow{D} 42-4+D=0 \Rightarrow D=$

$\boxed{\pi: 21x+5y-2z-38=0}$

B coplanario con A, C y D $\Leftrightarrow B\in\pi \Leftrightarrow 21a+10-38=0 \Leftrightarrow 21a=28 \Leftrightarrow \boxed{a=\frac{4}{3}}$

b) $\vec{AB}(a-1,-1,1)$, $\vec{AC}(0,2,5)$, $\vec{AD}(1,-3,3)$ $\Big\}$ $14=\frac{1}{6}\left|[\vec{AB},\vec{AC},\vec{AD}]\right| \Leftrightarrow \left\|\begin{matrix} a-1 & -1 & 1 \\ 0 & 2 & 5 \\ 1 & -3 & 3 \end{matrix}\right\| = 84 \Leftrightarrow$

$\Leftrightarrow |6a-6-5-2+15a-15|=84 \Leftrightarrow |21a-28|=84 \Leftrightarrow$ $\begin{cases} 21a-28=84 \Leftrightarrow \boxed{a_1=\frac{16}{3}} \\ 21a-28=-84 \Leftrightarrow \boxed{a_2=\frac{-8}{3}} \end{cases}$

c) $\vec{AB}(a-1,-1,1)$, $\vec{CD}(1,-5,-2)$ $A=|\vec{AB}\wedge\vec{CD}|=|(7,2a-1,-5a+6)|=\sqrt{86} \Leftrightarrow$

$\Leftrightarrow 49+(2a-1)^2+(-5a+6)^2=86 \Leftrightarrow 4a^2-4a+1+25a^2-60a+36-37=0 \Leftrightarrow$

$\Leftrightarrow 29a^2-64a=0 \Leftrightarrow a(29a-64)=0 \Leftrightarrow$ $\boxed{\begin{matrix} a_1=0 \\ a_2=\frac{64}{29} \end{matrix}}$

② $P(1,-2,0)$ $Q(0,1,-3)$ $r: \begin{cases} x-2y+z=0 \\ x-y=0 \end{cases}$ $\vec{u}_r(1,1,1)$ $P_r(0,0,0)$

a) $\vec{PQ}(-1,3,-3)$

$\begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ -1 & 3 & -3 \\ 1 & 1 & 1 \end{vmatrix} = 6\vec{i}-2\vec{j}-4\vec{k}$ $\vec{n}(3,-1,-2)$ $\boxed{\pi: 3x-y-2z=0}$

b) $x+y+z+D=0 \xrightarrow{Q} D=2$ $\pi: x+y+z+2=0$ Plano $\perp$ a r por Q

$B=r\cap\pi$ $3\lambda+2=0 \Leftrightarrow \lambda=\frac{-2}{3}$ $B\left(\frac{-2}{3},\frac{-2}{3},\frac{-2}{3}\right)$

$\vec{BQ}\left(\frac{2}{3},\frac{5}{3},\frac{-7}{3}\right)$ $\vec{u}_s(2,5,-7)$

$$\boxed{s: \begin{cases} x=2\lambda \\ y=1+5\lambda \\ z=-3-7\lambda \end{cases}}$$

c) $P_r(\lambda,\lambda,\lambda)$ $\quad d(P,P_r)=d(Q,P_r) \iff d^2(P,P_r)=d^2(Q,P_r) \iff$

$\iff \cancel{(\lambda-1)^2}+(\lambda+2)^2+\cancel{\lambda^2}=\cancel{\lambda^2}+\cancel{(\lambda-1)^2}+(\lambda+3)^2 \iff \cancel{\lambda^2}+4\lambda+4=\cancel{\lambda^2}+6\lambda+9 \iff$

$\iff 2\lambda=-5 \iff \lambda=\frac{-5}{2}$ $\qquad \boxed{P_r\left(\frac{-5}{2},\frac{-5}{2},\frac{-5}{2}\right)}$

3) $r:\begin{cases} x+y-z-4=0 \\ x+2y-7=0 \end{cases}$ $\quad \vec{u}_r(2,-1,1)$ $\quad P_r(7,0,3)$ $\quad \Big| \quad s:\begin{cases} x-2=0 \\ y+5=0 \end{cases}$ $\quad \vec{u}_s(0,0,1)$ $\quad P_s(2,-5,0)$ $\qquad \vec{P_rP_s}(-5,-5,-2)$

a) $\vec{u}_r \not\parallel \vec{u}_s \Rightarrow$ r y s se cortan o se cruzan

$$\begin{vmatrix} -5 & -5 & -3 \\ 2 & -1 & 1 \\ 0 & 0 & 1 \end{vmatrix} = 15 \neq 0 \Rightarrow \text{r y s se cruzan.}$$

b) $d(r,s)=\frac{|[\vec{P_rP_s},\vec{u}_r,\vec{u}_s]|}{|\vec{u}_r \wedge \vec{u}_s|}=\frac{15}{\sqrt{5}}=\frac{15\sqrt{5}}{5}=\boxed{3\sqrt{5}\ u}$

$$\begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 2 & -1 & 1 \\ 0 & 0 & 1 \end{vmatrix} = -\vec{i}-2\vec{j} \qquad |(-1,-2,0)|=\sqrt{5}$$

c) $2x-y+z+D=0 \xrightarrow{P} D=-3$ $\qquad \pi: 2x-y+z-3=0$ Plano $\perp$ a r por P

$M=\pi\cap r \Rightarrow 2\cdot(7+2\lambda)-(-\lambda)+(3+\lambda)-3=0 \Rightarrow 14+4\lambda+\lambda+\lambda=0 \Rightarrow \boxed{\lambda=\frac{-7}{3}}$

$M=\left(\frac{7}{3},\frac{7}{3},\frac{2}{3}\right)=\left(\frac{x+1}{2},\frac{y+2}{2},\frac{z+3}{2}\right) \Rightarrow \begin{cases} 14=3x+3 \Rightarrow x=\frac{11}{3} \\ 14=3y+6 \Rightarrow y=\frac{8}{3} \\ 4=3z+9 \Rightarrow z=\frac{-5}{3} \end{cases}$ $\qquad \boxed{P'\left(\frac{11}{3},\frac{8}{3},\frac{-5}{3}\right)}$

4) $r:\begin{cases} x+y-5z+3=0 \\ -2x+z-1=0 \end{cases}$ $\quad \vec{u}_r(1,9,2)$ $\quad P_r(0,2,1)$ $\qquad s: x+1=\frac{y-13}{a}=\frac{z+5}{b}$ $\quad P_s(-1,13,-5)$

a) $r \parallel s \iff \vec{u}_r \parallel \vec{u}_s \iff \frac{1}{1}=\frac{9}{a}=\frac{2}{b} \iff \boxed{\begin{matrix} a=9 \\ b=2 \end{matrix}}$

$\vec{P_rP_s}(-1,11,-6)$

$$\begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ -1 & 11 & -6 \\ 1 & 9 & 2 \end{vmatrix} = 76\vec{i}-4\vec{j}-20\vec{k} \qquad \vec{n}(19,-1,-5) \qquad 19x-y-5z+D=0 \xrightarrow{P_r} D=7$$

