1)

a) $\begin{vmatrix} 0 & 0 & 1/2 \\ 0 & 1 & 0 \\ 2 & 0 & 0 \end{vmatrix} = -1$; $A^t = \begin{pmatrix} 0 & 0 & 2 \\ 0 & 1 & 0 \\ 1/2 & 0 & 0 \end{pmatrix}$

$Adj(A^t) = \begin{pmatrix} 0 & 0 & -1/2 \\ 0 & -1 & 0 \\ -2 & 0 & 0 \end{pmatrix}$; $A^{-1} = \begin{pmatrix} 0 & 0 & 1/2 \\ 0 & 1 & 0 \\ 2 & 0 & 0 \end{pmatrix}$

b) $A^{-1} = A \Rightarrow A^2 = A \cdot A = A \cdot A^{-1} = I_3$

$A^3 = A^2 \cdot A = I_3 \cdot A = A$

$A^4 = A^2 \cdot A^2 = I_3 \cdot I_3 = I_3$

$A^n = \begin{cases} I_3 & \text{si } n \text{ par} \\ A & \text{si } n \text{ impar} \end{cases}$

c) $A^4 + A^2 - A = I_3 + I_3 - A =$

$= \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} + \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} - \begin{pmatrix} 0 & 0 & 1/2 \\ 0 & 1 & 0 \\ 2 & 0 & 0 \end{pmatrix} = \underbrace{\begin{pmatrix} 2 & 0 & -1/2 \\ 0 & 1 & 0 \\ -2 & 0 & 2 \end{pmatrix}}_{B}$

$X = \begin{pmatrix} 2 & 3 & 2 \\ 1 & 1 & 1 \end{pmatrix} \underbrace{\begin{pmatrix} 2 & 0 & -1/2 \\ 0 & 1 & 0 \\ -2 & 0 & 2 \end{pmatrix}^{-1}}_{B^{-1}}$ $|B| = 4 - 1 = 3 \neq 0$

$B^t = \begin{pmatrix} 2 & 0 & -2 \\ 0 & 1 & 0 \\ -1/2 & 0 & 2 \end{pmatrix}$

$Adj(B^t) = \begin{pmatrix} 2 & 0 & 1/2 \\ 0 & 3 & 0 \\ 2 & 0 & 2 \end{pmatrix}$; $B^{-1} = \begin{pmatrix} \frac{2}{3} & 0 & \frac{1}{6} \\ 0 & 1 & 0 \\ \frac{2}{3} & 0 & \frac{2}{3} \end{pmatrix}$

$X = \begin{pmatrix} 2 & 3 & 2 \\ 1 & 1 & 1 \end{pmatrix} \begin{pmatrix} \frac{2}{3} & 0 & \frac{1}{6} \\ 0 & 1 & 0 \\ \frac{2}{3} & 0 & \frac{2}{3} \end{pmatrix} = \begin{pmatrix} \frac{8}{3} & 3 & 5/3 \\ 4/3 & 1 & 5/6 \end{pmatrix}$

2) a) $\left(\begin{array}{ccc|c} 6 & -2 & 1 & 7 \\ 3 & -1 & 3 & 1 \\ 3 & -1 & 8 & -4 \end{array}\right) \begin{array}{l} -2F2+F1 \\ \to F2 \\ -2F3+F1 \\ \to F3 \end{array} \left(\begin{array}{ccc|c} 6 & -2 & 1 & 7 \\ 0 & 0 & -5 & 5 \\ 0 & 0 & -15 & 15 \end{array}\right)$

$\begin{array}{l} -3F2+F3 \\ \to F3 \end{array} \left(\begin{array}{ccc|c} 6 & -2 & 1 & 7 \\ 0 & 0 & -5 & 5 \\ 0 & 0 & 0 & 0 \end{array}\right) \quad \left.\begin{array}{r} 6x-2y+z=7 \\ -5z=5 \end{array}\right\} \to \underline{\underline{z=-1}}$

$6x-2y-1=7; \quad 6x-2y=8; \quad 3x-y=4$

$y=3x-4; \quad$ Sol: $(\lambda, 3\lambda-4, -1) \quad \lambda \in \mathbb{R}$

$\lambda+(3\lambda-4)-1=3; \quad \lambda+3\lambda-4-1=3$

$\lambda+3\lambda=3+4+1; \quad 4\lambda=8; \quad \lambda=\frac{8}{4}=2$

$\underline{\underline{\text{Sol: } (2, 2, -1)}}$

b) $\begin{vmatrix} 1 & -1 & 1 & -1 \\ 1 & 1-x & -1 & 1 \\ -1 & 1 & 2-x & -1 \\ 1 & -1 & 1 & 3-x \end{vmatrix} = \begin{array}{l} C2+C1 \\ \to C2 \\ C_3-C_1 \\ \to C_3 \\ C_4+C_1 \\ \to C_4 \end{array} = \begin{vmatrix} 1 & 0 & 0 & 0 \\ 1 & 2-x & -2 & 2 \\ -1 & 0 & 3-x & -2 \\ 1 & 0 & 0 & 4-x \end{vmatrix} =$

$= \begin{vmatrix} 2-x & -2 & 2 \\ 0 & 3-x & -2 \\ 0 & 0 & 4-x \end{vmatrix} = (2-x)(3-x)(4-x) = 0$

$\Rightarrow \underline{\underline{x = 2, 3, 4}}$

3)

a) $A = \begin{pmatrix} x & b & c \\ a & x & 1 \\ b & c & x \end{pmatrix} \quad A^t = \begin{pmatrix} x & a & b \\ b & x & c \\ c & 1 & x \end{pmatrix}$

$A = A^t \Rightarrow \begin{array}{l} x = x \\ b = a \\ c = b \end{array} \;\Bigg|\; \begin{array}{l} a = b \\ x = x \\ 1 = c \end{array} \;\Bigg|\; \begin{array}{l} b = c \\ c = 1 \\ x = x \end{array}$

$\underline{\underline{a = b = c = 1}} \qquad \underline{\underline{x = x}}$

$A = \begin{pmatrix} x & 1 & 1 \\ 1 & x & 1 \\ 1 & 1 & x \end{pmatrix}$

$|A| = x^3 + 1 + 1 - x - x - x = x^3 - 3x + 2 = 0$

$$\begin{array}{c|cccc} & 1 & 0 & -3 & 2 \\ 1 & & 1 & 1 & -2 \\ \hline & 1 & 1 & -2 & 0 \end{array}$$

$x^2 + x - 2 = 0$

$x = \frac{-1 \pm \sqrt{1+8}}{2} = \begin{cases} 1 \\ -2 \end{cases}$

No tiene inversa para $\underline{\underline{x = 1 \text{ y } x = -2}}$

b) $\begin{vmatrix} a+b & a & a \\ a & a+b & a \\ a & a & a+b \end{vmatrix} \underset{\to F3}{\overset{F3-F2}{=}} \begin{vmatrix} a+b & a & a \\ a & a+b & a \\ 0 & -b & b \end{vmatrix} =$

$\underset{\to C3}{\overset{C3+C2}{=}} \begin{vmatrix} a+b & a & 2a \\ a & a+b & 2a+b \\ 0 & -b & 0 \end{vmatrix} =$

$= b \begin{vmatrix} a+b & 2a \\ a & 2a+b \end{vmatrix} = b\left[(a+b)(2a+b) - 2a^2\right] =$

$= b\left[\cancel{2a^2} + ab + 2ab + b^2 - \cancel{2a^2}\right] =$

$= b\left[b^2 + 3ab\right] = \boxed{b^2(b+3a)}$

4)

$$\begin{vmatrix} 1 & 3-m & 1 \\ 1 & 1+m & 1 \\ 1 & 1 & 1+m \end{vmatrix} = (1+m)^2 + 1 + (3-m) - (1+m) - 1 - (3-m)(1+m) =$$

$$= 1 + m^2 + 2m + 1 + 3 - m - 1 - m - 1 - 3 - 3m + m + m^2 =$$

$$= 2m^2 - 2m = \underline{\underline{2m(m-1)}}$$

$$2m(m-1) = 0 \Rightarrow \begin{matrix} m = 0 \\ m = 1 \end{matrix}$$

$\underline{\underline{m=0}}$

$$\left(\begin{array}{ccc|c} 1 & 3 & 1 & 0 \\ 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 0 \end{array}\right) \begin{matrix} \xrightarrow{F2-F1} F2 \\ F3-F1 \\ \rightarrow F3 \end{matrix} \left(\begin{array}{ccc|c} 1 & 3 & 1 & 0 \\ 0 & -2 & 0 & 0 \\ 0 & -2 & 0 & 0 \end{array}\right) \begin{matrix} F3-F2 \\ \rightarrow F3 \end{matrix}$$

$$\left.\begin{matrix} \text{rang}A = 2 \\ \text{rang}A/B = 2 \end{matrix}\right\} < 3 \qquad \underline{\underline{S.C.I}}$$

$$\left(\begin{array}{ccc|c} 1 & 3 & 1 & 0 \\ 0 & -2 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right) \qquad \left.\begin{matrix} x + 3y + z = 0 \\ -2y = 0 \end{matrix}\right\} \rightarrow y = 0$$

$$x + z = 0; \quad x = -z$$

$$(-\lambda, 0, \lambda) \quad \lambda \in \mathbb{R}$$

Si $m = 0 \rightarrow S.C.I \rightarrow (-\lambda, 0, \lambda)$

$\underline{\underline{m=1}}$

$$\left(\begin{array}{ccc|c} 1 & 2 & 1 & 1 \\ 1 & 2 & 1 & 2 \\ 1 & 1 & 2 & 0 \end{array}\right) \begin{matrix} F2-F1 \\ \rightarrow F2 \\ F3-F1 \\ \rightarrow F3 \end{matrix} \left(\begin{array}{ccc|c} 1 & 2 & 1 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & -1 & 1 & -1 \end{array}\right) F2 \rightleftarrows F3$$

$$\left(\begin{array}{ccc|c} 1 & 2 & 1 & 1 \\ 0 & -1 & 1 & -1 \\ 0 & 0 & 0 & 1 \end{array}\right) \qquad \begin{matrix} \text{rang}A = 2 \\ \text{rang}A/B = 3 \end{matrix}$$

$\underline{\underline{\text{S Incompatible}}}$

Si $m \neq 0, 1 \Rightarrow$ S.C.D

$$X = \frac{\begin{vmatrix} m & 3-m & 1 \\ 2m & 1+m & 1 \\ 0 & 1 & 1+m \end{vmatrix}}{2m(m-1)} = \frac{\begin{aligned} &m(1+m)^2 + 2m - m - 2m(3-m)(1+m) = \\ &= m(1+m^2+2m) + 2m - m - 2m(3+3m-m-m^2) = \\ &= m + m^3 + 2m^2 + 2m - m - 6m - 6m^2 + 2m^2 + 2m^3 \end{aligned}}{2m(m-1)} =$$

$$= \frac{3m^3 - 2m^2 - 4m}{2m(m-1)} = \frac{\cancel{m}(3m^2 - 2m - 4)}{2\cancel{m}(m-1)} =$$

$3m^2 - 2m - 4 = 0$; $\quad m = \frac{+2 \pm \sqrt{4+48}}{6}$ (NO son exactas)

$$= \frac{3m^2 - 2m - 4}{2(m-1)} \quad //$$

$$y = \frac{\begin{vmatrix} 1 & m & 1 \\ 1 & 2m & 1 \\ 1 & 0 & 1+m \end{vmatrix}}{2m(m-1)} = \frac{\begin{aligned} &2m(1+m) + m - 2m - m(1+m) = \\ &= 2m + 2m^2 + m - 2m - m - m^2 = \\ &= m^2 \end{aligned}}{2m(m-1)} = \frac{m^2}{2\cancel{m}(m-1)} = \frac{m}{2(m-1)} \quad //$$

$$z = \frac{\begin{vmatrix} 1 & 3-m & m \\ 1 & 1+m & 2m \\ 1 & 1 & 0 \end{vmatrix}}{2m(m-1)} = \frac{\begin{aligned} &m + 2m(3-m) - m(1+m) - 2m = \\ &= m + 6m - 2m^2 - m - m^2 - 2m = \\ &= -3m^2 + 4m \end{aligned}}{2m(m-1)} = \frac{\cancel{m}(-3m+4)}{2\cancel{m}(m-1)} = \frac{-3m+4}{2(m-1)} \quad //$$

1) a) $T \in \mathcal{M}_3 \qquad |T| = \sqrt{2}$

$$|M| = \begin{vmatrix} 1 & 2 & 1 \\ 2 & 1 & 1 \\ 2 & 1 & -1 \end{vmatrix} = -1+2+4-2-1+4 = 6$$

i) $\left|\frac{1}{2}T\right| = \left(\frac{1}{2}\right)^3 |T| = \frac{\sqrt{2}}{8}$ //

ii) $|M^4| = |M|^4 = 6^4 = 1296$ //

iii) $|TM^3T^{-1}| = |T|\,|M|^3 \underbrace{|T^{-1}|}_{\frac{1}{|T|}} = |M|^3 = 216$ //

b)
$$\left\{\begin{array}{l} 6x+3y+2z=470 \\ 3x+4y+6z=370 \\ x+3y+2z=160 \end{array}\right.$$

$$\left(\begin{array}{ccc|c} 6 & 3 & 2 & 470 \\ 3 & 4 & 6 & 370 \\ 1 & 3 & 2 & 160 \end{array}\right) \begin{array}{l} -2F_2+F_1 \to F_2 \\ -6F_3+F_1 \to F_3 \end{array}$$

$$\left(\begin{array}{ccc|c} 6 & 3 & 2 & 470 \\ 0 & -5 & -10 & -270 \\ 0 & -15 & -10 & -490 \end{array}\right) \begin{array}{l} -3F_2+F_3 \\ \to F_3 \end{array}$$

$$\left(\begin{array}{ccc|c} 6 & 3 & 2 & 470 \\ 0 & -5 & -10 & -270 \\ 0 & 0 & 20 & 320 \end{array}\right)$$

$$\left\{\begin{array}{l} 6x+3y+2z=470 \\ -5y-10z=-270 \\ 20z=320 \end{array}\right.$$

$6x+66+32=470; \quad x = \frac{372}{6} = 62$

$-5y-160=-270; \quad -5y=-110; \quad y=22$

$z = \frac{320}{20} = 16$

$(62, 22, 16)$

2)

a) $B = \begin{pmatrix} 0 & -2 \\ 1 & 3 \end{pmatrix} - \begin{pmatrix} k & 0 \\ 0 & k \end{pmatrix} = \begin{pmatrix} -k & -2 \\ 1 & 3-k \end{pmatrix}$

$|B| = -k(3-k)+2 = -3k+k^2+2 = k^2-3k+2 = 0$

$k = \frac{3 \pm \sqrt{9-8}}{2} = \frac{3\pm 1}{2} = \begin{cases} 2 \text{ //} \\ 1 \text{ //} \end{cases}$