$\boxed{\pi: 19x-y-5z+7=0}$

$d(r,s)=d(P_r,s)=\frac{|\vec{P_rP_s}\wedge\vec{u}_s|}{|\vec{u}_s|}=\frac{|(76,-4,-20)|}{|(1,9,2)|}=\frac{\sqrt{6192}}{\sqrt{86}}=\sqrt{72}=\boxed{6\sqrt{2}\ u}$

b) se cortan

$$\begin{vmatrix} -1 & 11 & -6 \\ 1 & 9 & 2 \\ 1 & a & b \end{vmatrix} = -9b-6a+22+54+2a-11b=-4a-20b+76=0$$

$\vec{u}_r\cdot\vec{u}_s=0 \iff 1+9a+2b=0$

$\begin{cases} a+5b=19 \\ 9a+2b=-1 \end{cases}$ $\quad \begin{cases} 9a+45b=171 \\ 9a+2b=-1 \end{cases}$

$43b=172 \Rightarrow \boxed{b=4}$

$a+20=19$ $\quad \boxed{a=-1}$

$P_r(0,2,1)$ $\quad$ $P_s(-1,13,-5)$ $\quad$ $\begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & 9 & 2 \\ 1 & -1 & 4 \end{vmatrix} = 38\vec{i} - 2\vec{j} - 10\vec{k}$ $\quad$ $\vec{n}(19,-1,-5)$

$\vec{u_r}(1,9,2)$ $\quad$ $\vec{u_s}(1,-1,4)$

$$19x - y - 5z + D = 0 \xrightarrow{P_r} D = 7 \qquad \boxed{\pi: 19x - y - 5z + 7 = 0}$$

$$r: \begin{cases} x = \lambda \\ y = 2 + 9\lambda \\ z = 1 + 2\lambda \end{cases} \qquad s: \begin{cases} x = -1 + t \\ y = 13 - t \\ z = -5 + 4t \end{cases} \qquad \begin{cases} \lambda = -1 + t \\ 2 + 9\lambda = 13 - t \\ 1 + 2\lambda = -5 + 4t \end{cases}$$

$$2 + 9(-1+t) = 13 - t \Rightarrow -7 + 9t = 13$$

$$10t = 20 \Rightarrow \boxed{t = 2} \quad \boxed{\lambda = 1}$$

$$Q = r \cap s \Leftrightarrow \boxed{Q(1,11,3)}$$

1) a) i) $\lim_{x\to\infty}\left(\frac{x}{\sqrt{x-1}}-\frac{x}{\sqrt{x+1}}\right)=(\infty-\infty)=\lim_{x\to\infty}\frac{x(\sqrt{x+1}-\sqrt{x-1})}{\sqrt{x^2-1}}=$

$=\lim_{x\to\infty}\frac{x(\sqrt{x+1}-\sqrt{x-1})(\sqrt{x+1}+\sqrt{x-1})}{\sqrt{x^2-1}\,(\sqrt{x+1}+\sqrt{x-1})}=\lim_{x\to\infty}\frac{x\,(x+1-x+1)}{\sqrt{x^2-1}\,(\sqrt{x+1}+\sqrt{x-1})}=$

$=\lim_{x\to\infty}\frac{2x}{\sqrt{x^2-1}\,(\sqrt{x+1}+\sqrt{x-1})}=\frac{\infty}{\infty}\to\lim_{x\to\infty}\frac{2x}{x(\sqrt{x}+\sqrt{x})}=0$

ii) $\lim_{x\to 1}\left(\frac{3}{x^3-1}-\frac{1}{x-1}\right)=(\infty-\infty)=\lim_{x\to 1}\frac{3-(x^2+x+1)}{x^3-1}=\lim_{x\to 1}\frac{-x^2-x+2}{x^3-1}=($

$=\lim_{x\to 1}\frac{-(x+2)\cancel{(x-1)}}{\cancel{(x-1)}(x^2+x+1)}=\frac{-3}{3}=-1$

b) $\lim_{x\to\infty}\left(\frac{x^2+5x}{x^2+1}\right)^{ax}=\{1^\infty\}=\lim_{x\to\infty}\left(1+\frac{x^2+5x}{x^2+1}-1\right)^{ax}=\lim_{x\to\infty}\left(1+\frac{\cancel{x^2}+5x-\cancel{x^2}-1}{x^2+1}\right)^{ax}$

$=\lim_{x\to\infty}\left(1+\frac{1}{\frac{x^2+1}{5x-1}}\right)^{ax}=\lim_{x\to\infty}\left(1+\frac{1}{\frac{x^2+1}{5x-1}}\right)^{\frac{x^2+1}{5x-1}\cdot\frac{5x-1}{x^2+1}\cdot ax}=e^{\lim_{x\to\infty}\frac{5ax^2-ax}{x^2+1}}=e^{5a}$

$5a=-5\Rightarrow\boxed{a=-1}$

2) a) i) $x^2+3x=0\Leftrightarrow x=\begin{cases}0\\-3\end{cases}$ $\quad$ $+\ (-3)\ -\ (0)\ +$ $\quad$ $-\ (0)\ +$

$f(x)=\begin{cases}\frac{x^2+3x}{1-x} & x\le -3\\ \frac{-x^2-3x}{1-x} & -3<x\le 0\\ \frac{x^2+3x}{1+x} & x>0\end{cases}$

ii) $f(-3)=0$, $\lim_{x\to -3^-}\frac{x^2+3x}{1-x}=0$, $\lim_{x\to -3^+}\frac{x^2+3x}{1+x}=0$ $\Big\}$ f cta en $x=-3$

$f(0)=0$, $\lim_{x\to 0^-}\frac{-x^2-3x}{1-x}=0$, $\lim_{x\to 0^+}\frac{x^2+3x}{1+x}=0$ $\Big\}$ f cta en $x=0$

$f'(x)=\begin{cases}\frac{-x^2+2x+3}{(1-x)^2} & x<-3\\ \frac{x^2-2x-3}{(1-x)^2} & -3<x<0\\ \frac{x^2+2x+3}{(1+x)^2} & x>0\end{cases}$

$\boxed{x=-3}$

$\lim_{x\to -3^-}f'(x)=\frac{-3}{4}$, $\lim_{x\to -3^+}f'(x)=\frac{3}{4}$ $\Big\}$ No deriv. $x=-3$

$\boxed{x=0}$

$\lim_{x\to 0^-}f'(x)=-3$, $\lim_{x\to 0^+}f'(x)=3$ $\Big\}$ No deriv. $x=0$

b) $f(x)=\frac{ax+b}{cx-1}$ $\quad$ $\frac{c}{2}-1=0\Rightarrow\boxed{c=2}$

$f(x)=\frac{ax+b}{2x-1}$ $\quad$ $f(1)=a+b=-1$

$f'(x)=\frac{-a-2b}{(2x-1)^2}$ $\quad$ $f'(1)=-a-2b=5$

$-b=4\rightarrow\boxed{b=-4}$ $\quad$ $\boxed{a=-1-b=3}$

③ a) $f(x) = \begin{cases} 5+2\operatorname{sen}(ax) & x\leq 0 \\ ax^2+bx+b & x>0 \end{cases}$ $\quad f'(x) = \begin{cases} 2a\cos(ax) & x\leq 0 \\ 2ax+b & x>0 \end{cases}$

$f(0) = 5$

$\left.\begin{array}{l} \lim_{x\to 0^-} f(x) = 5 \\ \lim_{x\to 0^+} f(x) = b \end{array}\right\}$ f cta em $x=0$ $\Leftrightarrow \boxed{b=5}$

$\left.\begin{array}{l} \lim_{x\to 0^-} f'(x) = 2a \\ \lim_{x\to 0^+} f'(x) = b \end{array}\right\}$ f derivável em $x=0$ $\Leftrightarrow 2a = b = 5 \Leftrightarrow \boxed{a=\frac{5}{2}}$

b) $5x+5=6 \Leftrightarrow x=\frac{1}{5}$ $\quad \left(\frac{1}{5}, \frac{61}{10}\right)$

④ a) i) $y = x\operatorname{arcsen}\frac{1}{x} + \sqrt{x^2-1}$

$y' = \operatorname{arcsen}\frac{1}{x} + \cancel{x}\frac{1}{\sqrt{1-\left(\frac{1}{x}\right)^2}}\cdot\frac{-1}{x^{\cancel{2}}} + \frac{1}{\cancel{2}\sqrt{x^2-1}}\cdot\cancel{2}x =$

$= \operatorname{arcsen}\frac{1}{x} - \frac{1}{\sqrt{x^2-1}} + \frac{x}{\sqrt{x^2-1}} = \operatorname{arcsen}\frac{1}{x} + \frac{x-1}{\sqrt{x^2-1}} =$

$= \operatorname{arcsen}\frac{1}{x} + \frac{\sqrt{x^2-1}}{x+1}$

ii) $y = (\operatorname{sen}(3x-1))^{2x+1}$

$\ln y = (2x+1)\ln(\operatorname{sen}(3x-1)) \Leftrightarrow \frac{y'}{y} = 2\ln(\operatorname{sen}(3x-1)) + (2x+1)\operatorname{cotg}(3x-1)\cdot 3$

$y' = (\operatorname{sen}(3x-1))^{2x+1}\left[2\ln(\operatorname{sen}(3x-1)) + (6x+3)\operatorname{cotg}(3x-1)\right]$

b) $f(x) = e^{ax^2+b}$

$f(1) = e^{a+b} = 1 \Leftrightarrow a+b=0 \Leftrightarrow a=-b$

$f'(x) = 2ax\,e^{ax^2+b}$

$f'(1) = 2a\,e^{a+b} = 2a = -2 \Leftrightarrow \boxed{a=-1}$ $\quad \boxed{b=1}$

1) a) i) $\lim_{x\to 0} \frac{e^{\operatorname{sen} x} - e^x + \operatorname{sen}^2 x}{2x^2} = \left(\frac{0}{0}\right) \underset{L'H}{=} \lim_{x\to 0} \frac{\cos x\, e^{\operatorname{sen} x} - e^x + 2\operatorname{sen} x \cos x}{4x} =$

$= \left(\frac{0}{0}\right) \underset{L'H}{=} \lim_{x\to 0} \frac{-\operatorname{sen} x\, e^{\operatorname{sen} x} + \cos^2 x\, e^{\operatorname{sen} x} - e^x + 2\cos 2x}{4} = \frac{1}{2}$

a) ii) $\lim_{x\to 1} \left(\sqrt{2x-1}\right)^{\frac{x}{x-1}} = \{1^\infty\} = \lim_{x\to 1} \left(1 + \frac{1}{\frac{1}{\sqrt{2x-1}-1}}\right)^{\frac{1}{\sqrt{2x-1}-1}\left(\sqrt{2x-1}-1\right)\frac{x}{x-1}}$

$= e^{\lim_{x\to 1} \frac{x\left(\sqrt{2x-1}-1\right)}{x-1}} = e^{\lim_{x\to 1}\left(\sqrt{2x-1} - 1 + x\frac{1}{\sqrt{2x-1}}\right)} =$

$= e^{\lim_{x\to 1} \frac{2x-1-\sqrt{2x-1}+x}{\sqrt{2x-1}}} = e^{\lim_{x\to 1} \frac{3x-1-\sqrt{2x-1}}{\sqrt{2x-1}}} = e^{\frac{1}{1}} = e$

b) $\pi = \frac{1}{3}\pi r^2 h \Rightarrow 3 = r^2 h \Rightarrow h = \frac{3}{r^2}$

$g^2 = h^2 + r^2 = \left(\frac{3}{r^2}\right)^2 + r^2 \Rightarrow g = \sqrt{\frac{9}{r^4} + r^2} \Rightarrow g = \frac{\sqrt{9+r^6}}{r^2}$

$g' = \frac{\frac{3r^{\cancel{7}6}}{\sqrt{9+r^6}} - 2\cancel{r}\sqrt{9+r^6}}{r^{\cancel{4}3}} = \frac{3r^6 - 18 - 2r^6}{r^3\sqrt{9+r^6}} = \frac{r^6-18}{r^3\sqrt{9+r^6}} = 0$

$\Leftrightarrow r^6 = 18 \Leftrightarrow r = \sqrt[6]{18} = 1'619$ signo de g' $\underset{\searrow}{-}\ 1'619\ \underset{\nearrow}{+}$

$g = \frac{\sqrt{9+18}}{\sqrt[6]{18^2}} = \frac{\sqrt{27}}{\sqrt[3]{18}} = \frac{3\sqrt{3}}{\sqrt[3]{18}} \simeq 1'98\text{ dm}$

$3 = r^2 h \Rightarrow r^2 = \frac{3}{h}$

$g^2 = h^2 + r^2 = h^2 + \frac{3}{h} = \frac{h^3+3}{h} \Rightarrow g = \sqrt{\frac{h^3+3}{h}} \Rightarrow g' = \frac{\frac{3h^2 h - h^3 - 3}{h^2}}{2\sqrt{\frac{h^3+3}{h}}} \Rightarrow$

$\Rightarrow g' = \frac{2h^3-3}{2h^2\sqrt{\frac{h^3+3}{h}}} = 0 \Leftrightarrow 2h^3 - 3 = 0 \Leftrightarrow h^3 = \frac{3}{2} \Leftrightarrow h = \sqrt[3]{\frac{3}{2}} \simeq 1'144$

Signo de g' $\underset{\searrow}{-}\ 1'144\ \underset{\nearrow}{+}$ $g = \sqrt{\frac{\frac{3}{2}+3}{\sqrt[3]{\frac{3}{2}}}} = 1'98$

2) a) $y = \dfrac{4x^3}{ax^2+b}$ $\quad y = 2x+3$

$\lim\limits_{x\to\infty} \dfrac{4x^3}{ax^3+bx} = \left(\dfrac{\infty}{\infty}\right) = \dfrac{4}{a} = m$ $\Bigg\} \Rightarrow \dfrac{4}{a} = 2 \Rightarrow \boxed{a=2}$

$\lim\limits_{x\to 2^-} f(x) = -\infty \Leftrightarrow \lim\limits_{x\to 2^-} \dfrac{4x^3}{2x^2+b} = -\infty \Leftrightarrow 2\cdot 2^2 + b = 0 \Leftrightarrow \boxed{b=-8}$

b) $y = \ln(1+x^2)$

$y' = \dfrac{2x}{1+x^2} = 0 \Leftrightarrow x = 0$ $\quad$ $-$ | 0 | $+$ (↘ 0 ↗) $\quad$ $x=0$ mínimo

$\nexists$ máx.