$\exists\, B^{-1}$ si $k \neq 1, 2$

b) $k=3 \Rightarrow B = \begin{pmatrix} -3 & -2 \\ 1 & 0 \end{pmatrix} \qquad |B| = 2$

$B^t = \begin{pmatrix} -3 & 1 \\ -2 & 0 \end{pmatrix} \qquad \text{Adj}(B^t) = \begin{pmatrix} 0 & 2 \\ -1 & -3 \end{pmatrix}$

$B^{-1} = \begin{pmatrix} 0 & 1 \\ -1/2 & -3/2 \end{pmatrix}$ //

c) $A^2 = \begin{pmatrix} 0 & -2 \\ 1 & 3 \end{pmatrix} \begin{pmatrix} 0 & -2 \\ 1 & 3 \end{pmatrix} = \begin{pmatrix} -2 & -6 \\ 3 & 7 \end{pmatrix}$

$\alpha \begin{pmatrix} -2 & -6 \\ 3 & 7 \end{pmatrix} + \beta \begin{pmatrix} 0 & -2 \\ 1 & 3 \end{pmatrix} = \begin{pmatrix} -2 & 0 \\ 0 & -2 \end{pmatrix}$

$-2\alpha = -2 \rightarrow \alpha = 1$ //

$-6\alpha - 2\beta = 0 \rightarrow \beta = -3$ //

$3\alpha + \beta = 0 \rightarrow$ se cumple

$7\alpha + 3\beta = -2 \rightarrow$ se cumple

d) i) $P = I - M$; $M^2 = M$

$P^2 = (I-M)(I-M) = I^2 - IM - MI + M^2 =$

$= I^2 - M - M + M = I - M = P$

$M \cdot P = M(I-M) = M \cdot I - M^2 = M - M = 0$

$P \cdot M = (I-M) \cdot M = IM - M^2 = M - M = 0$

3) a) $|A| = \begin{vmatrix} 3y+5 & 7 & 12 \\ 2y+3 & 3 & 6 \\ 3y+4 & 2 & 6 \end{vmatrix} = \begin{vmatrix} 3y & 7 & 12 \\ 2y & 3 & 6 \\ 3y & 2 & 6 \end{vmatrix} + \begin{vmatrix} 5 & 7 & 12 \\ 3 & 3 & 6 \\ 4 & 2 & 6 \end{vmatrix}$

$= y \begin{vmatrix} 3 & 7 & 12 \\ 2 & 3 & 6 \\ 3 & 2 & 6 \end{vmatrix} + \begin{vmatrix} 5 & 7 & 12 \\ 3 & 3 & 6 \\ 4 & 2 & 6 \end{vmatrix} =$

$-2F2 + F1 \rightarrow F2$ $\quad$ $-2F2 + F1 \rightarrow F2$

$-2F3 + F1 \rightarrow F3$ $\quad$ $-2F3 + F1 \rightarrow F3$

$= \frac{y}{4} \underbrace{\begin{vmatrix} 3 & 7 & 12 \\ -1 & 1 & 0 \\ -3 & 3 & 0 \end{vmatrix}}_{= 0} + \frac{1}{4} \underbrace{\begin{vmatrix} 5 & 7 & 12 \\ -1 & 1 & 0 \\ -3 & 3 & 0 \end{vmatrix}}_{= 0} = 0$ //

$F3 = 3F2$ $\quad$ $F3 = 3F2$

b)

i) $|M(\lambda)| = -4 + 2\lambda^2 + 6\lambda - \lambda^2 - 8\lambda + 6 =$

$= \lambda^2 - 2\lambda + 2$

$\lambda^2 - 2\lambda + 2 = 0 \Rightarrow \lambda = \dfrac{2 \pm \sqrt{4-8}}{2} \rightarrow \nexists \text{ sol}$

ii) $M(0) = \begin{pmatrix} 4 & 3 & 0 \\ 2 & 1 & 2 \\ 0 & 0 & -1 \end{pmatrix}$ $\quad |M(0)| = -4 + 6 = 2$

$(M(0))^t = \begin{pmatrix} 4 & 2 & 0 \\ 3 & 1 & 0 \\ 0 & 2 & -1 \end{pmatrix}$ $\quad \mathrm{Adj}(M(0))^t = \begin{pmatrix} -1 & 3 & 6 \\ 2 & -4 & -8 \\ 0 & 0 & -2 \end{pmatrix}$

$(M(0))^{-1} = \begin{pmatrix} -1/2 & 3/2 & 6/2 \\ 2/2 & -4/2 & -8/2 \\ 0 & 0 & -2/2 \end{pmatrix}$

$|A| = 8^2 - 2\cdot 8 + 2 = 64 - 16 + 2 = 50$

$|B| = 4^2 - 2\cdot 4 + 2 = 16 - 8 + 2 = 10$

$|C| = 3^2 - 2\cdot 3 + 2 = 9 - 6 + 2 = 5$

$|A \cdot B^{-1} \cdot C^{-1}| = |A|\,|B^{-1}|\,|C^{-1}| = |A| \cdot \dfrac{1}{|B|} \cdot \dfrac{1}{|C|} =$

$= \dfrac{|A|}{|B|\,|C|} = \dfrac{50}{5 \cdot 10} = \dfrac{50}{50} = 1$

4) $|A| = \begin{vmatrix} 1 & a & 1 \\ 1 & 1 & a \\ 1 & a & a^2 \end{vmatrix} = a^2 + a + a^2 - 1 - a^2 - a^3 =$

$= -a^3 + a^2 + a - 1 =$

$= -(a-1)^2(a+1) = 0$

$$\begin{array}{r|rrrr} & -1 & 1 & 1 & -1 \\ 1 & & -1 & 0 & 1 \\ \hline & -1 & 0 & 1 & \boxed{0} \end{array}$$

$-a^2 + 1 = 0 \; ; \; a^2 = 1 \Rightarrow a = \pm 1$

$a = 1$

$a = -1$

Si $a = 1$

$$\left(\begin{array}{ccc|c} 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \end{array}\right) \begin{array}{l} F2-F1 \\ \to F2 \\ F3-F1 \\ \to F3 \end{array} \left(\begin{array}{ccc|c} 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right)$$

rang $A = 1$; rang $A|B = 1$. S.C.I $\quad GI = 3-1 = 2$ //

$x+y+z=1$; $x = 1-y-z$; $(1-\lambda-\mu, \lambda, \mu)$ $\lambda,\mu \in \mathbb{R}$

Si $a = -1$

$$\left(\begin{array}{ccc|c} 1 & -1 & 1 & -1 \\ 1 & 1 & -1 & 1 \\ 1 & -1 & 1 & 1 \end{array}\right) \begin{array}{l} F2-F1 \\ \to F2 \\ F3-F1 \\ \to F3 \end{array} \left(\begin{array}{ccc|c} 1 & -1 & 1 & -1 \\ 0 & 2 & -2 & 2 \\ 0 & 0 & 0 & 2 \end{array}\right)$$

rang $A = 2$; rang $A|B = 3$ $\quad$ S.I

Si $a \neq 1,-1 \to$ S.C.D

$$a^3 + a + a^4 - a^2 - a^3 - a^3 = a^4 - a^3 - a^2 + a = a(a^3 - a^2 - a + 1)$$

$$x = \frac{\begin{vmatrix} a & a & 1 \\ 1 & 1 & a \\ a^2 & a & a^2 \end{vmatrix}}{-(a-1)^2(a+1)} = \frac{a(a-1)^2(a+1)}{-(a-1)^2(a+1)} = -a \text{ //}$$

$$a^2 + a^2 + a^2 - 1 - a^3 - a^3 = -2a^3 + 3a^2 - 1$$

$$\begin{array}{c|cccc} & -2 & 3 & 0 & -1 \\ 1 & & -2 & 1 & 1 \\ \hline & -2 & 1 & 1 & 0 \end{array}$$

$$\frac{-1 \pm \sqrt{1+8}}{-4} = \frac{-1 \pm 3}{-4} = \begin{cases} -\frac{1}{2} \\ 1 \end{cases}$$

$$y = \frac{\begin{vmatrix} 1 & a & 1 \\ 1 & 1 & a \\ 1 & a^2 & a^2 \end{vmatrix}}{-(a-1)^2(a+1)} = \frac{-2(a-1)^2(a+\frac{1}{2})}{-(a-1)^2(a+1)} = \frac{2(a+1/2)}{a+1} \text{ //}$$

$$a^2 + a^2 + a - a - a - a^3 = -a^3 + 2a^2 - a = -a(a^2 - 2a + 1)$$

$$z = \frac{\begin{vmatrix} 1 & a & a \\ 1 & 1 & 1 \\ 1 & a & a^2 \end{vmatrix}}{-(a-1)^2(a+1)} = \frac{\cancel{-}a\,\cancel{(a-1)^2}}{\cancel{-}\cancel{(a-1)^2}(a+1)} = \frac{a}{a+1} \text{ //}$$

$$\left(-a, \frac{2a+1}{a+1}, \frac{a}{a+1}\right)$$

1) a) $\overline{w} = (x, y, z)$

$\overline{w} \perp \overline{u} \Rightarrow -2x + 3y + z = 0$

$\overline{w} \perp \overline{v} \Rightarrow -x + y = 0$

$|\overline{w}| = \sqrt{2} \Rightarrow \sqrt{x^2+y^2+z^2} = \sqrt{2}$

$$\left\{ \begin{array}{l} -2x + 3y + z = 0 \\ y = x \\ x^2 + y^2 + z^2 = 2 \end{array} \right.$$

$$\left. \begin{array}{l} -2x + 3x + z = 0 \\ x^2 + x^2 + z^2 = 2 \end{array} \right\} \begin{array}{l} \rightarrow x + z = 0 \\ \rightarrow 2x^2 + z^2 = 2 \end{array} \rightarrow z = -x$$

$2x^2 + (-x)^2 = 2$; $3x^2 = 2$; $x^2 = \frac{2}{3}$; $x = \pm\sqrt{\frac{2}{3}}$

$\overline{w}_1 \left(\sqrt{\frac{2}{3}}, \sqrt{\frac{2}{3}}, -\sqrt{\frac{2}{3}}\right)$ y $\overline{w}_2 \left(-\sqrt{\frac{2}{3}}, -\sqrt{\frac{2}{3}}, \sqrt{\frac{2}{3}}\right)$

$$V = |[\overline{u}, \overline{v}, \overline{w}]| = \left| \begin{vmatrix} -2 & 3 & 1 \\ -1 & 1 & 0 \\ \sqrt{\frac{2}{3}} & \sqrt{\frac{2}{3}} & -\sqrt{\frac{2}{3}} \end{vmatrix} \right| =$$

$$= \left| 2\sqrt{\frac{2}{3}} - \sqrt{\frac{2}{3}} - \sqrt{\frac{2}{3}} - 3\sqrt{\frac{2}{3}} \right| = 3\sqrt{\frac{2}{3}} = \frac{3\sqrt{2}}{\sqrt{3}} = \sqrt{6}\ u^3$$

b) $\overline{u}(x, y, z)$

$\overline{u} \cdot \overline{v} = 0 \Rightarrow -x + 2z = 0$

$$\cos 120^\circ = \frac{-y}{\sqrt{x^2+y^2+z^2}\ \sqrt{0^2+(-1)^2+0^2}}; \quad -\frac{1}{2} = \frac{-y}{\underbrace{\sqrt{x^2+y^2+z^2}}_{\sqrt{5}}} \Rightarrow y = \frac{\sqrt{5}}{2}$$

$\Downarrow$

$z = \pm\frac{\sqrt{3}}{2}$

$x = \pm\sqrt{3}$

$\sqrt{x^2+y^2+z^2} = 2y$; $x^2 + y^2 + z^2 = 4y^2$

$$\left. \begin{array}{l} x^2 - 3y^2 + z^2 = 0 \\ x^2 + y^2 + z^2 = 5 \\ -x + 2z = 0 \end{array} \right\} \quad \boxed{\left(\pm\sqrt{3}, \frac{\sqrt{5}}{2}, \pm\frac{\sqrt{3}}{2}\right)}$$

$\rightarrow x = 2z$

2) $\overline{u_r} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & 1 & -1 \\ 2 & 1 & -2 \end{vmatrix} = (-1, 0, -1)$; $P_r(0, 8, 3)$

$\overline{u_s} = \overline{PQ} = (2, 2, 1)$

a) $r: \begin{cases} x = -t \\ y = 8 \\ z = 3 - t \end{cases}$ $\qquad s: \begin{cases} x = 3 + 2t \\ y = 10 + 2t \\ z = 5 + t \end{cases}$

b) $\overrightarrow{u_r} \nparallel \overrightarrow{u_s}$ $\qquad P_r(0, 8, 3) \quad P_s(3, 10, 5)$

$\overline{P_rP_s}(3, 2, 2)$;

$[\overline{u_r}, \overline{u_s}, \overline{P_rP_s}] = \begin{vmatrix} -1 & 0 & -1 \\ 2 & 2 & 1 \\ 3 & 2 & 2 \end{vmatrix} = -4 - 4 + 6 + 2 = 0$

SE CORTAN

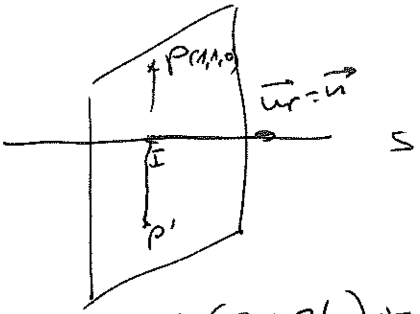
$$\begin{aligned} -t_1 &= 3 + 2t_2 \\ 8 &= 10 + 2t_2 \\ 3 - t_1 &= 5 + t_2 \end{aligned} \quad \begin{cases} t_1 = -1 \\ t_2 = -1 \\ \text{se cumple} \end{cases}$$

$H(1, 8, 4)$

c) $\cos(\widehat{r, s}) = \dfrac{|-2 - 1|}{\sqrt{2} \cdot \sqrt{9}} = \dfrac{3}{\sqrt{2} \cdot 3} = \dfrac{1}{\sqrt{2}} = \dfrac{\sqrt{2}}{2}$