$y'' = \dfrac{2(1+x^2) - 2x\cdot 2x}{(1+x^2)^2} = \dfrac{2+2x^2-4x^2}{(1+x^2)^2} = \dfrac{2-2x^2}{(1+x^2)^2} = 0 \Leftrightarrow x = \begin{cases} -1 \\ 1 \end{cases}$

$-$ | -1 | $+$ | 1 | $-$ ($\cap$ $\cup$ $\cap$)

$\left.\begin{array}{l} x=-1 \\ x=1 \end{array}\right\}$ Ptos de inflexión.

3) a) $\displaystyle\int (2x+1)\ln(x-1)\,dx = (x^2+x)\ln(x-1) - \int \frac{x^2+x}{x-1}\,dx =$

$u = \ln(x-1)$ $\quad du = \dfrac{1}{x-1}dx$

$dv = (2x+1)dx$ $\quad v = x^2+x$

$$\begin{array}{r|l} x^2+x & x-1 \\ \hline -x^2+x & x+2 \\ 2x & \\ -2x+2 & \\ 2 & \end{array}$$

$\displaystyle = (x^2+x)\ln(x-1) - \int (x+2)\,dx - \int \frac{2}{x-1}\,dx = (x^2+x)\ln(x-1) - \frac{x^2}{2} - 2x -$

$\displaystyle - 2\ln(x-1) + K = (x^2+x-2)\ln(x-1) - \frac{x^2}{2} - 2x + K$

b) $\displaystyle\int e^{2x}\,\text{sen}(3x+5)\,dx = \frac{1}{2}e^{2x}\,\text{sen}(3x+5) - \frac{3}{2}\int e^{2x}\cos(3x+5)\,dx =$

$u = \text{sen}(3x+5)$ $\quad du = 3\cos(3x+5)$

$dv = e^{2x}dx$ $\quad v = \frac{1}{2}e^{2x}$

$u = \cos(3x+5)$ $\quad du = -3\,\text{sen}(3x+5)$

$dv = e^{2x}dx$ $\quad v = \frac{1}{2}e^{2x}$

$\displaystyle = \frac{e^{2x}\,\text{sen}(3x+5)}{2} - \frac{3}{4}e^{2x}\cos(3x+5) - \frac{9}{4}\int e^{2x}\,\text{sen}(3x+5)\,dx \Rightarrow$

$\displaystyle \Rightarrow \int e^{2x}\,\text{sen}(3x+5)\,dx = \frac{\cancel{4}}{13}\left(\frac{e^{2x}(2\,\text{sen}(3x+5) - 3\cos(3x+5))}{\cancel{4}}\right) + K$

4) a) $\displaystyle\int_0^2 x\sqrt[3]{x^2-4}\,dx = \frac{1}{2}\int_{-4}^{0}\sqrt[3]{t}\,dt = \frac{1}{2}\,\frac{t^{4/3}}{\frac{4}{3}}\Bigg|_{-4}^{0} = \frac{3}{8}\left(-\sqrt[3]{256}\right) =$

$t = x^2-4 \quad dt = 2x\,dx$

$x=0 \to t=-4$

$x=2 \to t=0$

$\displaystyle = \frac{-3}{8}\,4\sqrt[3]{4} = \frac{-3}{2}\sqrt[3]{4}$

b) $f(x) = x^2 e^{3x-1} \qquad g(x) = e^{3x-1} \qquad (f-g)(x) = (x^2-1)e^{3x-1} = 0 \iff x = \begin{cases}-1\\ 1\end{cases}$

$$\int_{-1}^{1}(1-x^2)e^{3x-1}\,dx \underset{\substack{u=1-x^2 \quad du=-2x\,dx\\ dv=e^{3x-1}dx \quad v=\frac{1}{3}e^{3x-1}}}{=} \frac{(1-x^2)e^{3x-1}}{3}\Bigg|_{-1}^{1} + \frac{2}{3}\int_{-1}^{1} x\,e^{3x-1}\,dx =$$

$$= \frac{2}{3}\int_{-1}^{1} x\,e^{3x-1}\,dx \underset{\substack{u=x \quad du=dx\\ dv=e^{3x-1}dx \quad v=\frac{1}{3}e^{3x-1}}}{=} \frac{2x\,e^{3x-1}}{9}\Bigg|_{-1}^{1} - \frac{2}{9}\int_{-1}^{1} e^{3x-1}\,dx =$$

$$= \frac{2e^2}{9} + \frac{2e^{-4}}{9} - \frac{2}{27}e^{3x-1}\Bigg|_{-1}^{1} = \frac{2e^2}{9} + \frac{2e^{-4}}{9} - \frac{2e^2}{27} + \frac{2e^{-4}}{27} =$$

$$= \frac{4e^2}{27} + \frac{8e^{-4}}{27} = \frac{4e^6+8}{27e^4}$$

① a) $f(x)=\begin{cases} x^2+2 & x\le 0 \\ \sqrt{ax+b} & 0<x\le 2 \\ \frac{3}{\sqrt2}-\frac{x}{2\sqrt2} & x>2\end{cases}$ $\qquad f'(x)=\begin{cases} 2x & x\le 0 \\ \frac{-1}{2\sqrt{4-x}} & 0<x\le 2 \\ \frac{-1}{2\sqrt2} & x>2\end{cases}$

$\boxed{x=0}$ $f(0)=2$

$\left.\begin{array}{l}\lim_{x\to 0^-} f(x)=2 \\ \lim_{x\to 0^+} f(x)=\sqrt{b}\end{array}\right\}\ \boxed{b=4}$

$\boxed{x=2}$ $f(2)=\sqrt{2a+4}$

$\left.\begin{array}{l}\lim_{x\to 2^-} f(x)=\sqrt{2a+4} \\ \lim_{x\to 2^+} f(x)=\sqrt{2}\end{array}\right\}\ 2a+4=2 \Leftrightarrow \boxed{a=-1}$

b) $\boxed{x=0}$

$\left.\begin{array}{l}\lim_{x\to 0^-} f'(x)=0 \\ \lim_{x\to 0^+} f'(x)=\frac{-1}{4}\end{array}\right\}$ No derivable en $x=0$

$\boxed{x=2}$

$\left.\begin{array}{l}\lim_{x\to 2^-} f'(x)=\frac{-1}{2\sqrt2} \\ \lim_{x\to 2^+} f'(x)=\frac{-1}{2\sqrt2}\end{array}\right\}$ Derivable en $x=2$

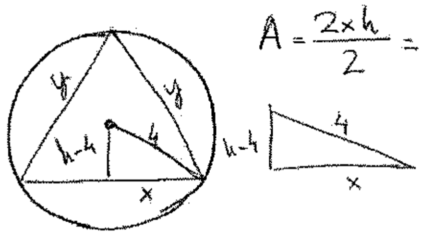
② a) i) $\lim_{x\to 0}\frac{tg\,x-\operatorname{sen}x}{x-\operatorname{sen}x}=\left(\frac{0}{0}\right)\underset{L'H}{=}\lim_{x\to 0}\frac{\frac{1}{\cos^2 x}-\cos x}{1-\cos x}=\lim_{x\to 0}\frac{1-\cos^3 x}{\cos^2 x\,(1-\cos x)}=\left(\frac{0}{0}\right)$

$=\lim_{x\to 0}\frac{\cancel{(1-\cos x)}(\cos^2 x+\cos x+1)}{\cos^2 x\,\cancel{(1-\cos x)}}=\frac{3}{1}=3$

ii) $\lim_{x\to 0}(x+\cos x)^{\frac{2}{\operatorname{sen}x}}=\{1^\infty\}=e^{\lim_{x\to 0}\frac{2\ln(x+\cos x)}{\operatorname{sen}x}}=$

$A=(x+\cos x)^{\frac{2}{\operatorname{sen}x}}\Rightarrow \ln A=\frac{2}{\operatorname{sen}x}\ln(x+\cos x)\Rightarrow A=e^{\frac{2\ln(x+\cos x)}{\operatorname{sen}x}}$

$\underset{L'H}{=}e^{\lim_{x\to 0}\frac{2\,\frac{1-\operatorname{sen}x}{x+\cos x}}{\cos x}}=e^{\lim_{x\to 0}\frac{2-2\operatorname{sen}x}{\cos x(x+\cos x)}}=e^{\frac{2-0}{1(0+1)}}=e^2$

b) 

$A=\frac{2x\,h}{2}=x\left(4+\sqrt{16-x^2}\right)=4x+x\sqrt{16-x^2}$

$(h-4)^2=16-x^2\Rightarrow h-4=\sqrt{16-x^2}\Rightarrow h=4+\sqrt{16-x^2}$

$A'(x)=4+\sqrt{16-x^2}+x\cdot\frac{-\cancel{2}x}{\cancel{2}\sqrt{16-x^2}}=\frac{4\sqrt{16-x^2}+16-x^2-x^2}{\sqrt{16-x^2}}=0\Leftrightarrow$

$\Leftrightarrow 4\sqrt{16-x^2}=2x^2-16\Leftrightarrow \cancel{256}-16x^2=4x^4-64x^2+\cancel{256}\Leftrightarrow 4x^4-48x^2=0\Leftrightarrow$

$\Leftrightarrow x=\begin{cases}\cancel{0}\\ 2\sqrt3\\ \cancel{-2\sqrt3}\end{cases}$ $\qquad \overset{+}{\nearrow}\ 2\sqrt3\ \overset{-}{\searrow}$

Base $4\sqrt3$
Altura 6
Lado igual $4\sqrt3$

$\Big\}$ $\triangle$ equilátero de lado $4\sqrt3$

③ a) $y=\frac{x\ln x}{x-1}$ $\text{Dom} f=(0,\infty)-\{1\}$

$$y'=\frac{(\ln x+1)(x-1)-x\ln x}{(x-1)^2}=\frac{\cancel{x\ln x}-\ln x+x-1-\cancel{x\ln x}}{(x-1)^2}=0 \iff \ln x=x-1 \iff x=e^{x-1}\iff x=1$$

$0 \quad + \quad 1 \quad +$ $\nearrow \quad \nearrow$ Crece en Dom f   No hay extremos relativos

Asíntotas

AV $x=1$

$$\lim_{x\to 1}\frac{x\ln x}{x-1}=\left(\frac{0}{0}\right)\underset{L'H}{=}\lim_{x\to 1}\frac{\ln x+1}{x}=1$$ No hay AV

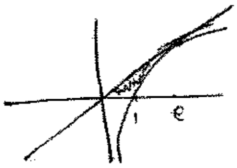
AH

$$\lim_{x\to\infty}\frac{x\ln x}{x-1}=\left(\frac{\infty}{\infty}\right)\underset{L'H}{=}\lim_{x\to\infty}\frac{\ln x+1}{1}=\infty$$ No hay AH

AO

$$\lim_{x\to\infty}\frac{\ln x}{x-1}=\left(\frac{\infty}{\infty}\right)\underset{L'H}{=}\lim_{x\to\infty}\frac{1}{x}=0$$ No hay AO

b) $y=\ln x$ $y'=\frac{1}{x}$ $(e,1)$ $m=\frac{1}{e}$ $\boxed{y=\frac{x}{e}}$ → Recta tg a $y=\ln x$ en $(e,1)$



$$A=\int_0^1\frac{x}{e}\,dx+\int_1^e\left(\frac{x}{e}-\ln x\right)dx=\frac{1}{e}\left[\frac{x^2}{2}\right]_0^1+\frac{1}{e}\left[\frac{x^2}{2}\right]_1^e-\int_1^e\ln x\,dx=$$

$$\underset{\substack{u=\ln x \quad du=\frac{1}{x}dx\\ dv=dx \quad v=x}}{=}\frac{e}{2}-x\ln x\Big|_1^e+\int_1^e dx=\frac{e}{2}-e+e-1=\frac{e}{2}-1=\frac{e-2}{2}$$

④ a) $$\int\frac{6x^3-x}{1+x^4}\underset{\substack{t=1+x^4 \;|\; t=x^2\\ dt=4x^3dx \;|\; dt=2x\,dx}}{=}\frac{6}{4}\int\frac{dt}{t}-\frac{1}{2}\int\frac{dt}{1+t^2}=\frac{3}{2}\ln|t|-\frac{1}{2}\text{arctg}\,t+K=$$

$$=\frac{3}{2}\ln(1+x^4)-\frac{1}{2}\text{arctg}\,x^2+K$$

b) $$\int\frac{2x^3-3x^2+12}{x^2-4}\,dx=\int(2x-3)\,dx+4\int\frac{2x}{x^2-4}\,dx=x^2-3x+4\ln|x^2-4|+K$$

$$\begin{array}{l|l}
2x^3-3x^2\qquad+12 & x^2-4\\
\hline
-2x^3\qquad\;\; -8x & 2x-3\\
\hline
-3x^2+8x+12 & \\
-\;-3x^2\qquad +12 & \\
\hline
8x &
\end{array}$$

① a) $AB = \begin{pmatrix} K & 0 & -1 \\ 3K & K & 2K-2 \\ 1 & 1 & 2 \end{pmatrix}$ $|AB| = \cancel{2K^2} - 3K + K - \cancel{2K^2} + 2K = 3K - 3K = 0 \quad \forall K \quad \nexists (AB)^{-1}$

b) $BA = \begin{pmatrix} K & -1 \\ 3 & K+2 \end{pmatrix}$ $|BA| = K^2 + 2K + 3 \neq 0 \quad \forall K$ $(BA)^{-1} = \frac{1}{K^2+2K+3} \begin{pmatrix} K+2 & 1 \\ -3 & K \end{pmatrix}$

$BA = \begin{pmatrix} 1 & -1 \\ 3 & 3 \end{pmatrix}$ $|BA| = 6$ $(BA)^{-1} = \frac{1}{6} \begin{pmatrix} 3 & 1 \\ -3 & 1 \end{pmatrix}$

c) $(\lambda, -3\lambda, \lambda) \quad \lambda \in \mathbb{R}$

② a) $AX = X - B \Rightarrow B = X - AX = (I_3 - A)X \Rightarrow (I_3 - A)^{-1} B = X$

$I_3 - A = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} - \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ -1 & 0 & 0 \end{pmatrix} = \begin{pmatrix} 1 & 0 & -1 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{pmatrix}$ $|I_3 - A| = 1 + 1 = 2$