$(\widehat{r, s}) = 45^\circ$

d)



$$2x + 2y + z + D = 0$$

$$2 + 2 + D = 0; \quad D = -4$$

$$\pi: \ 2x + 2y + z - 4 = 0$$

$$2(3+2t) + 2(10+2t) + (5+t) - 4 = 0$$

$$6 + 4t + 20 + 4t + 5 + t - 4 = 0; \quad 9t = -27$$

$$t = -3; \quad I(-3, 4, 2)$$

$$\frac{x+1}{2} = -3 \rightarrow x = -7 \qquad \underline{\underline{P'(-7, 7, 4)}}$$

$$\frac{y+1}{2} = 4 \longrightarrow y = 7$$

$$\frac{z+0}{2} = 2 \longrightarrow z = 4$$

3) a) $\vec{u} = \vec{AB} = (-2, 4, -4)$

$\vec{v} = \vec{AC} = (0, 6, -3)$

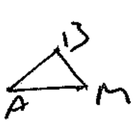
$$\vec{n} = \vec{u} \times \vec{v} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ -2 & 4 & -4 \\ 0 & 6 & -3 \end{vmatrix} = (12, -6, -12) \text{ vale } (2, -1, -2)$$

$$\pi: \ 2x - y - 2z + D = 0; \quad 2 \cdot 4 - (-4) - 2 \cdot 9 + D = 0$$

$$8 + 4 - 18 + D = 0; \quad D = 6$$

$$\underline{\underline{\pi: \ 2x - y - 2z + 6 = 0}}$$

b) $$d(M, \pi) = \frac{|2 \cdot 0 - 2 - 2 \cdot 3 + 6|}{\sqrt{2^2 + (-1)^2 + (-2)^2}} = \underline{\underline{\frac{2}{3} \ u}}$$



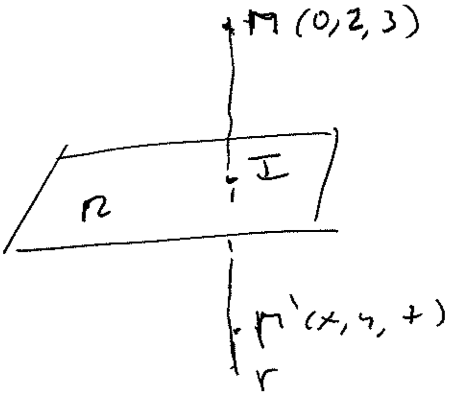
$\vec{AB} = (-2, 4, -4)$

$\vec{AM} = (-4, 6, -6)$

$$\vec{AB} \times \vec{AM} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ -2 & 4 & -4 \\ -4 & 6 & -6 \end{vmatrix} = (0, 4, 4)$$

$$A_{TRIANG} = \frac{\sqrt{0^2 + 4^2 + 4^2}}{2} = \frac{\sqrt{32}}{2} = \frac{4\sqrt{2}}{2} = \underline{\underline{2\sqrt{2} \ u^2}}$$

c)



$\overline{u_r} = \overline{n} = (2, -1, -2)$

$$r: \begin{cases} x = 2t \\ y = 2-t \\ z = 3-2t \end{cases}$$

$$2(2t) - (2-t) - 2(3-2t) + 6 = 0$$

$$4t - 2 + t - 6 + 4t + 6 = 0; \quad 9t = 2; \quad t = \frac{2}{9}$$

$$I\left(\frac{4}{9}, \frac{16}{9}, \frac{23}{9}\right)$$

d) $\frac{x+0}{2} = \frac{4}{9} \longrightarrow 9x = 8; \quad x = \frac{8}{9}$

$\frac{y+2}{2} = \frac{16}{9} \longrightarrow 9y + 18 = 32; \quad y = \frac{14}{9}$

$\frac{z+3}{2} = \frac{23}{9} \longrightarrow 9z + 27 = 46; \quad z = \frac{19}{9}$

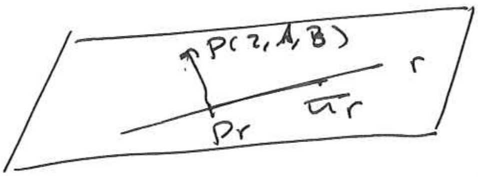
$$M'\left(\frac{8}{9}, \frac{14}{9}, \frac{19}{9}\right)$$

4) $r: \begin{cases} 2x - y + z - 3 = 0 \\ x - y + z - 2 = 0 \end{cases}$ $\quad P_r(1,-1,0)$

$$\overline{u_r} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 2 & -1 & 1 \\ 1 & -1 & 1 \end{vmatrix} = (0,-1,-1)$$

a) $$\frac{x-1}{0} = \frac{y+1}{-1} = \frac{z}{-1}$$

b)



$P(2,1,3) \quad P_r(1,-1,0)$

$\overline{P_rP}\ (1,2,3)$

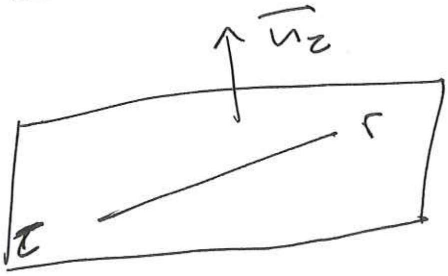
$$\overline{n} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 0 & -1 & -1 \\ 1 & 2 & 3 \end{vmatrix} = (-1,-1,1) \text{ vale } (1,1,-1)$$

$\pi: x + y - z + D = 0$

$2 + 1 - 3 + D = 0 \rightarrow D = 0$

$\pi: x + y - z = 0$

c)



$\overline{u_r}\ (0,-1,-1)$

$\overline{u_r} \perp \overline{n_2} \quad n_2(3,-1,-a)$

$3 \cdot 0 + (-1)(-1) - a(-1) = 0$

$1 + a = 0 \quad ; \quad a = -1$

$\tau: 3x - y + z - b = 0$

$P_r \in \tau \quad ; \quad 3 \cdot 1 - (-1) + 0 - b = 0$

$3 + 4 - b = 0 \quad ; \quad b = 4$

1) a) $\bar{u} + k\bar{v} = \left(\frac{\sqrt{2}}{2}, -k, -\frac{\sqrt{2}}{2}\right)$

$\bar{u} - k\bar{v} = \left(\frac{\sqrt{2}}{2}, k, -\frac{\sqrt{2}}{2}\right)$

$\cos 60^\circ = \frac{1}{2} = \frac{\bar{u}\cdot\bar{v}}{|\bar{u}|\,|\bar{v}|} =$

$= \frac{\frac{1}{2} - k^2 + \frac{1}{2}}{\sqrt{\frac{1}{2}+k^2+\frac{1}{2}}\ \sqrt{\frac{1}{2}+k^2+\frac{1}{2}}} = \frac{1-k^2}{1+k^2}$

$1+k^2 = 2(1-k^2)$; $1+k^2 = 2-2k^2$

$3k^2 = 1$; $k^2 = \frac{1}{3}$; $k = \pm\sqrt{\frac{1}{3}} = \pm\frac{\sqrt{3}}{3}$ //

b) $\bar{u}+\bar{v} = (1, m+1, m-1)$; $\bar{w}-\bar{v} = (1, m, 1)$

$(\bar{u}+\bar{v}) \wedge (\bar{w}-\bar{v}) = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & m+1 & m-1 \\ 1 & m & 1 \end{vmatrix} = (m+1-m^2+m,\ -1\cancel{+}m-1,\ m-m-1)$

$= (-m^2+2m+1,\ m-2,\ -1)$

$1\cdot(-m^2+2m+1) + 1(m-2) + 1\cdot(-1) =$

$= -m^2+2m+1+m-2-1 = -m^2+3m-2$

$m = \frac{-3\pm\sqrt{9-8}}{-2} = \frac{-3\pm 1}{-2} = \begin{cases} 1 \\ 2 \end{cases}$ //

2) a) i) $P_r(0, 1, -1)$ $\quad P_s(-1, 0, 0)$

$\overrightarrow{P_rP_s}\,(-1, -1, 1)$ $\quad \overline{u_r}\,(1, 2, 2)$

$\overline{u_s}\,(1, 1, 2)$

$$\begin{vmatrix} 1 & 2 & 2 \\ 1 & 1 & 2 \\ -1 & -1 & 1 \end{vmatrix} = 1 - 2 - 4 + 2 + 2 - 2 = -3 \neq 0$$

<u>Se cruzan</u>

ii) $r: \begin{cases} x = t \\ y = 1 + 2t \\ z = -1 + 2t \end{cases}$ $\quad \overrightarrow{P_rP}\,(1-t, -2t, -2t)$

$\sqrt{(1-t)^2 + (-2t)^2 + (-2t)^2} = 1$; $\quad (1-t)^2 + (-2t)^2 + (-2t)^2 = 1$

$1 + t^2 - 2t + 4t^2 + 4t^2 = 1$; $\quad 9t^2 - 2t = 0$

$t(9t-2) = 0$ $\begin{cases} t = 0 \longrightarrow P_{r1}(0, 1, -1) \\ t = 2/9 \longrightarrow P_{r2}\left(\frac{2}{9}, \frac{13}{9}, \frac{-5}{9}\right) \end{cases}$

b) $\pi_{//}: x + y + z + D = 0$

Corte eje OX: $(-D, 0, 0)$ $\quad \overline{u}\,(D, -D, 0)$

" " OY: $(0, -D, 0)$

" " OZ: $(0, 0, -D)$ $\quad \overline{v}\,(D, 0, -D)$

$$\begin{vmatrix} \overline{\imath} & \overline{\jmath} & \overrightarrow{k} \\ D & -D & 0 \\ D & 0 & -D \end{vmatrix} = (D^2, D^2, D^2)$$

$\dfrac{\sqrt{D^4+D^4+D^4}}{2} = 8\sqrt{3}$; $\sqrt{3D^4} = 16\sqrt{3}$

$3D^4 = 768$; $D^4 = \dfrac{768}{3} = 256$; $D = \pm\sqrt{256} = \underline{\underline{\pm 4}}$

3) a) $\overline{u_r} = \begin{vmatrix} \bar{i} & \bar{j} & \bar{k} \\ 1 & 1 & 1 \\ 2 & -2 & 1 \end{vmatrix} = (3, 1, -4)$

$P_r(0,1,1)$ $\quad \overrightarrow{P_rP} = (2, 2, -1)$

$P(2,3,0)$

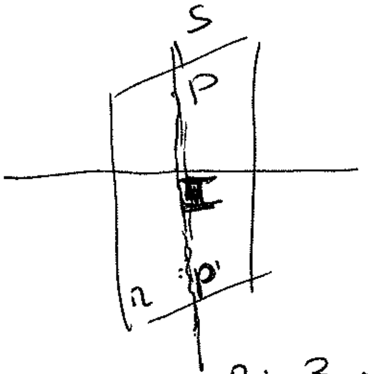
$\overline{u_r} \wedge \overline{P_rP} = \begin{vmatrix} \bar{i} & \bar{j} & \bar{k} \\ 3 & 1 & -4 \\ 2 & 2 & -1 \end{vmatrix} = (7, -5, 4)$

$7x - 5y + 4z + D = 0$; $\quad 14 - 15 + D = 0$; $D = 1$

$\boxed{\pi : 7x - 5y + 4z + 1 = 0}$

$\overline{u_r} = \overline{n}$

b)



$3x + y - 4z + D = 0$

$6 + 3 + D = 0$; $D = -9$

$\pi : 3x + y - 4z - 9 = 0$

$r: \begin{cases} x = 3t \\ y = 1 + t \\ z = 1 - 4t \end{cases}$

$3(3t) + (1+t) - 4(1-4t) - 9 = 0$

$9t + 1 + t - 4 + 16t - 9 = 0$

$26t = 12$; $t = \dfrac{12}{26} = \dfrac{6}{13}$

$I = \left(\dfrac{18}{13}, \dfrac{19}{13}, -\dfrac{11}{13}\right)$

$$\frac{x+2}{2} = \frac{18}{13} \qquad 13x+26 = 36; \quad x = \frac{10}{13}$$

$$\frac{y+3}{2} = \frac{19}{13} \qquad 13y+39 = 38; \quad y = \frac{-1}{13}$$

$$\frac{z+0}{2} = -\frac{11}{13} \qquad 13z = -22; \quad z = -\frac{22}{13}$$

$$P'\left(\frac{10}{13}, \frac{-1}{13}, \frac{-22}{13}\right)$$

c) $\vec{u}_s = \overrightarrow{PI} = \left(\frac{18}{13} - 2, \frac{19}{13} - 3, \frac{-11}{13}\right)$

$$\vec{u}_s = \left(\frac{-8}{13}, \frac{-20}{13}, \frac{-11}{13}\right) \simeq (8, 20, 11)$$

$$\boxed{s: \frac{x-2}{8} = \frac{y-3}{20} = \frac{z}{11}}$$

4) a) $\vec{u}_r(-2, 1, m) \qquad \vec{n}(2, 1, -1)$

$$\vec{u}_r \cdot \vec{n} = -4 + 1 - m = -3 - m$$

$$-3 - m = 0; \quad \boxed{m = -3}$$

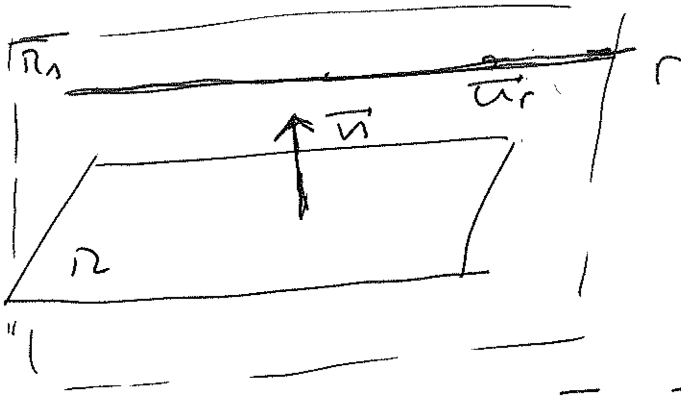
Si $m \neq -3$ son secantes

Si $m = -3$

$P_r(5, 0, 6)$; $2 \cdot 5 + 0 - 6 = 10 - 6 \neq 0$

Si $m = -3$ paralela no contenida

b)



$$\vec{n}_1 = \vec{u}_r \wedge \vec{u} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ -2 & 1 & -3 \\ 2 & 1 & -1 \end{vmatrix} = (2,-8,-4)$$

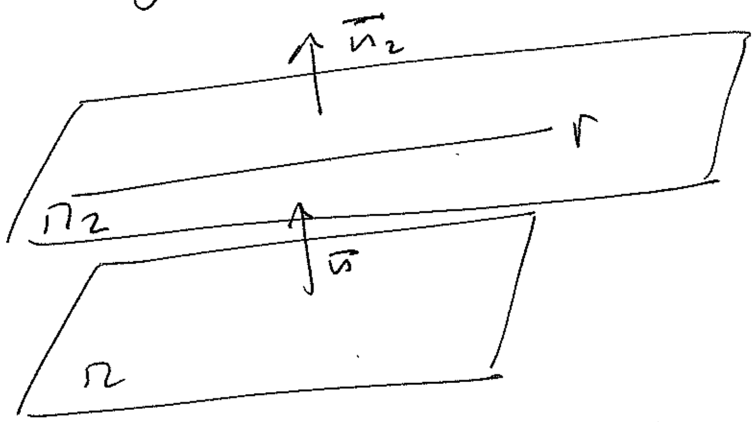
$2x-8y-4z+D=0$; $2\cdot 5-8\cdot 0-4\cdot 6+D=0$