$(I_3 - A)^{-1} = \frac{1}{2} \begin{pmatrix} 1 & 0 & 1 \\ 0 & 2 & 0 \\ -1 & 0 & 1 \end{pmatrix}$ $X = \frac{1}{2} \begin{pmatrix} 1 & 0 & 1 \\ 0 & 2 & 0 \\ -1 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 0 & -1 & -1 \end{pmatrix} = \frac{1}{2} \begin{pmatrix} 1 & -1 & 0 \\ 0 & 2 & 2 \\ -1 & -1 & -2 \end{pmatrix}$

b) $MN = \begin{pmatrix} y & 0 & 0 \\ x & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$ $NM = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & x \\ 0 & 0 & y \end{pmatrix}$ $\boxed{\begin{matrix} x = 0 \\ y = 1 \end{matrix}}$

$M^{2019} = M \quad (M^2 = I_3)$

③ a) $\begin{vmatrix} a & 1 & 0 & 1 \\ 1 & a & 1 & 0 \\ 0 & 1 & a & 1 \\ 1 & 0 & 1 & a \end{vmatrix} = \begin{vmatrix} a & 1 & 0 & 1 \\ 1 & a & 1 & 0 \\ 0 & 1 & a & 1 \\ 0 & -a & 0 & a \end{vmatrix} = a \begin{vmatrix} a & 1 & 0 \\ 1 & a & 1 \\ -a & 0 & a \end{vmatrix} - \begin{vmatrix} 1 & 0 & 1 \\ 1 & a & 1 \\ -a & 0 & a \end{vmatrix} =$

$= a(a^3 - a - a) - (a^2 + a^2) = a^4 - 2a^2 - 2a^2 = \boxed{a^4 - 4a^2} = 0 \Leftrightarrow \begin{cases} a = 0 \\ a = 2 \\ a = -2 \end{cases}$

b) $\begin{vmatrix} -x & 0 & 0 & 1 \\ 1 & -x & 0 & 0 \\ 0 & 1 & -x & 0 \\ 0 & 0 & 1 & -x \end{vmatrix} = -x(-x^3) - \begin{vmatrix} 0 & 0 & 1 \\ 1 & -x & 0 \\ 0 & 1 & -x \end{vmatrix} = x^4 - 1 = 0 \Leftrightarrow (x^2+1)(x^2-1) = 0 \Leftrightarrow$

$\Leftrightarrow x = \begin{cases} 1 \\ -1 \end{cases}$

$m \neq 0 \quad |A| = 6m^2 + \cancel{12m} + 16 - \cancel{12m} - 4m^2 - 24 = 2m^2 - 8 = 0 \Leftrightarrow m = \pm 2$

$m \neq 2$ y -2 SCD

$\frac{16 \pm \sqrt{256-240}}{6} = \frac{16 \pm 4}{6} = \begin{cases} \frac{10}{3} \\ 2 \end{cases}$

$$x = \frac{\begin{vmatrix} 0 & 2 & 6 \\ 2 & m & 4 \\ m-2 & m & 6 \end{vmatrix}}{2m^2-8} = \frac{-6m^2+32m-40}{2m^2-8} = \frac{-2(3m^2-16m+20)}{2m^2-8} = \frac{-\cancel{2}(3m-10)\cancel{(m-2)}}{\cancel{2}(m+2)\cancel{(m-2)}} = \boxed{\frac{10-3m}{m+2}}$$

$$y = \frac{\begin{vmatrix} m & 0 & 6 \\ 2 & 2 & 4 \\ 2 & m-2 & 6 \end{vmatrix}}{2m^2-8} = \frac{-4m^2+32m-48}{2m^2-8} = \frac{-4(m^2-8m+12)}{2(m+2)(m-2)} = \frac{-\cancel{4}^{2}\cancel{(m-2)}(m-6)}{\cancel{2}(m+2)\cancel{(m-2)}} = \boxed{\frac{2(6-m)}{m+2}}$$

$$z = \frac{\begin{vmatrix} m & 2 & 0 \\ 2 & m & 2 \\ 2 & m & m-2 \end{vmatrix}}{2m^2-8} = \frac{m^3-4m^2-4m+16}{2(m+2)(m-2)} = \frac{\cancel{(m-2)}(m-4)\cancel{(m+2)}}{2\cancel{(m+2)}\cancel{(m-2)}} = \boxed{\frac{m-4}{2}}$$

$$\begin{array}{c|cccc} & 1 & -4 & -4 & 16 \\ 2 & & 2 & -4 & -16 \\ \hline & 1 & -2 & -8 & 0 \end{array}$$

$\boxed{m=2}$ SCI

$$\left(\begin{array}{ccc|c} 2 & 2 & 6 & 0 \\ 2 & 2 & 4 & 2 \\ 2 & 2 & 6 & 0 \end{array}\right) \sim \left(\begin{array}{ccc|c} 2 & 2 & 6 & 0 \\ 2 & 2 & 4 & 2 \\ 0 & 0 & 0 & 0 \end{array}\right) \sim \left(\begin{array}{ccc|c} 1 & 1 & 3 & 0 \\ 1 & 1 & 2 & 1 \\ 0 & 0 & 0 & 0 \end{array}\right)$$

$$\sim \begin{pmatrix} 1 & 1 & 3 & 0 \\ 0 & 0 & -1 & -1 \\ 0 & 0 & 0 & 0 \end{pmatrix} \quad \begin{array}{l} x+y=3 \\ z=-1 \end{array} \quad \begin{array}{l} y=\lambda \\ x=3-\lambda \end{array} \quad (3-\lambda, \lambda, -1)$$

$\boxed{M=-2}$ $\left(\begin{array}{ccc|c} -2 & 2 & 6 & 0 \\ 2 & -2 & 4 & 2 \\ 2 & -2 & 6 & -4 \end{array}\right) \sim \left(\begin{array}{ccc|c} -1 & 1 & 3 & 0 \\ 1 & -1 & 2 & 1 \\ 1 & -1 & 3 & -2 \end{array}\right) \sim \left(\begin{array}{cccc} -1 & 1 & 3 & 0 \\ 0 & 0 & 5 & 1 \\ 0 & 0 & 6 & -2 \end{array}\right)$

$\sim \left(\begin{array}{ccc|c} -1 & 1 & 3 & 0 \\ 0 & 0 & 5 & 1 \\ 0 & 0 & 0 & -16 \end{array}\right)$ SI

① a) $\begin{vmatrix} 1 & m & 1 \\ 1 & 3 & 1 \\ m & 1 & 1 \end{vmatrix} = 3 + \cancel{1} + m^2 - 3m - \cancel{1} - m = m^2 - 4m + 3 = 0 \iff m = \begin{cases} 1 \\ 3 \end{cases}$

$m \neq 1, 3$ Rango $A = 3 =$ Rango $\bar{A} =$ nº incógnitas

$m = 1$

$$\begin{pmatrix} 1 & 1 & 1 & 4 \\ 1 & 3 & 1 & 5 \\ 1 & 1 & 1 & 4 \end{pmatrix} \underset{F_3 - F_1 \to F_3}{\overset{F_2 - F_1 \to F_2}{\sim}} \begin{pmatrix} 1 & 1 & 1 & 4 \\ 0 & 2 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix} \text{ Rango } A = 2 = \text{Rango } \bar{A} < 3 \Rightarrow SCI$$