$10-24+D=0$; $D=14$

$2x-8y-4z+14=0$

$$\boxed{\pi_1 \equiv x-4y-2z+7=0}$$

c)



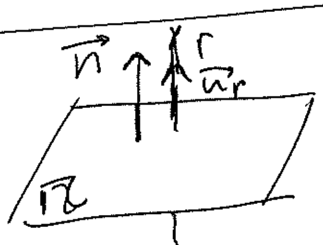
$\vec{n}_2 = \vec{u} = (2,1,-1)$

$2x+y-z+D=0$; $2\cdot 5+0-6+D=0$

$10-6+D=0$; $D=-4$

$$\boxed{\pi_2 : 2x+y-z-4=0}$$

d)



$\vec{u}_r \parallel \vec{u}$

$\frac{-2}{2} = \frac{1}{1} = \frac{m}{-1}$

$-1 \neq 1$

NO PUEDE SER

1) a) $\begin{vmatrix} 1 & 0 & 1 \\ 0 & -2 & 2 \\ m & 1 & n \end{vmatrix} = -2n + 2m - 2 = 0$

$$\left.\begin{array}{l} m - n = 1 \\ m + n = 0 \end{array}\right\} \begin{array}{l} m = 1/2 \\ n = -1/2 \end{array} //$$

b) $\sqrt{m^2+1+n^2} = 1$; $m^2 + n^2 = 0$

$$-\frac{\sqrt{2}}{2} = \frac{-2 + 2n}{\sqrt{8} \cdot 1} ; \quad -2 = -2 + 2n ; \quad \underline{\underline{n = 0 \, , \, m = 0}}$$

c) $\bar{u} \wedge \vec{v} = \begin{vmatrix} \bar{i} & \bar{j} & \vec{k} \\ 1 & 0 & 1 \\ 0 & -2 & 2 \end{vmatrix} = (2, -2, -2)$

$\bar{u} \wedge \overline{w} = \begin{vmatrix} \bar{i} & \bar{j} & \vec{k} \\ 1 & 0 & 1 \\ m & 1 & n \end{vmatrix} = (-1, m-n, 1)$

$\vec{v} \wedge \overline{w} = \begin{vmatrix} \bar{i} & \vec{j} & \vec{k} \\ 0 & -2 & 2 \\ m & 1 & n \end{vmatrix} = (-2n-2, 2m, 2m)$

$(\bar{u} \wedge \vec{v}) \perp (\bar{u} \wedge \vec{w}) \Rightarrow$ ~~-2-2m+2n-2=0~~

$-2 - 2m + 2n - 2 = 0$

$-2m + 2n = 4 \rightarrow \underline{-m + n = 2}$

$(\bar{u} \wedge \vec{v}) \perp (\vec{v} \wedge \vec{w}) \Rightarrow -4n - 4 - 4m - 4m = 0$

$-8m - 4n = 4 \rightarrow \underline{-2m - n = 1}$

$$\left.\begin{array}{l} -m + n = 2 \\ -2m - n = 1 \end{array}\right\} \quad \underline{\underline{m = -1 ; \quad n = 1}}$$

2) $\vec{u}_r = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & 1 & -2 \\ 3 & 0 & -1 \end{vmatrix} = (-1, -5, -3)$

$P_r(0, -4, -1)$

$\vec{u}_s = (2, -1, 6)$ $\qquad P_s(-1, 2, -4)$

b)

$r: \begin{cases} x = -t_1 \\ y = -4 - 5t_1 \\ z = -1 - 3t_1 \end{cases}$

$s: \begin{cases} x = -1 + 2t_2 \\ y = 2 - t_2 \\ z = -4 + 6t_2 \end{cases}$ $\qquad \begin{cases} -t_1 = -1 + 2t_2 \\ -4 - 5t_1 = 2 - t_2 \\ -1 - 3t_1 = -4 + 6t_2 \end{cases}$ $\begin{cases} t_1 = -1 \\ t_2 = 1 \end{cases}$

$\boxed{I = (1, 1, 2)}$

a) $\overrightarrow{P_rP_s}(-1, 6, -3)$ $\qquad \begin{vmatrix} -1 & -5 & -3 \\ 2 & -1 & 6 \\ -1 & 6 & -3 \end{vmatrix} =$

$= -3 - 36 + 30 + 3 + 36 - 30 = 0$

<u>Son secantes</u>

c) $\cos(\widehat{\bar r, \bar s}) = \dfrac{|-1\cdot 2 - 5\cdot(-1) - 3\cdot 6|}{\sqrt{(-1)^2+(-5)^2+(-3)^2}\ \sqrt{2^2+(-1)^2+6^2}} =$

$= \dfrac{|-2+5-18|}{\sqrt{35}\ \sqrt{41}} = \dfrac{15}{\sqrt{35}\ \sqrt{41}}$; $(\widehat{r, s}) = 66'6^\circ$

d) $\bar n = \begin{vmatrix} \bar i & \bar j & \bar k \\ -1 & -5 & -3 \\ 2 & -1 & 6 \end{vmatrix} = (-33, 0, 11)$

$-33x + 11z + D = 0$; $-33 + 22 + D = 0$; $\underline{\underline{D = 11}}$

$-33x + 11z + 11 = 0$; $\boxed{3x - z - 1 = 0}$

3) a) $\vec n(m, -3, 2)$; $P_r\left(0, 1, \frac{2}{m}\right)$

$\bar u_r = \begin{vmatrix} \bar i & \bar j & \bar k \\ 3 & 1 & 0 \\ 2 & -1 & m \end{vmatrix} = (m, -3m, -5)$

$\bar n \cdot \bar u_r = m^2 + 9m - 10 = 0$ $\begin{cases} m = 1 \\ m = -10 \end{cases}$

$\underline{\underline{m = 1}}$

$P_r(0, 1, +2)$ $\qquad \pi: x - 3y + 2z - 1 = 0$

$0 - 3\cdot 1 + 2\cdot(+2) - 1 = -3 + 4 - 1 = 0$

<u>Paralela contenida</u>

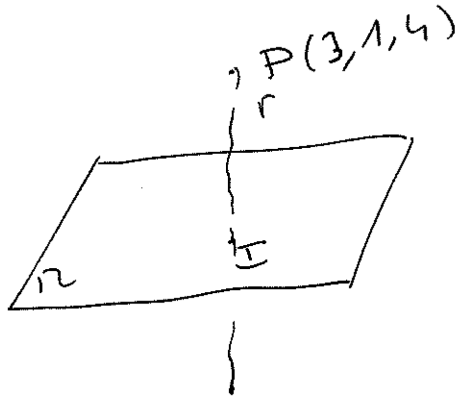
$m = -10;$ $\quad P_r\left(0, 1, +\frac{1}{5}\right)$ $\quad \pi: -10x - 3y + 2z - 1 = 0$

$$-10 \cdot 0 - 3 \cdot 1 + 2 \cdot \frac{1}{5} - 1 = -3 + \frac{2}{5} - 1 \neq 0$$

Paralela no contenida

Si $m \neq +1, -10 \Rightarrow$ **Secante**

b)



$\overline{u_r} = \overline{n} = (1, 0, -1)$

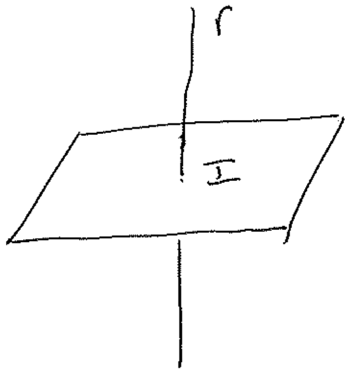
$$r: \begin{cases} x = 3 + t \\ y = 1 \\ z = 4 - t \end{cases}$$

$(3+t) - (4-t) - 3 = 0;$ $\quad 3 + t - 4 + t - 3 = 0$

$2t = 4;$ $\quad t = 2;$ $\quad I(5, 1, 2)$

$$d(P, \pi) = \frac{|3 - 4 - 3|}{\sqrt{1^2 + 0^2 + (-1)^2}} = \frac{4}{\sqrt{2}} = \frac{4\sqrt{2}}{2} = \underline{\underline{2\sqrt{2}\,u}}$$

4) a)



$$\overline{u_r} = \overline{n} = (1,-1,1)$$

$$\pi: x - y + z = 0$$

$$(1+t)-(2-t)+t = 0; \quad 1+t-2+t+t=0;$$

$$3t=1; \quad t = 1/3 \Rightarrow I = \left(\frac{4}{3}, \frac{5}{3}, \frac{1}{3}\right)$$

b) $\overline{u_r}\,(a,2,1)$ $\qquad \overline{u_s} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & 1 & 0 \\ 0 & 1 & -1 \end{vmatrix} = (-1,1,1)$

$$\overline{u_r} \perp \overline{u_s} \Rightarrow a(-1)+2\cdot 1+1\cdot 1 = 0$$

$$-a+2+1 = 0; \quad -a=-3; \quad \boxed{a=3}$$

$P_r\,(1,1,-1)$ $\qquad P_s\,(b,0,-1)$

$\overline{P_rP_s}\,(b-1,-1,0)$

$$\begin{vmatrix} 3 & 2 & 1 \\ -1 & 1 & 1 \\ b-1 & -1 & 0 \end{vmatrix} = 1+2(b-1)-(b-1)+3=0$$

$$1+2b-2-b+1+3=0$$

$$\boxed{b=-3}$$

1) a) $\lim\limits_{x\to 3} \dfrac{x-1-\sqrt{x+1}}{x^3-27} = \lim\limits_{x\to 3} \dfrac{\left((x-1)-\sqrt{x+1}\right)\left((x-1)+\sqrt{x+1}\right)}{(x^3-27)\left((x-1)+\sqrt{x+1}\right)} =$

$= \lim\limits_{x\to 3} \dfrac{(x-1)^2-(x+1)}{(x^3-27)\left((x-1)+\sqrt{x+1}\right)} =$

$$\begin{array}{r|rrrr} & 1 & 0 & 0 & -27 \\ 3 & & 3 & 9 & 27 \\ \hline & 1 & 3 & 9 & 0 \end{array}$$

$= \lim\limits_{x\to 3} \dfrac{x^2+1-2x-x-1}{(x-3)(x^2+3x+9)\left((x-1)+\sqrt{x+1}\right)} =$

$= \lim\limits_{x\to 3} \dfrac{x^2-3x}{(x-3)(x^2+3x+9)\left((x-1)+\sqrt{x+1}\right)} = \lim\limits_{x\to 3} \dfrac{x\cancel{(x-3)}}{\cancel{(x-3)}(x^2+3x+9)\left((x-1)+\sqrt{x+1}\right)}$

$= \lim\limits_{x\to 3} \dfrac{x}{(x^2+3x+9)\left((x-1)+\sqrt{x+1}\right)} = \dfrac{3}{(3^2+3\cdot 3+9)\left((3-1)+\sqrt{3+1}\right)} =$

$= \dfrac{3}{27\cdot 4} = \dfrac{3}{108} = \dfrac{1}{36}$ //

b) $\lim\limits_{x\to 2} \left(\dfrac{(3x+1)+(x^2-2x)}{3x+1}\right)^{\frac{k+1}{k(x^2-4)}} =$

$= \lim\limits_{x\to 2} \left(1+\dfrac{x^2-2x}{3x+1}\right)^{\frac{k+1}{k(x^2-4)}} = \lim\limits_{x\to 2} \left(1+\dfrac{1}{\frac{3x+1}{x^2-2x}}\right)^{\frac{k+1}{k(x^2-4)}} =$

$= \lim\limits_{x\to 2} \left(1+\dfrac{1}{\frac{3x+1}{x^2-2x}}\right)^{\frac{k+1}{k(x^2-4)}\cdot\frac{3x+1}{x^2-2x}\cdot\frac{x^2-2x}{3x+1}} =$

$= e^{\lim\limits_{x\to 2} \frac{(k+1)x\cancel{(x-2)}}{k(x+2)\cancel{(x-2)}(3x+1)}} = e^{\lim\limits_{x\to 2} \frac{(k+1)x}{k(x+2)(3x+1)}} =$

$= e^{\frac{(k+1)\cdot 2}{28k}} = e^{1/4}$

$\dfrac{(k+1)\cdot 2}{28k} = \dfrac{1}{4}$; $8(k+1) = 28k$; $8k+8 = 28k$

$8 = 20k$; $k = \dfrac{8}{20} = \dfrac{2}{5}$ //

c) $$\lim_{x \to -\infty} \left(\frac{(x^2+1)(\sqrt{x^2-1}-1)-(x^2-1)(\sqrt{x^2+1}+1)}{(\sqrt{x^2+1}+1)(\sqrt{x^2-1}-1)} \right) =$$

$$= \lim_{x \to -\infty} \frac{x^2\sqrt{x^2-1}-x^2+\sqrt{x^2-1}-1-x^2\sqrt{x^2+1}-x^2+\sqrt{x^2+1}+1}{\sqrt{x^2+1}\sqrt{x^2-1}-\sqrt{x^2+1}+\sqrt{x^2-1}-1} =$$

$$= \lim_{x \to -\infty} \frac{\cancel{x^3}-x^2+x-1-\cancel{x^3}-x^2+x+1}{x^2-x+x-1} =$$

$$= \lim_{x \to -\infty} \frac{-2x^2+2x}{x^2-1} = -2 \,//$$

$-$ $+$

2

2) $x-2=0$

a)
$$f(x) = \begin{cases} \dfrac{ax+(-x+2)}{x+(-x+2)} & \text{si } x < 2 \\[2mm] \dfrac{ax+(x-2)}{x+(x-2)} & \text{si } x \geq 2 \end{cases}$$

$$f(x) = \begin{cases} \dfrac{ax-x+2}{2} & \text{si } x < 2 \\[2mm] \dfrac{ax+x-2}{2x-2} & \text{si } x \geq 2 \end{cases}$$