$m = 3$

$$\begin{pmatrix} 1 & 3 & 1 & 4 \\ 1 & 3 & 1 & 5 \\ 3 & 1 & 1 & 4 \end{pmatrix} \underset{F_3 - 3F_1 \to F_3}{\overset{F_2 - F_1 \to F_2}{\sim}} \begin{pmatrix} 1 & 3 & 1 & 4 \\ 0 & 0 & 0 & 1 \\ 0 & -8 & -2 & -8 \end{pmatrix} \text{ Rango } A = 2 \neq 3 = \text{Rango } \bar{A} \Rightarrow SI$$

b) $$\begin{vmatrix} 3 & x & x & x \\ x & 3 & x & x \\ x & x & 3 & x \\ x & x & x & 3 \end{vmatrix} \overset{F_4 - F_3 \to F_4}{=\!=} \begin{vmatrix} 3 & x & x & x \\ x & 3 & x & x \\ x & x & 3 & x \\ 0 & 0 & x-3 & 3-x \end{vmatrix} = (x-3) \begin{vmatrix} 3 & x & x & x \\ x & 3 & x & x \\ x & x & 3 & x \\ 0 & 0 & 1 & -1 \end{vmatrix} \overset{F_3 - F_2 \to F_3}{=\!=}$$

$$= (x-3) \begin{vmatrix} 3 & x & x & x \\ x & 3 & x & x \\ 0 & x-3 & 3-x & 0 \\ 0 & 0 & 1 & -1 \end{vmatrix} = (x-3)^2 \begin{vmatrix} 3 & x & x & x \\ x & 3 & x & x \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 1 & -1 \end{vmatrix} \overset{F_2 - F_1 \to F_2}{=\!=} (x-3)^2 \begin{vmatrix} 3 & x & x & x \\ x-3 & 3-x & 0 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 1 & -1 \end{vmatrix} =$$

$$= (x-3)^3 \begin{vmatrix} 3 & x & x & x \\ 1 & -1 & 0 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 1 & -1 \end{vmatrix} =\!=\!= (x-3)^3 \cdot \left(-1 \begin{vmatrix} 3 & x & x \\ 1 & -1 & 0 \\ 0 & 1 & 0 \end{vmatrix} - 1 \begin{vmatrix} 3 & x & x \\ 1 & -1 & 0 \\ 0 & 1 & -1 \end{vmatrix} \right) =$$

$$= (x-3)^3 \left(-x - 3 - x - x \right) = (x-3)^3 (-3x - 3) = -3(x-3)^3 (x+1) = 0 \iff$$

$\iff x = \begin{cases} -1 \\ 3 \end{cases}$

② $A(1,-3,1)$ $B(2,3,1)$ $C(1,3,-1)$

a) $\begin{vmatrix} x-1 & y+3 & z-1 \\ 1 & 6 & 0 \\ -1 & 0 & -2 \end{vmatrix} = -12x+12+6z-\cancel{6}+2y+\cancel{6}=0$

$-12x+2y+6z+12=0 \Rightarrow \boxed{6x-y-3z-6=0}$

b) $d(O,\pi) = \dfrac{|-6|}{\sqrt{6^2+(-1)^2+(-3)^2}} = \dfrac{6}{\sqrt{46}} = \dfrac{6\sqrt{46}}{46} = \boxed{\dfrac{3\sqrt{46}}{23}\ u}$

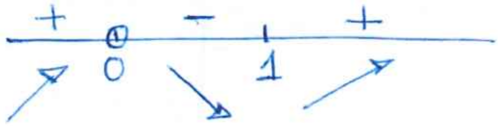
c) $\vec{OA}=(1,-3,1)$ $\vec{OB}=(2,3,1)$ $\vec{OC}=(1,3,-1)$

$\begin{vmatrix} 1 & -3 & 1 \\ 2 & 3 & 1 \\ 1 & 3 & -1 \end{vmatrix} = -3+\cancel{6}-3-3-3-\cancel{6}=-12$

$V_{tetraedro} = \dfrac{1}{6}|-12| = \boxed{2u^3}$

③ a) $f(x)=xe^{\frac{1}{x}}$ $\quad Dom f = \mathbb{R}\setminus\{0\}$

$f'(x)=e^{\frac{1}{x}}+x\dfrac{-1}{x^2}e^{\frac{1}{x}} = e^{\frac{1}{x}}\left(1-\dfrac{1}{x}\right)=e^{\frac{1}{x}}\left(\dfrac{x-1}{x}\right)=0 \Leftrightarrow \boxed{x=1}$



Crece en $(-\infty,0)\cup(1,\infty)$

Decrece en $(0,1)$

Mínimo en $x=1$ $(1,e)$

Asíntotas

AV ¿$x=0$?

$\lim\limits_{x\to 0} xe^{\frac{1}{x}} = (0\cdot\infty) = \lim\limits_{x\to 0}\dfrac{e^{\frac{1}{x}}}{\frac{1}{x}} = \left(\dfrac{\infty}{\infty}\right) \underset{L'H}{=} \lim\limits_{x\to 0}\dfrac{\cancel{\frac{-1}{x^2}}e^{\frac{1}{x}}}{\cancel{\frac{-1}{x^2}}} = \lim\limits_{x\to 0} e^{\frac{1}{x}} =$

$= \begin{cases} \lim\limits_{x\to 0^-} e^{\frac{1}{x}} = 0 \\ \lim\limits_{x\to 0^+} e^{\frac{1}{x}} = +\infty \end{cases}$ AV $x=0$ por la dcha

AH

$\lim\limits_{x\to +\infty} xe^{\frac{1}{x}} = +\infty$ No tiene por la dcha.

$\lim\limits_{x\to -\infty} xe^{\frac{1}{x}} = -\infty$ No tiene por la izq

Geometría

① $\vec{u}(1,-1,0)$ $\vec{v}(0,1,2)$ $\vec{w}(1+a, 2a, 2-3a)$

a) $\begin{vmatrix} 1 & -1 & 0 \\ 0 & 1 & 2 \\ 1+a & 2a & 2-3a \end{vmatrix} = \cancel{2}-3a-\cancel{2}-2a-4a = -9a = 0 \Leftrightarrow \boxed{a=0}$

b) $1+a-2a=0 \Leftrightarrow \boxed{a=1}$

$2a+4-6a=0 \Leftrightarrow 4-4a=0 \Leftrightarrow \boxed{a=1}$

c) $\frac{1}{6}|-9a| = \frac{1}{6} \Leftrightarrow |-9a| = 1 \Leftrightarrow$
$-9a=1 \Leftrightarrow \boxed{a=\frac{-1}{9}}$
$-9a=-1 \Leftrightarrow \boxed{a=\frac{1}{9}}$

② $\pi: x+2y+z=1$ $A(3,1,2)$

a) $r: \left.\begin{array}{l} x=3+\lambda \\ y=1+2\lambda \\ z=2+\lambda \end{array}\right\}$ $r\cap\pi \Rightarrow 3+\lambda+2+4\lambda+2+\lambda=1 \Rightarrow 6\lambda=-6 \Rightarrow \lambda=-1$

Punto de $\cap$ de r y π $\boxed{(2,-1,1)}$

b) $\pi_2 \parallel \pi_1 \Rightarrow \pi_2: x+2y+z+D=0$

$A(0,0,-D)$, $B(-D,0,0)$, $C(0,\frac{-D}{2},0)$

$\vec{AB}(-D,0,D)$

$\vec{BC}(D,\frac{-D}{2},0)$ $\quad A_\Delta = \frac{1}{2}|\vec{AB}\wedge\vec{BC}| = \frac{1}{2}\left|\left(\frac{D^2}{2}, D^2, \frac{D^2}{2}\right)\right| =$