~~FUNCIONES~~ ~~CONEXIONES~~

$$f'(x) = \begin{cases} \dfrac{a-1}{2} & \text{si } x < 2 \\[2mm] \dfrac{(a+1)(2x-2)-2(ax+x-2)}{(2x-2)^2} & \text{si } x \geq 2 \end{cases}$$

$$\frac{2a\cancel{x}-2a+2\cancel{x}-2-2\cancel{a x}-2\cancel{x}+4}{(2x-2)^2} = \frac{-2a+2}{(2x-2)^2}$$

$$f'(x) = \begin{cases} \dfrac{a-1}{2} & si \quad x < 2 \\ \dfrac{-2a+2}{(2x-2)^2} & si \quad x > 2 \end{cases}$$

$$\lim_{x \to 2^-} f(x) = a$$

$$\lim_{x \to 2^+} f(x) = a$$

$$f(2) = a$$

Continua

$$\lim_{x \to 2^-} f'(x) = \frac{a-1}{2}$$

$$\lim_{x \to 2^+} f'(x) = \frac{-2a+2}{4} = \frac{-a+1}{2}$$

$$\frac{a-1}{2} = \frac{-a+1}{2}; \quad 2a-2 = -2a+2$$

$$4a = 4; \quad \boxed{a = 1}$$

b)

$$f(x) = \begin{cases} \dfrac{x+2}{2} & si \quad x < 2 \\ \dfrac{3x-2}{2x-2} & si \quad x \geq 2 \end{cases}$$

$$f'(x) = \begin{cases} \dfrac{1}{2} & si \quad x < 2 \\ \dfrac{-2}{(2x-2)^2} & si \quad x > 2 \end{cases}$$

$\boxed{\text{CONTINUIDAD}}$

FUNCIONES

NADA

CONEXIONES

$x = 2$

$$\lim_{x \to 2^-} f(x) = 2$$
$$\lim_{x \to 2^+} f(x) = 2$$
$$f(2) = 2$$

Continua

DERIVABILIDAD

FUNCIONES

NADA

CONEXIONES

$x = 2$

$$\lim_{x \to 2^-} f'(x) = \frac{1}{2}$$
$$\lim_{x \to 2^+} f'(x) = -\frac{1}{2}$$

~~No derivable~~

No derivable

Continua en $\mathbb{R}$; Derivable en $\mathbb{R} \setminus \{2\}$

3) a) $f(x) = \begin{cases} \ln(x^2+a^2) & \text{si } x < 0 \\ \dfrac{bx + \ln a + 1}{x+1} & \text{si } x \geq 0 \end{cases}$

$$f'(x) = \begin{cases} \dfrac{2x}{x^2+a^2} & \text{si } x < 0 \\ \dfrac{b(x+1) - 1(bx + \ln a + 1)}{(x+1)^2} & \text{si } x > 0 \end{cases}$$

$$\frac{\cancel{bx} + b - \cancel{bx} - \ln a - 1}{(x+1)^2}$$

$$f'(x) = \begin{cases} \dfrac{2x}{x^2+a^2} & \text{si } x < 0 \\ \dfrac{b - \ln a - 1}{(x+1)^2} & \text{si } x > 0 \end{cases}$$

③

CONTINUIDAD

FUNCIONES

NADA

CONEXIONES

$x = 0 \to$ CTA si $\underline{a = e}$

$$\lim_{x \to 0^-} f(x) = \ln a^2 = 2 \ln a$$

$$\lim_{x \to 0^+} f(x) = \ln a + 1$$

$$f(0) = \ln a + 1$$

$$2 \ln a = \ln a + 1 ; \quad \ln a = 1 \Rightarrow \boxed{a = e}$$

DERIVABILIDAD

FUNCIONES

NADA

CONEXIONES

$x = 0$

$$\lim_{x \to 0^-} f'(x) = 0$$

$$\lim_{x \to 0^+} f'(x) = b - \ln a - 1$$

$$b - \ln a - 1 = 0 \Rightarrow b = \ln a + 1$$

Como $a = e \Rightarrow b = \ln e + 1 = 1 + 1 = 2$ $\boxed{b = 2}$

b) $x - y + 2 = 0 \Rightarrow y = x + 2 \Rightarrow m_{tg} = 1$

$f(1) = 0$ y $f'(1) = 1$

$$f(1) = \frac{b + \ln a + 1}{1 + 1} = \frac{b + \ln a + 1}{2} = 0$$

$$\underline{b = -\ln a - 1}$$

$$f'(1) = \frac{b - \ln a - 1}{(1+1)^2} = \frac{b - \ln a - 1}{4} = 1$$

$$\underline{\underline{b - \ln a - 1 = 4}}$$

$$\left.\begin{aligned} b &= -\ln a - 1 \\ b &= 4 + \ln a + 1 \end{aligned}\right\} \quad \begin{aligned} -\ln a - 1 &= 5 + \ln a \\ -6 &= 2\ln a \\ \ln a &= -\frac{6}{2} = -3 \end{aligned}$$

$$\boxed{a = e^{-3}}$$

$$b = -(-3) - 1 = 3 - 1; \qquad \boxed{b = 2}$$

4) a) i) $y = \ln \sqrt[3]{\dfrac{x+2}{x-2}} = \dfrac{1}{3}\left(\ln(x+2) - \ln(x-2)\right)$

$$y' = \frac{1}{3}\left(\frac{1}{x+2} - \frac{1}{x-2}\right) = \frac{1}{3}\left(\frac{x-2-x-2}{(x+2)(x-2)}\right) =$$

$$= \frac{1}{3} \cdot \frac{-4}{x^2-4} = \frac{-4}{3(x^2-4)} \quad //$$

ii) $y' = \dfrac{1}{1 + \left(\dfrac{x^2+1}{x^2-1}\right)^2} \cdot \dfrac{2x(x^2-1) - 2x(x^2+1)}{(x^2-1)^2} =$

$$= \frac{2x^3 - 2x - 2x^3 - 2x}{\dfrac{(x^2-1)^2 + (x^2+1)^2}{\cancel{(x^2-1)^2}} \cdot \cancel{(x^2-1)^2}} =$$

$$= \frac{-4x}{x^4 + 1 - \cancel{2x^2} + x^4 + 1 + \cancel{2x^2}} = \frac{-4x}{2(x^4+1)} = \frac{-2x}{x^4+1} \quad //$$

b) $$(f\circ g)(x) = f(g(x)) = \frac{(\sqrt{x^2-1})^2+1}{(\sqrt{x^2-1})^2} = \frac{x^2-1+1}{x^2-1} =$$

$$= \frac{x^2}{x^2-1}$$ //

4

$$(f\circ g)'(x) = \frac{2x(x^2-1)-2x\cdot x^2}{(x^2-1)^2} = \frac{\cancel{2x^3}-2x-\cancel{2x^3}}{(x^2-1)^2} =$$

$$= \frac{-2x}{(x^2-1)^2}$$

$$f'(x) = \frac{2x\cdot x^2-2x(x^2+1)}{(x^2)^2} = \frac{\cancel{2x^3}-\cancel{2x^3}-2x}{x^4} = -\frac{2}{x^3}$$ //

$$g'(x) = \frac{1}{2\sqrt{x^2-1}}\cdot 2x = \frac{x}{\sqrt{x^2-1}}$$ //

$$f'(x)\circ g'(x) = f'(g'(x)) = \frac{-2}{\left(\frac{x}{\sqrt{x^2-1}}\right)^3} =$$

$$= \frac{-2(\sqrt{x^2-1})^3}{x^3} = \frac{-2(x^2-1)\sqrt{x^2-1}}{x^3}$$

NO ES CIERTO

1) a)

i) $L = \lim_{x\to 0} \left(\frac{1}{x^2}\right)^{tg\,x} =$ $\ln L = \ln \lim_{x\to 0} \left(\frac{1}{x^2}\right)^{tg\,x} =$

$= \lim_{x\to 0} \ln\left(\frac{1}{x^2}\right)^{tg\,x} = \lim_{x\to 0} tg\,x \cdot \ln\left(\frac{1}{x^2}\right) =$

$$= \lim_{x\to 0} \frac{\ln\left(\frac{1}{x^2}\right)}{\frac{1}{tg\,x}} \underset{\text{L'HôP}}{\overset{\uparrow}{=}} \lim_{x\to 0} \frac{\frac{1}{\frac{1}{x^2}} \cdot \frac{-2}{x^3}}{\frac{\frac{-1}{\cos^2 x}}{tg^2 x}} =$$

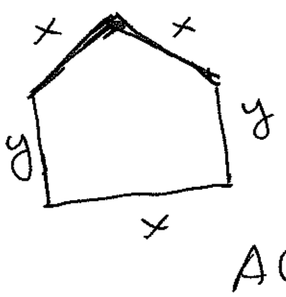
$$= \lim_{x\to 0} \frac{\frac{-2x^2}{x^3}}{\frac{-1}{tg^2 x \cdot \cos^2 x}} = \lim_{x\to 0} \frac{\frac{-2}{x}}{\frac{-1}{sen^2 x}} = \lim_{x\to 0} \frac{2\,sen^2 x}{x} =$$

$$\underset{\text{L'HôP}}{\overset{\uparrow}{=}} \lim_{x\to 0} \frac{4\,sen\,x \cdot \cos x}{1} = 0 \Rightarrow \underline{\underline{L = e^0 = 1}}$$

ii) $$\lim_{x\to 0} \frac{-tg^2 x}{1-\cos x} \underset{\text{L'HôP}}{\overset{\uparrow}{=}} \lim_{x\to 0} \frac{-2\,tg\,x \cdot \frac{1}{\cos^2 x}}{sen\,x} =$$

$$= \lim_{x\to 0} \frac{-2\,\cancel{sen\,x}}{\cancel{sen\,x}\,\cos^3 x} = \lim_{x\to -2} \frac{-2}{\cos^3 x} = \underline{\underline{-2}}$$

b)



$A(x,y) = xy + \frac{\sqrt{3}}{4}x^2 \to$ máximo

$3x + 2y = 12$, $y = \frac{12-3x}{2}$

$A(x) = x\left(\frac{12-3x}{2}\right) + \frac{\sqrt{3}}{4}x^2 =$

$= \frac{12x - 3x^2}{2} + \frac{\sqrt{3}x^2}{4} = \frac{24x - 6x^2 + \sqrt{3}x^2}{4} =$

$= \frac{(\sqrt{3}-6)x^2 + 24x}{4}$ – máximo

$A'(x) = \frac{2(\sqrt{3}-6)x + 24}{4} = 0$

$2(\sqrt{3}-6)x + 24 = 0$

$x = \frac{-24}{2(\sqrt{3}-6)} = \frac{12}{6-\sqrt{3}}$ //

$x \in [0, 4]$

$A(0) = 0$

$A(4) = \dfrac{(\sqrt{3}-6)\cdot 16 + 80}{4} \simeq 2'92$

$A\left(\dfrac{12}{6-\sqrt{3}}\right) = \dfrac{(\sqrt{3}-6)\cdot\left(\frac{12}{6-\sqrt{3}}\right)^2 + 20\left(\frac{12}{6-\sqrt{3}}\right)}{4} =$

$= \dfrac{\frac{144}{6-\sqrt{3}} - \frac{240}{6-\sqrt{3}}}{4} = \dfrac{96}{4(6-\sqrt{3})} = 5'62$

$x = \dfrac{12}{6-\sqrt{3}} \simeq 2'81$; $y = \dfrac{12 - \frac{36}{6-\sqrt{3}}}{2} =$

$= \dfrac{36 - 12\sqrt{3}}{2(6-\sqrt{3})} = \dfrac{18-6\sqrt{3}}{6-\sqrt{3}} \simeq 1'78$

2) i) $D = \mathbb{R}$

ii) Corte ejes

$OX: x^2 - 2x = 0$ → $x = 0 \to (0,0)$; $x = 2 \to (2,0)$

$OY: f(0) = 0 \to (0,0)$

iii) Asíntotas

Horiz: $\lim_{x\to+\infty} \dfrac{x^2-2x}{e^x} = 0$ $\boxed{y=0}$ Dcha

$\lim_{x\to-\infty} \dfrac{x^2-2x}{e^x} = +\infty$

Vert: No hay

Oblic: $m = \lim_{x\to\pm\infty} \dfrac{\frac{x^2-x}{e^x}}{x} = \lim_{x\to\pm\infty} \dfrac{x^2-x}{x\cdot e^x} =$

$= \lim_{x\to\pm\infty} \dfrac{x-1}{e^x} =$ → 0 ; → $-\infty$

No hay

iv) Extremos relativos.

$$f'(x) = \frac{(2x-2)\cdot e^x - (x^2-2x)e^x}{e^{2x}} =$$

$$= \frac{e^x(-x^2+4x-2)}{e^{2x}} = \frac{-x^2+4x-2}{e^x} = 0$$

$-x^2+4x-2=0$; $x^2-4x+2=0$; $x = \frac{4 \pm \sqrt{16-8}}{2} = \frac{4 \pm 2\sqrt{2}}{2} = \begin{cases} 2+\sqrt{2} \\ 2-\sqrt{2} \end{cases}$

$E_1(\underbrace{2+\sqrt{2}}_{3'41}, 0'15)$ Máx relat $E_2(\underbrace{2-\sqrt{2}}_{0'58}, -0'46)$ Mín relat

$$f''(x) = \frac{(-2x+4)e^x - (-x^2+4x-2)e^x}{e^{2x}} =$$

$$= \frac{e^x(x^2-6x+6)}{e^{2x}} = \frac{(x^2-6x+6)}{e^x} = 0$$

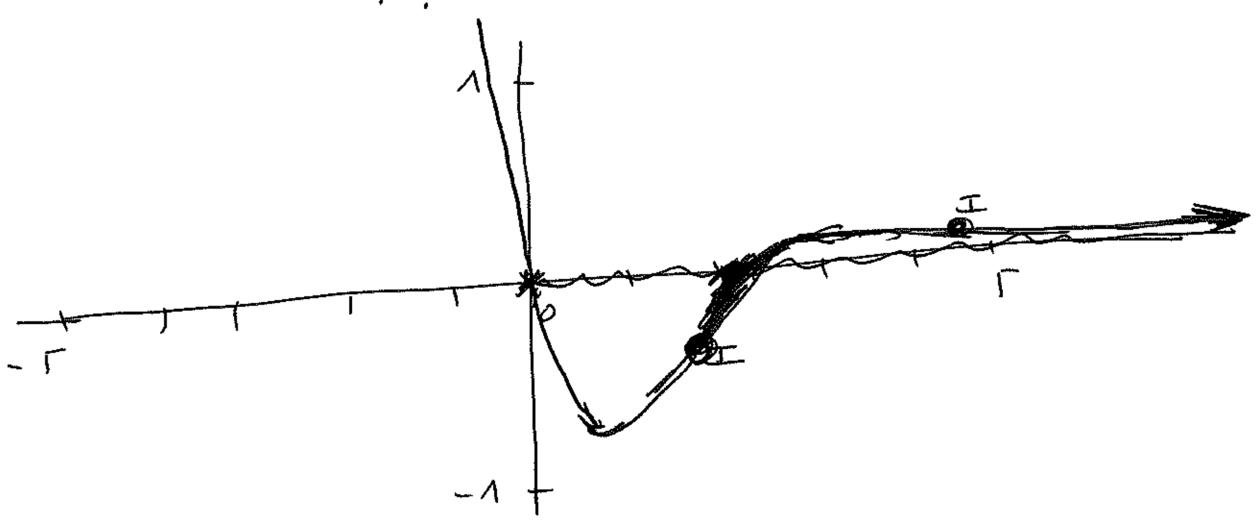
$x^2-6x+6=0$; $x = \frac{6 \pm \sqrt{36-24}}{2} = \frac{6 \pm 2\sqrt{3}}{2} = \begin{cases} 3+\sqrt{3} \\ 3-\sqrt{3} \end{cases}$

$I_1(\underbrace{3+\sqrt{3}}_{4'73}, 0'11)$ $I_2(\underbrace{3-\sqrt{3}}_{1'26}, -0'26)$

v) Simetrías

$$f(-x) = \frac{(-x)^2-2(-x)}{e^{-x}} = \frac{x^2+2x}{e^{-x}} = \begin{cases} f(x) \to \text{NO} \\ -f(x) \to \text{NO} \end{cases}$$

No tiene



3) a)

$$\begin{array}{c|cccc} & 1 & -4 & 5 & -2 \\ 1 & & 1 & -3 & 2 \\ \hline & 1 & -3 & 2 & \boxed{0} \end{array}$$

$x^2-3x+2=0 \to 1,\ 2$

$$\frac{x^2+1}{x^3-4x^2+5x-2} = \frac{A}{(x-1)} + \frac{B}{(x-1)^2} + \frac{C'}{(x-2)} =$$

$$= \frac{A(x-1)(x-2)+B(x-2)+C'(x-1)^2}{(x-1)^2(x-2)}$$

$x=1 \to \quad 2=-B \to B=-2$

$x=2 \to \quad 5 = C' \to C'=5$

$x=0 \to \quad 1 = 2A-2B+C' \to A=-4$

$$\int \frac{x^2+1}{x^2-4x^2+5x-2}\,dx = \int \frac{-4}{(x-1)}\,dx + \int \frac{-2}{(x-1)^2}\,dx + \int \frac{5}{(x-2)}\,dx$$

$$= \underline{\underline{-4\ln(x-1) + 2/(x-1) + 5\ln(x-2) + cte}}$$

b) $J = \int \ln(1+x^2)\,dx$;

$u = \ln(1+x^2) \qquad v = x$

$du = \dfrac{2x\,dx}{1+x^2} \qquad dv = dx$

$$I = x\ln(1+x^2) - \int \frac{2x^2}{1+x^2}\,dx$$

$$2x^2 = 2(1+x^2)-2$$

$$\int \frac{2x^2}{1+x^2}\,dx = \int 2\,dx - \int \frac{2\,dx}{1+x^2} = 2x - 2\,\mathrm{arctg}\,x$$

$$J = \underline{\underline{x\ln(1+x^2) - 2x + 2\,\mathrm{arctg}\,x + cte}}$$

4) a) $I = \int x\sqrt{1+3x^2}\,dx$

$$1+3x^2 = t$$
$$6x\,dx = dt$$
$$x\,dx = \frac{dt}{6}$$

$$I = \int \sqrt{t}\cdot\frac{dt}{6} = \frac{1}{6}\int t^{1/2}\,dt =$$

$$= \frac{1}{6}\cdot\frac{t^{3/2}}{3/2} = \frac{2\sqrt{t^3}}{18} = \frac{\sqrt{t^3}}{9} = \frac{\sqrt{(1+3x^2)^3}}{9}$$

$$\left[\frac{\sqrt{(1+3x^2)^3}}{9}\right]_0^{\sqrt{5}} = \left(\frac{\sqrt{16^3}}{9} - \frac{\sqrt{1}}{9}\right) = \frac{64}{9} - \frac{1}{9} = \frac{63}{9} =$$

$$= 7$$

b) $\frac{x^2-2x}{e^x} = 0$; $x^2-2x=0 \;\; \begin{cases} x=0 \\ x=2 \end{cases}$

0		2		3
	$f<0$		$f>0$	

$$A = -\int_0^2 \frac{x^2-2x}{e^x}\,dx + \int_2^3 \frac{x^2-2x}{e^x}\,dx$$

$$I = \int \frac{x^2-2x}{e^x}\,dx = \int (x^2-2x)\cdot e^{-x}\,dx$$

$$I = -(x^2-2x)e^{-x} + \underbrace{\int (2x-2)e^{-x}\,dx}_{I_1}$$

$u = x^2-2x$ $\quad v = -e^{-x}$

$du = (2x-2)dx$ $\quad dv = e^{-x}dx$

$$I_1 = -(2x-2)e^{-x} + \int 2e^{-x}\,dx =$$

$u_1 = 2x-2$ $\quad v_1 = -e^{-x}$

$$= -(2x-2)e^{-x} - 2e^{-x}$$

$du_1 = 2dx$ $\quad dv_1 = e^{-x}dx$

$$I = -(x^2-2x)\,e^{-x} - (2x-2)\,e^{-x} - 2e^{-x} =$$

$$= e^{-x}\cdot(-x^2) = -x^2\cdot e^{-x} = \frac{-x^2}{e^x}$$

$$A = -\left[\frac{-x^2}{e^x}\right]_0^2 + \left[\frac{-x^2}{e^x}\right]_2^3 =$$

$$= -\left[\frac{-4}{e^2} - 0\right] + \left[\frac{-9}{e^3} - \frac{-4}{e^2}\right] =$$

$$= \frac{4}{e^2} - \frac{9}{e^3} + \frac{4}{e^2} = \frac{8}{e^2} - \frac{9}{e^3} =$$

$$= \frac{(8e-9)}{e^3}\,\mu^2$$

1)

a) $$f(x)=\begin{cases} \dfrac{a}{x+1} & \text{si } x<1 \\ \dfrac{bx^2+1}{x} & \text{si } x\geq 1 \end{cases}$$

$$f'(x)=\begin{cases} \dfrac{-a}{(x+1)^2} & \text{si } x<1 \\ \dfrac{bx^2-1}{x^2} & \text{si } x>1 \end{cases}$$

$$\left.\begin{array}{l} \lim\limits_{x\to 1^-} f(x)=\dfrac{a}{2} \\ \lim\limits_{x\to 1^+} f(x)=b+1 \\ f(1)=b+1 \end{array}\right\} \quad b+1=\frac{a}{2} \qquad \boxed{a=2b+2}$$

$$\left.\begin{array}{l} \lim\limits_{x\to 1^-} f'(x)=-\dfrac{a}{4} \\ \lim\limits_{x\to 1^+} f'(x)=b-1 \end{array}\right\} \quad \begin{array}{l} b-1=-\dfrac{a}{4} \\ 4b-4=-a \\ \boxed{a=4-4b} \end{array}$$

$$2b+2=4-4b\ ;\quad 6b=2\ ;\quad b=\frac{1}{3}$$

$$\boxed{a=\frac{8}{3}\ ;\quad b=\frac{1}{3}}$$

$$a=\frac{2}{3}+2=\frac{8}{3}$$

b) $$f(x)=\begin{cases} \dfrac{4}{x+1} & \text{si } x<1 \\ \dfrac{x^2+1}{x} & \text{si } x\geq 1 \end{cases} \qquad f'(x)=\begin{cases} \dfrac{-4}{(x+1)^2} & \text{si } x<1 \\ \dfrac{x^2-1}{x^2} & \text{si } x\geq 1 \end{cases}$$

FUNCIONES

$x=-1 \to$ D.I.S.I

CONEXIONES

$x=1 \to$ CONTINUA NO DERIVABLE

$$\lim_{x \to -1^-} f(x) = -\infty \qquad \lim_{x \to 1^-} f(x) = 2$$

$$\lim_{x \to -1^+} f(x) = +\infty \qquad \lim_{x \to 1^+} f(x) = 2$$

$$f(-1) = \nexists \qquad f(1) = 2$$

$$\lim_{x \to 1^-} f'(x) = -1$$

$$\lim_{x \to 1^+} f'(x) = 0$$

2) a) i) $$\lim_{x \to 0} \frac{\operatorname{tg} x - \operatorname{sen} x}{x \operatorname{sen} x} \overset{\text{L'Hôp}}{=} \lim_{x \to 0} \frac{\frac{1}{\cos^2 x} - \cos x}{\operatorname{sen} x + x\cos x} =$$

$$= \lim_{x \to 0} \frac{1 - \cos^3 x}{\operatorname{sen} x \cos^2 x + x\cos^3 x} \underset{\text{L'Hôp}}{=}$$

$$\overset{\text{L'Hôp}}{=} \frac{3\cos^2 x \operatorname{sen} x}{\underbrace{\cos^3 x}_{1} - \underbrace{2\cos x \operatorname{sen}^2 x}_{0} + \underbrace{\cos^3 x}_{1} - \underbrace{3x\cos^2 x \operatorname{sen} x}_{0}} = \frac{0}{2} = \underline{\underline{0}}$$

ii) $$L = \lim_{x \to 2} \left(\frac{e^{x-2} + 1}{x} \right)^{\frac{1}{x-2}}$$

$$\ln L = \ln \lim_{x \to 2} \left(\frac{e^{x-2} + 1}{x} \right)^{\frac{1}{x-2}} = \lim_{x \to 2} \ln \left(\frac{e^{x-2} + 1}{x} \right)^{\frac{1}{x-2}} =$$

$$= \lim_{x \to 2} \frac{1}{x-2} \cdot \ln \left(\frac{e^{x-2} + 1}{x} \right) =$$

$$= \lim_{x \to 2} \frac{\ln \left(\frac{e^{x-2} + 1}{x} \right)}{x - 2} \overset{\text{L'Hôp}}{=} \lim_{x \to 2} \frac{\frac{1}{\frac{e^{x-2}+1}{x}} \cdot \frac{e^{x-2} \cdot x - e^{x-2} - 1}{x^2}}{1}$$

$$= \lim_{x \to 2} \frac{x\,(x e^{x-2} - e^{x-2} - 1)}{e^{x-2} + 1} =$$

$$= \frac{2\cdot(2-1-1)}{2} = \frac{0}{2} = 0 \; ; \qquad l = e^{0} = 1$$

b)

$$D = \sqrt{x^2 + 4x^2 + y^2} \rightarrow \text{mínimo} \qquad 2x^2y = 8$$

$$y = \frac{4}{x^2}$$

$$D(x) = \sqrt{x^2 + 4x^2 + \frac{16}{x^4}} \text{ — mínimo}$$

$$D(x) = \sqrt{5x^2 + \frac{16}{x^4}} = \sqrt{\frac{5x^6 + 16}{x^4}} = \frac{\sqrt{5x^6+16}}{x^2}$$

$$D'(x) = \frac{\frac{30x^5}{2\sqrt{5x^6+16}}\cdot x^2 - 2x\sqrt{5x^6+16}}{x^4} =$$

$$= \frac{15x^7 - 2x(5x^6+16)}{x^4\sqrt{5x^6+16}} = 0$$

$$15x^7 - 10x^7 - 32x = 0 \; ; \quad 5x^7 - 32x = 0$$

$$x\,(5x^6 - 32) = 0 \quad \begin{cases} \cancel{x = 0} \\ 5x^6 = 32 \; ; \quad x = \sqrt[6]{\frac{32}{5}} \end{cases}$$

$x \in (0, +\infty)$

$$D(0) = +\infty \; ; \qquad D(\infty) = \lim_{x \to +\infty} \frac{\sqrt{5x^6+16}}{x^2} = +\infty$$

$$x = \sqrt[6]{\frac{32}{5}} \qquad y = \frac{4}{\sqrt[3]{\frac{32}{5}}} = \frac{4\sqrt[3]{5}}{\sqrt[3]{32}}$$

3) a) $f(x) = \dfrac{x^2}{2-x}$

- $D = \mathbb{R} - \{2\}$

- A.H: $\lim\limits_{x \to \pm\infty} \dfrac{x^2}{2-x} = \begin{cases} -\infty \\ +\infty \end{cases}$

- A.V: $\boxed{x = 2}$ $\quad \lim\limits_{x \to 2^-} f(x) = +\infty$; $\lim\limits_{x \to 2^+} f(x) = -\infty$

- A.O: $m = \lim\limits_{x \to \pm\infty} \dfrac{\frac{x^2}{2-x}}{x} = \lim\limits_{x \to \pm\infty} \dfrac{x^2}{2x - x^2} = \underline{\underline{-1}}$

$$n = \lim_{x \to \pm\infty} \left(\frac{x^2}{2-x} - (-1) \cdot x \right) = \lim_{x \to \pm\infty} \left(\frac{x^2}{2-x} + x \right) =$$

$$= \lim_{x \to \pm\infty} \left(\frac{\cancel{x^2} + 2x - \cancel{x^2}}{2-x} \right) = \lim_{x \to \pm\infty} \frac{2x}{2-x} = -2$$

$$\boxed{y = -x - 2}$$

- Extr Relat

$$f'(x) = \frac{2x(2-x) + 1 \cdot x^2}{(2-x)^2} = \frac{4x - 2x^2 + x^2}{(2-x)^2} =$$

$$= \frac{-x^2 + 4x}{(2-x)^2} = 0$$

$-x^2 + 4x = 0$
- $x = 0$ — $E_1(0, 0)$ Mín relat
- $x = 4$ — $E_2(4, -8)$ Máx relat

$f' < 0$		$f' > 0$		$f' > 0$		$f' < 0$
Decrece	0	Crece	2	Crece	4	Decrece
	↓ Mín relat		↓ A.V		↓ Máx relat	

(3)

$$f''(x) = \frac{(-2x+4)(2-x)^2 + 2(2-x)(-x^2+4x)}{(2-x)^4} =$$

$$= \frac{(2-x)\left((-2x+4)(2-x) + 2(-x^2+4x)\right)}{(2-x)^4} =$$

$$= \frac{-\cancel{4x} + \cancel{2x^2} + 8 - \cancel{4x} - \cancel{2x^2} + \cancel{8x}}{(2-x)^3} = \frac{8}{(2-x)^3} \neq 0$$

No tiene puntos de inflexión

b) $$f'(x) = \frac{3x^2 \cdot \operatorname{arctg} x + \frac{x^3}{1+x^2}}{3} + \frac{\frac{2x}{x^2+1}}{6} - \frac{2x}{6} =$$

$$= \frac{3x^2(1+x^2)\operatorname{arctg} x + x^3}{3(1+x^2)} + \frac{x}{3(x^2+1)} - \frac{x}{3} =$$

$$= \frac{3x^2(1+x^2)\operatorname{arctg} x + x^3 + x - x(x^2+1)}{3(x^2+1)} =$$

$$= \frac{3x^2(1+x^2)\operatorname{arctg} x + \cancel{x^3} + \cancel{x} - \cancel{x^3} - \cancel{x}}{3(x^2+1)} =$$

$$= \frac{\cancel{3}x^2\cancel{(1+x^2)}\operatorname{arctg} x}{\cancel{3}\cancel{(x^2+1)}} = \underline{\underline{x^2 \operatorname{arctg} x}}$$