$\vec{CA}(0,\frac{+D}{2},-D)$ $\quad = \frac{1}{2}\sqrt{\frac{D^4}{4}+D^4+\frac{D^4}{4}} = \frac{1}{4}\sqrt{6}\,D^2 = \sqrt{6} \Leftrightarrow D^2=4 \Leftrightarrow D=\pm 2$

$\boxed{x+2y+z+2=0}$ $\boxed{x+2y+z-2=0}$

③ a) $\frac{2}{m} = \frac{-1}{4} = \frac{n}{2}$ $\boxed{m=-8}$ $\boxed{n=\frac{-1}{2}}$

$\left.\begin{array}{l} x=1-8\lambda \\ y=4\lambda \\ z=1+2\lambda \end{array}\right\}$ $2(1-8\lambda)-4\lambda-\frac{1}{2}(1+2\lambda)=0 \Rightarrow 4-32\lambda-8\lambda-1-2\lambda=0$

$-42\lambda=-3 \Rightarrow \lambda=\frac{1}{14}$ $\boxed{\text{Pto de intersección}\left(\frac{3}{7},\frac{2}{7},\frac{8}{7}\right)}$

b) $2m-4+2n=0 \Rightarrow m-2+n=0$ $\quad m-2-2=0 \Rightarrow \boxed{m=4}$

$P(1,0,1)$ $\quad 2\cdot 1+n=0 \Rightarrow \boxed{n=-2}$

4) $A(1,0,2)$ $B(-1,2,4)$ $\quad \frac{x+2}{2} = y-1 = \frac{z-1}{3}$ $\quad P(-2,1,1)$ $\vec{u}(2,1,3)$

a) $\vec{AB}(-2,2,2) \parallel (1,-1,-1)$

$$\begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & -1 & -1 \\ 2 & 1 & 3 \end{vmatrix} = -2\vec{i} - 5\vec{j} + 3\vec{k} \qquad 2x+5y-3z+D=0 \xrightarrow{A} 2-6+D=0 \ \boxed{D=4}$$

$$\boxed{2x+5y-3z+4=0}$$

b) $(x-1)^2 + y^2 + (z-2)^2 = (x+1)^2 + (y-2)^2 + (z-4)^2 \iff -2x + \cancel{1} - 4z + \cancel{4} = 2x + \cancel{1} - 4y + \cancel{4} - 8z + 16$

$\iff 4x - 4y - 4z + 16 = 0 \iff \boxed{x - y - z + 4 = 0}$

c) $\vec{u}_{AB}(1,-1,-1)$ $\qquad r \not\parallel r_{AB} \Rightarrow$ Las rectas se cortan o se cruzan

$\vec{u}_r(2,1,3)$

$$\begin{vmatrix} \vec{AP} \\ \vec{u}_r \\ \vec{u}_{AB} \end{vmatrix} = \begin{vmatrix} -3 & 1 & -1 \\ 2 & 1 & 3 \\ 1 & -1 & -1 \end{vmatrix} = 3+2+3+1-9+2 = 2 \neq 0 \Rightarrow \boxed{r \text{ y } r_{AB} \text{ se cruzan}}$$

(10)

$$\lim_{x\to\infty} \frac{x e^{\frac{1}{x}}}{x} = \lim_{x\to\infty} e^{\frac{1}{x}} = e^0 = 1$$

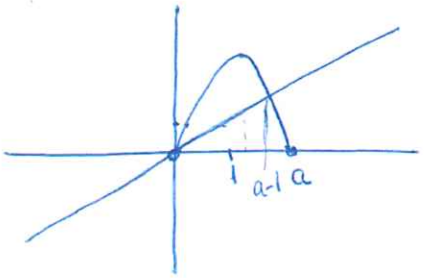
$$\lim_{x\to\infty} \left(xe^{\frac{1}{x}} - x\right) = \lim_{x\to\infty} \left[x\left(e^{\frac{1}{x}}-1\right)\right] = (\infty\cdot 0) = \lim_{x\to\infty} \frac{e^{\frac{1}{x}}-1}{\frac{1}{x}} = \left(\frac{0}{0}\right) =$$

$$\underset{L'H}{=} \lim_{x\to\infty} \frac{\cancel{\frac{-1}{x^2}}\, e^{\frac{1}{x}}}{\cancel{\frac{-1}{x^2}}} = 1 \qquad \boxed{y=x+1}$$

b) $$\lim_{x\to\infty}\left(\frac{5-3x}{6-3x}\right)^{\frac{x^2-2}{x}} = (1^\infty) = \lim_{x\to\infty}\left(1+\frac{5-\cancel{3x}-6+\cancel{3x}}{6-3x}\right)^{\frac{x^2-2}{x}} =$$

$$= \lim_{x\to\infty}\left(1+\frac{1}{3x-6}\right)^{3x-6\,\cdot\,\frac{x^2-2}{3x^2-6x}} = e^{\lim_{x\to\infty}\frac{x^2-2}{3x^2-6x}} = e^{\frac{1}{3}} = \sqrt[3]{e}$$

(4) a) $y=-x^2+ax \qquad y=x \qquad$ Vértice $x=\frac{-a}{2\cdot(-1)}=\frac{a}{2}$



$$x=-x^2+ax \iff x^2+(1-a)x=0 \iff x=\begin{cases}0\\ a-1\end{cases}$$

$$\int_0^{a-1}(-x^2+ax-x)\,dx = \int_0^{a-1}\left[-x^2+(a-1)x\right]dx =$$

$$= -\frac{x^3}{3}\Big|_0^{a-1} + (a-1)\frac{x^2}{2}\Big|_0^{a-1} = -\frac{(a-1)^3}{3}+\frac{(a-1)^3}{2} = \frac{(-2+3)(a-1)^3}{6} = \frac{(a-1)^3}{6} = \frac{4}{3} \iff$$

$$\iff (a-1)^3 = 8 \iff a-1=2 \iff \boxed{a=3}$$

b) $$\int \frac{x}{x^3-x^2-x+1}\,dx = \frac{-1}{4}\int\frac{dx}{x+1}+\frac{1}{4}\int\frac{dx}{x-1}+\frac{1}{2}\int\frac{dx}{(x-1)^2} = \frac{-1}{4}\ln|x+1|+\frac{1}{4}\ln|x-1|+$$

$$\frac{A}{x+1}+\frac{B}{x-1}+\frac{C}{(x-1)^2} = \frac{x}{x^3-x^2-x+1} \Rightarrow A(x-1)^2+B(x^2-1)+C(x+1)=x$$

$$\begin{array}{l} A+B=0 \\ -2A+C=1 \\ A-B+C=0 \end{array}\left\{\begin{array}{l} B=-A \\ C=1+2A \\ A+A+1+2A=0 \Rightarrow A=\frac{-1}{4} \quad B=\frac{1}{4} \quad C=\frac{1}{2}\end{array}\right.$$

$$+\frac{1}{2}\,\frac{-1}{x-1} = \boxed{\frac{1}{4}\ln\left|\frac{x-1}{x+1}\right| - \frac{1}{2(x-1)} + K}$$