4) a) $\displaystyle\int \frac{\operatorname{arc\,tg}(\ln x)}{x}\,dx$

$\ln x = t$

$\dfrac{dx}{x} = dt$

$\displaystyle I = \int \operatorname{arc\,tg} t \; dt$

$u = \operatorname{arc\,tg} t \qquad v = t$

$du = \dfrac{dt}{1+t^2} \qquad dv = dt$

$\displaystyle I = t \cdot \operatorname{arc\,tg} t - \underbrace{\int \frac{t}{1+t^2}\,dt}_{I_1}$

$1+t^2 = z$

$2t\,dt = dz$

$t\,dt = \dfrac{dz}{2}$

$\displaystyle I_1 = \int \frac{t}{1+t^2}\,dt$

$\displaystyle I = \int \frac{dz/2}{z} = \frac{1}{2}\int \frac{dz}{z} = \frac{1}{2}\ln z =$

$= \dfrac{1}{2}\ln(1+t^2)$

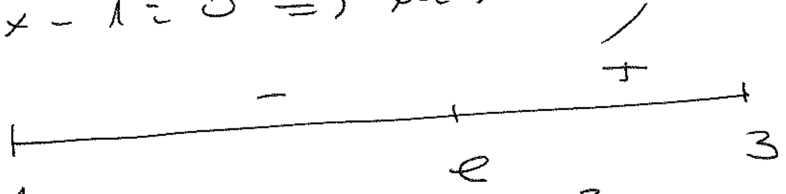
$I = t \cdot \operatorname{arc\,tg} t - \dfrac{1}{2}\ln(1+t^2)$

$$\boxed{I = (\ln x) \cdot \operatorname{arc\,tg}(\ln x) - \frac{1}{2}\ln\left(1+(\ln x)^2\right) + \text{cte}}$$

b) $D(x) = f(x) - g(x) = (\ln x + x) - (x+1) =$

$= \ln x + x - x - 1 = \ln x - 1$

$\ln x - 1 = 0 \Rightarrow \ln x = 1;$ $\quad \underline{\underline{x = e}}$



$$A = -\int_1^e (\ln x - 1)\,dx + \int_e^3 (\ln x - 1)\,dx$$

$u = \ln x - 1 \qquad v = x$

$I = \int (\ln x - 1)\,dx \qquad du = \frac{dx}{x} \qquad dv = dx$

$I = x(\ln x - 1) - \int \frac{\cancel{x}\,dx}{\cancel{x}} = x(\ln x - 1) - x =$

$= x(\ln x - 2)$

$A = -\left[x(\ln x - 2)\right]_1^e + \left[x(\ln x - 2)\right]_e^3 =$

$= -\left[e(\ln e - 2) - 1(\ln 1 - 2)\right] + \left[3(\ln 3 - 2) - e(\ln e - 2)\right] =$

$= -\left[-e + 2\right] + \left[3\ln 3 - 6 + e\right] =$

$= e - 2 + 3\ln 3 - 6 + e = \underline{\underline{2e + 3\ln 3 - 8\ u^2}}$

1) $\left.\begin{array}{l} x+y+z=20.400 \\ 0'06x+0'08y+0'1z=1624 \\ z=2x \end{array}\right\}$

$$\left(\begin{array}{ccc|c} 1 & 1 & 1 & 20.400 \\ 6 & 8 & 10 & 162.400 \\ -2 & 0 & 1 & 0 \end{array}\right) \sim \left(\begin{array}{ccc|c} 1 & 1 & 1 & 20.400 \\ 0 & 2 & 4 & 42.400 \\ 0 & 2 & 3 & 40.400 \end{array}\right)$$

$$\sim \left(\begin{array}{ccc|c} 1 & 1 & 1 & 20.400 \\ 0 & 2 & 4 & 42.400 \\ 0 & 0 & 1 & 2.400 \end{array}\right) \qquad \left.\begin{array}{r} x+y+z=20.400 \\ 2y+4z=42.400 \\ z=2.400 \end{array}\right\}$$

$(1.200,\ 16.400,\ 2.400)$

2) a) $A-\lambda I = \begin{pmatrix} 0 & 1 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix} - \begin{pmatrix} \lambda & 0 & 0 \\ 0 & 0 & \lambda \\ 0 & 0 & \lambda \end{pmatrix} = \begin{pmatrix} -\lambda & 1 & 1 \\ 1 & 1 & -\lambda \\ 1 & 0 & -\lambda \end{pmatrix}$

$(A-\lambda I)^2 = \begin{pmatrix} \lambda^2+2 & 1-\lambda & -3\lambda \\ 1-2\lambda & 2 & \lambda^2-\lambda+1 \\ -2\lambda & 1 & \lambda^2+1 \end{pmatrix}$

$\left.\begin{array}{r} \lambda^2+2=6 \\ 1-\lambda=-3 \\ -3\lambda=-4 \\ 1-2\lambda=-3 \\ 2=2 \\ \lambda^2-\lambda+1=1 \\ -2\lambda=-4 \\ 1=1 \\ \lambda^2+1=5 \end{array}\right\}$ ¡NO HAY NINGÚN VALOR QUE CUMPLA TODAS LAS ECUACIONES!

b) $\begin{vmatrix} x & 1/4 & 4 \\ y & 0 & 4 \\ z & 1/2 & 12 \end{vmatrix} \underset{|A|=|A^t|}{=} \begin{vmatrix} x & y & z \\ 1/4 & 0 & 1/2 \\ 4 & 4 & 12 \end{vmatrix} =$

$= \frac{1}{4} \cdot 4 \begin{vmatrix} x & y & z \\ 1 & 0 & 2 \\ 1 & 1 & 3 \end{vmatrix} = \frac{1}{4} \cdot 4 \cdot 1 = \underline{\underline{1}}$

3) a) $|A| = 6-4+6-1-4+36 = 39$

$|A| \neq 0 \rightarrow$ Es invertible

B no es cuadrada $\Rightarrow$ no es invertible

b) $A \cdot X = B \Rightarrow X = A^{-1} B$

$A^t = \begin{pmatrix} 2 & 4 & 1 \\ 3 & -1 & 1 \\ -1 & 2 & -3 \end{pmatrix}$ $\quad \text{Adj}(A^t) = \begin{pmatrix} 1 & 8 & 5 \\ 14 & -5 & -8 \\ 5 & 1 & -14 \end{pmatrix}$

$A^{-1} = \frac{1}{39} \begin{pmatrix} 1 & 8 & 5 \\ 14 & -5 & -8 \\ 5 & 1 & -14 \end{pmatrix}$

$X = \frac{1}{39} \begin{pmatrix} 1 & 8 & 5 \\ 14 & -5 & -8 \\ 5 & 1 & -14 \end{pmatrix} \begin{pmatrix} -10 \\ 11 \\ -4 \end{pmatrix} = \frac{1}{39} \begin{pmatrix} 58 \\ -163 \\ 17 \end{pmatrix}$

$X = \begin{pmatrix} 58/39 \\ -163/39 \\ 17/39 \end{pmatrix}$

4) $\begin{vmatrix} a & 2 & 2 \\ 1 & 1 & -a \\ 2 & 3 & 1 \end{vmatrix} = \cancel{a} + 6 - \cancel{4a} - 4 + \cancel{3a^2} - 2 =$ ②

$= 3a^2 - 3a = \underbrace{3a(a-1) = 0}_{\substack{a=0 \\ a=1}}$

$a = 0$

$\left(\begin{array}{ccc|c} 0 & 2 & 2 & 0 \\ 1 & 1 & 0 & 0 \\ 2 & 3 & 1 & 0 \end{array}\right) \sim \left(\begin{array}{ccc|c} 1 & 1 & 0 & 0 \\ 0 & 2 & 2 & 0 \\ 2 & 3 & 1 & 0 \end{array}\right) \sim \left(\begin{array}{ccc|c} 1 & 1 & 0 & 0 \\ 0 & 2 & 2 & 0 \\ 0 & 1 & 1 & 0 \end{array}\right)$

$\left(\begin{array}{ccc|c} 1 & 1 & 0 & 0 \\ 0 & 2 & 2 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right)$ rang A = rang A/B = 2 < 3 → S.C.I

$x + y = 0 \rightarrow y = -x$

$2y + 2z = 0 \rightarrow -2x + 2z = 0 \rightarrow z = x$

$(\lambda, -\lambda, \lambda) \quad \lambda \in \mathbb{R}$

$a = 1$

$\left(\begin{array}{ccc|c} 1 & 2 & 2 & 1 \\ 1 & 1 & -1 & 1 \\ 2 & 3 & 1 & 1 \end{array}\right) \sim \left(\begin{array}{ccc|c} 1 & 2 & 2 & 1 \\ 0 & 1 & 3 & 0 \\ 0 & -1 & -3 & -1 \end{array}\right) \sim \left(\begin{array}{ccc|c} 1 & 2 & 2 & 1 \\ 0 & 1 & 3 & 0 \\ 0 & 0 & 0 & -1 \end{array}\right)$

rang A = 2 ; rang A/B = 3 → S.I

Si $a \neq 0, 1$

$\cancel{a} + 6a - \cancel{2a^2} - \cancel{2a} + 3a^2 - \cancel{2a} =$

$= a^2 + 3a$

$$X = \frac{\begin{vmatrix} a & 2 & 2 \\ a & 1 & -a \\ a & 3 & 1 \end{vmatrix}}{3a(a-1)} = \frac{\cancel{a}(a+3)}{3\cancel{a}(a-1)} = \frac{a+3}{3(a-1)}$$

[illegible]

$$y = \frac{\begin{vmatrix} a & a & 2 \\ 1 & a & -a \\ 2 & a & 1 \end{vmatrix}}{3a(a-1)} = \frac{\cancel{a}(a^2-a-3)}{3\cancel{a}(a-1)} = \frac{a^2-a-3}{3(a-1)}$$

$$a^2 + 2a - 2a^2 - 4a + a^3 - a =$$

$$= a^3 - a^2 - 3a = a(a^2-a-3)$$

$$z = \frac{\begin{vmatrix} a & 2 & a \\ 1 & 1 & a \\ 2 & 3 & a \end{vmatrix}}{3a(a-1)} = \frac{\cancel{a}(-2a+3)}{3\cancel{a}(a-1)} = \frac{-2a+3}{3(a-1)}$$

$$a^2 + 3a + 4a - 2a - 3a^2 - 2a =$$

$$-2a^2 + 3a$$

1

1) $\vec{u_r} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & 1 & 1 \\ 1 & 0 & -1 \end{vmatrix} = (-1, 2, -1)$

$\vec{u_s} = (2, 2, 1)$

$P_r(0, -4, 4)$ $\quad P_s(0, -4, 4)$

$\vec{P_rP_s}(0, 0, 0)$

Son secantes $\quad I(0, -4, 4)$

$[\vec{u_r}, \vec{u_s}, \vec{P_rP_s}] = 0$

$\cos(\hat{r, s}) = \cos(\vec{u_r}, \vec{u_s}) = \dfrac{|\vec{u_r} \cdot \vec{u_s}|}{|\vec{u_r}|\,|\vec{u_s}|} =$

$= \dfrac{|-2+4-1|}{\sqrt{6}\ \sqrt{9}} = \dfrac{1}{3\sqrt{6}} = \dfrac{\sqrt{6}}{18}$

$(\hat{r, s}) = 82'17^\circ$

2) $\vec{u_r} = \vec{n} = (1, 1, 1)$ $\quad r: \begin{cases} x = \lambda \\ y = \lambda \\ z = \lambda \end{cases}$

$\lambda + \lambda + \lambda = 3; \quad 3\lambda = 3; \quad \lambda = 1$

$I(1, 1, 1)$

$\dfrac{0+x'}{2} = 1; \quad \dfrac{0+y'}{2} = 1; \quad \dfrac{0+z'}{2} = 1$

$x' = 2; \quad y' = 2; \quad z' = 2$

$O'(2, 2, 2)$

3) a) $[\overline{u}, \vec{v}, \overline{w}] = \begin{vmatrix} 1 & -1 & 1 \\ 1 & 0 & 0 \\ 2 & -2 & 1 \end{vmatrix} = -2+1=-1 \neq 0$

Son lineal^{te} independientes

b) $(0, 2, -3) = \alpha(1, -1, 1) + \beta(1, 0, 0) + \gamma(2, -2, 1)$

$$\left.\begin{aligned} 0 &= \alpha + \beta + 2\gamma \\ 2 &= -\alpha \quad\; - 2\gamma \\ -3 &= \alpha \quad\;\; + \gamma \end{aligned}\right\} \qquad \begin{aligned} \alpha &= -4 \\ \beta &= 2 \\ \gamma &= +1 \end{aligned}$$

4) a) $\overline{u_r} = \overline{AB} = (-1, 1, -1)$

$\overline{u_s} = \overline{AC} = (4, -1, -5)$

$$\overline{n} = \overline{u_r} \wedge \overline{u_s} = \begin{vmatrix} \overline{i} & \overline{j} & \vec{k} \\ -1 & 1 & -1 \\ 4 & -1 & -5 \end{vmatrix} = (-6, -9, -3) \sim (2, 3, 1)$$

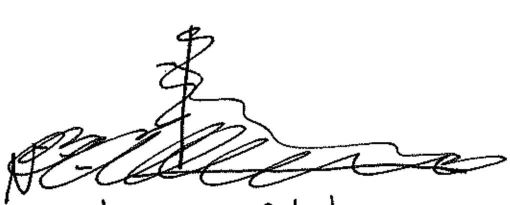
$2x + 3y + z + D = 0$; $\quad 8+3+1+D=0$; $D=-12$

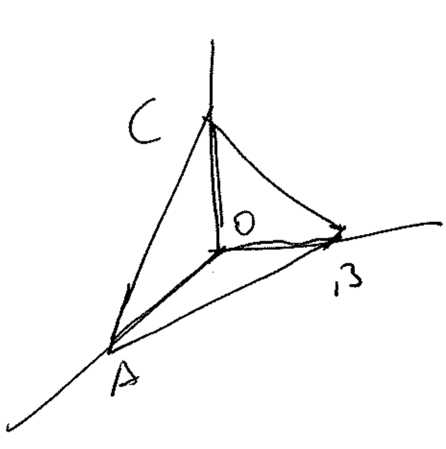
$$\boxed{\pi:\ 2x+3y+z-12=0}$$

b) $x=y=0 \Rightarrow z=12 \rightarrow C(0,0,12)$

$x=z=0 \Rightarrow y=4 \rightarrow B(0,4,0)$

$y=z=0 \Rightarrow x=6 \rightarrow A(6,0,0)$





$$V = \frac{\left|\begin{vmatrix} 0 & 0 & 12 \\ 0 & 4 & 0 \\ 6 & 0 & 0 \end{vmatrix}\right|}{6} = \underline{\underline{48\, u^3}}$$

1) a)

$$f(x) = \begin{cases} \dfrac{a}{x-1} & \text{si } x < 2 \\ ax^2+bx+1 & \text{si } x \geq 2 \end{cases}$$

$$f'(x) = \begin{cases} \dfrac{-a}{(x-1)^2} & \text{si } x < 2 \\ 2ax+b & \text{si } x > 2 \end{cases}$$

$$\lim_{x\to 2^-} f(x) = \frac{a}{2-1} = a$$

$$\lim_{x\to 2^+} f(x) = 4a+2b+1$$

$$f(2) = 4a+2b+1$$

$$a = 4a+2b+1$$

$$\boxed{3a+2b=-1}$$

$$\lim_{x\to 2^-} f'(x) = \frac{-a}{(2-1)^2} = -a$$

$$\lim_{x\to 2^+} f'(x) = 4a+b$$

$$-a = 4a+b$$

$$\boxed{5a+b=0}$$

$$\begin{cases} 3a+2b=-1 \\ 5a+b=0 \end{cases}$$

$a = +1/7$

$b = -5/7$

b)

$$f(x) = \begin{cases} \dfrac{1/7}{x-1} & \text{si } x < 2 \\ \dfrac{1}{7}x^2 - \dfrac{5x}{7} + 1 & \text{si } x \geq 2 \end{cases}$$

$$f'(x) = \begin{cases} \dfrac{-1/7}{(x-1)^2} & \text{si } x < 2 \\ \dfrac{2}{7}x - \dfrac{5}{7} & \text{si } x > 2 \end{cases}$$

FUNCIONES

$x = 1 \to$ D.I.S.I

(no derivable)

$\lim_{x \to 1^-} f(x) = -\infty$

$\lim_{x \to 1^+} f(x) = +\infty$

$f(1) = \nexists$

CONEXIONES

$x = 2 \to$ CONTINUA Y DERIVABLE

$\lim_{x \to 2^-} f(x) = \frac{1}{7}$

$\lim_{x \to 2^+} f(x) = \frac{4}{3} - 1 = \frac{1}{7}$

$f(2) = \frac{1}{7}$

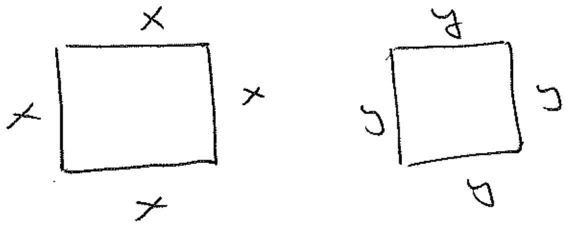
$\lim_{x \to 2^-} f'(x) = -1/7$

$\lim_{x \to 2^+} f'(x) = \frac{4}{7} - \frac{5}{7} = -\frac{1}{7}$

c) $f'(0) = \frac{-1/7}{(0-1)^2} = -\frac{1}{7}$; $m_t = -\frac{1}{7}$

$P\left(0, -\frac{1}{7}\right)$; $\boxed{y = -\frac{1}{7}x - \frac{1}{7}}$

a)



$$4x + 4y = 100$$
$$x + y = 25$$
$$y = 25 - x$$

$$c(x,y) = 2x^2 + 3y^2$$

$$c(x) = 2x^2 + 3(25-x)^2 = 2x^2 + 3(625 + x^2 - 50x) =$$
$$= 2x^2 + 1875 + 3x^2 - 150x = 5x^2 - 150x + 1875$$

$$c'(x) = 10x - 150 = 0 \; ; \quad x = \frac{150}{10} = 15 \; //$$
$$y = 25 - 15 = 10 \; //$$

$$c''(x) = 10 > 0 \rightarrow \text{mínimo}$$

b)
$$\lim_{x\to 0} \frac{1-\cos x}{(e^x-1)^2} \overset{\text{L'HÔP}}{=} \lim_{x\to 0} \frac{\operatorname{sen} x}{2(e^x-1)e^x} =$$

$$\overset{\text{L'HôP}}{=} \lim_{x\to 0} \frac{\cos x}{2[e^x \cdot e^x + (e^x-1)\cdot e^x]} =$$

$$= \lim_{x\to 0} \frac{\cos x}{2[2e^{2x} - e^x]} = \frac{1}{2[2-1]} = \frac{1}{2} \; //$$

3) a) $m = 1$; $m = \lim_{x \to \infty} \frac{x^2 + ax}{x^2 + \cancel{x}} = 1$

$$n = \lim_{x \to \infty} \left(\frac{x^2 + ax}{x + 1} - x \right) = \lim_{x \to \infty} \left(\frac{x^2 + ax - x^2 - \cancel{x}}{x + 1} \right) =$$

$$= \lim_{x \to \infty} \frac{\overset{(a-1) \cdot x}{\cancel{ax - x}}}{\cancel{x} + 1} = a - 1 \Rightarrow \boxed{a = 2}$$

$a - 1 = 1 \rightarrow$

b) $f(x) = \frac{x^2 + 2x}{x + 1}$

i) $D = \mathbb{R} \setminus \{-1\}$

ii) Corte ejes: OX : $(0, 0)$ y $(-2, 0)$; OY : $(0, 0)$

iii) Asíntotas

H: $\lim_{x \to \pm\infty} f(x) = +\infty$ / $-\infty$ no tiene

V: $x + 1 = 0$; $\boxed{x = -1}$

$\lim_{x \to -1^-} f(x) = +\infty$

$\lim_{x \to -1^+} f(x) = -\infty$

O: $m = \lim_{x \to \infty} \frac{x^2 + 2x}{x^2 + x} = 1$

$$n = \lim_{x \to \infty} \left(\frac{x^2 + 2x}{x + 1} - x \right) = \lim_{x \to \infty} \left(\frac{x^2 + 2x - x^2 - x}{x + 1} \right)$$

$$= \lim_{x \to \infty} \frac{x}{x + 1} = 1 ; \quad \boxed{y = x + 1}$$

iv) Extremos relativos.

$$f'(x) = \frac{(2x+2)(x+1) - 1(x^2+2x)}{(x+1)^2} =$$

$$= \frac{2x^2+2x+2x+2-x^2-2x}{(x+1)^2} = \frac{x^2+2x+2}{(x+1)^2}$$

$$x^2+2x+2=0; \quad x = \frac{-2 \pm \sqrt{4-8}}{2} \rightarrow \nexists \text{ sol}$$

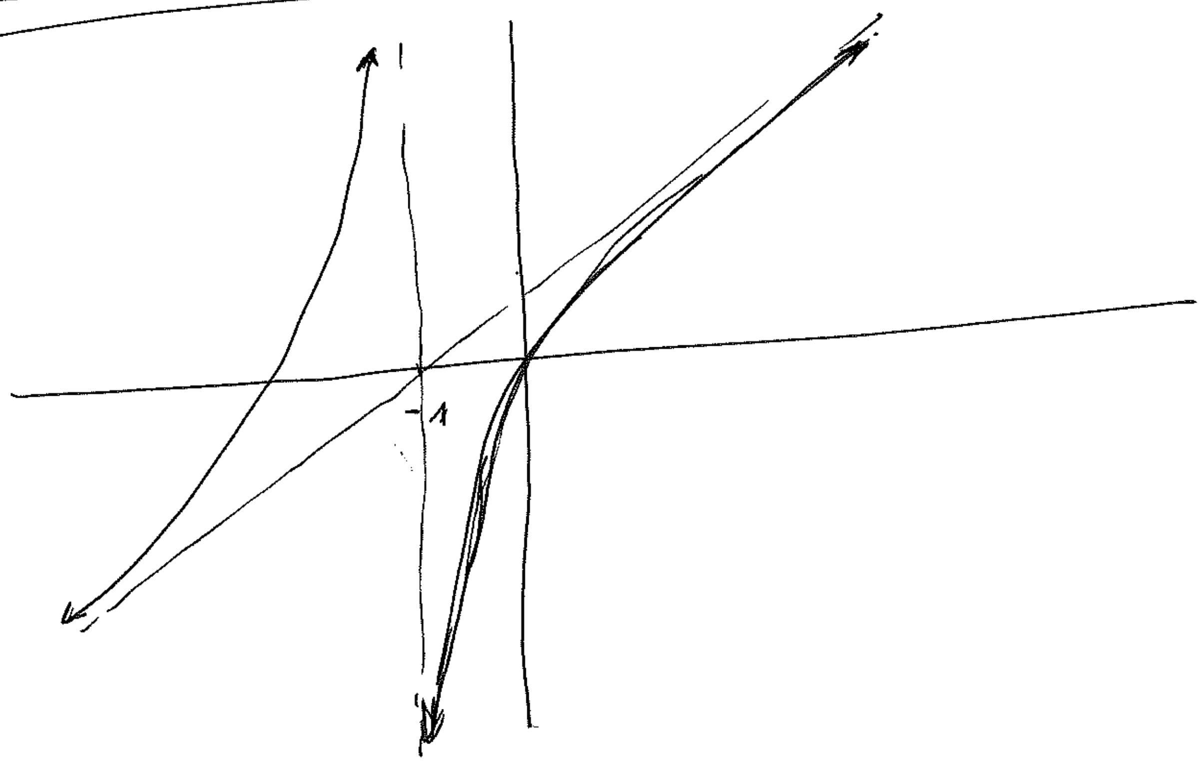
$$\begin{array}{ccc} f'>0 & & f'>0 \\ \hline \text{Creciente} & -1 & \text{Creciente} \end{array}$$

$$f''(x) = \frac{(2x+2)(x+1)^2 - 2(x+1)(x^2+2x+2)}{(x+1)^4} =$$

$$= \frac{(x+1)\left[(2x+2)(x+1) - 2(x^2+2x+2)\right]}{(x+1)^4} =$$

$$= \frac{\cancel{2x^2}+\cancel{2x}+\cancel{2x}+2-\cancel{2x^2}-\cancel{4x}-4}{(x+1)^3} = \frac{-2}{(x+1)^3} = 0$$

No tiene inflexiones



4) a) $I = \int x^2 \operatorname{sen} 2x \, dx$

$u = x^2 \quad v = \frac{-\cos 2x}{2}$

$du = 2x\,dx \quad dv = \operatorname{sen} 2x \, dx$

$$I = \frac{-x^2 \cos 2x}{2} - \int \frac{-\cos 2x}{2} \cdot 2x \, dx =$$

$$= \frac{-x^2 \cos 2x}{2} + \underbrace{\int x \cos 2x \, dx}_{I_1}$$

$u = x \quad v = \frac{\operatorname{sen} 2x}{2}$

$du = dx \quad dv = \cos 2x \, dx$

$$I_1 = \frac{x \operatorname{sen} 2x}{2} - \int \frac{\operatorname{sen} 2x}{2} \, dx =$$

$$= \frac{x \operatorname{sen} 2x}{2} - \frac{1}{2} \cdot \frac{-\cos 2x}{2} = \frac{x \operatorname{sen} 2x}{2} + \frac{\cos 2x}{4}$$

$$\boxed{I = \frac{-x^2 \cos 2x}{2} + \frac{x \operatorname{sen} 2x}{2} + \frac{\cos 2x}{4}}$$

b) $\ln x = t$

$\frac{dx}{x} = dt$

$$\int \frac{2t+1}{(1+t)(2-t)} \, dt$$

$$\frac{2t+1}{(1+t)(2-t)} = \frac{A}{(1+t)} + \frac{B}{(2-t)} = \frac{A(2-t) + B(1+t)}{(1+t)(2-t)}$$

$t = 2 \Rightarrow 5 = 3B \Rightarrow B = 5/3$

$t = -1 \Rightarrow -1 = 3A \Rightarrow A = -1/3$

$$\int \frac{-1/3}{1+t}\,dt + \int \frac{5/3}{2-t}\,dt =$$

$$= -\frac{1}{3}\ln(1+t) - 5/3 \ln(2-t) =$$

$$= \underline{\underline{-\frac{1}{3}\ln(1+\ln x) - \frac{5}{3}\ln(2-\ln x)}}$$

c) $D = x^2 - (2 - x^4) = x^2 - 2 + x^4 =$

$= x^4 + x^2 - 2$

$x^4 + x^2 - 2 = 0$; $\underline{\underline{x^2 = t}}$ $\quad t^2 + t - 2 = 0$

$$t = \frac{-1 \pm \sqrt{1+8}}{2} = \frac{-1 \pm 3}{2} = \begin{cases} 1 \\ -2 \end{cases}$$

$x = \pm\sqrt{1} \begin{cases} -1 \\ 1 \end{cases}$ $\qquad x = \pm\sqrt{-2}$ ∄ sol

D(x) > 0 | D(x) < 0 | D(x) > 0

−2 −1 1 2

$$A = \int_{-2}^{-1} (x^4 + x^2 - 2)\,dx - \int_{-1}^{1} (x^4 + x^2 - 2)\,dx + \int_{1}^{2} (x^4 + x^2 - 2)\,dx$$

$$= \left[\frac{x^5}{5} + \frac{x^3}{3} - 2x\right]_{-2}^{-1} - \left[\frac{x^5}{5} + \frac{x^3}{3} - 2x\right]_{-1}^{1} + \left[\frac{x^5}{5} + \frac{x^3}{3} - 2x\right]_{1}^{2} =$$

$$= \left[\left(-\frac{1}{5} - \frac{1}{3} + 2\right) - \left(-\frac{32}{5} - \frac{8}{3} + 4\right)\right] - \left[\left(\frac{1}{5} + \frac{1}{3} - 2\right) - \left(-\frac{1}{5} - \frac{1}{3} + 2\right)\right]$$

$$+ \left[\left(\frac{32}{5} + \frac{8}{3} - 4\right) - \left(\frac{1}{5} + \frac{1}{3} - 2\right)\right] =$$

$$= \left[-\frac{1}{5} - \frac{1}{3} + 2 + \frac{32}{5} + \frac{8}{3} - 4\right] - \left[\frac{1}{5} + \frac{1}{3} - 2 + \frac{1}{5} + \frac{1}{3} - 2\right] +$$

$$+ \left[\frac{32}{5} + \frac{8}{3} - 4 - \frac{1}{5} - \frac{1}{3} + 2\right] =$$

$$= \left[\frac{31}{5} + \frac{7}{3} - 2\right] - \left[\frac{2}{5} + \frac{2}{3} - 4\right] + \left[\frac{31}{5} + \frac{7}{3} - 2\right] =$$

$$= \left[\frac{93 + 35 - 30}{15}\right] - \left[\frac{6 + 10 - 60}{15}\right] + \left[\frac{93 + 35 - 30}{15}\right] =$$

$$= \frac{98}{15} + \frac{44}{15} + \frac{98}{15} = \frac{240}{15} = \frac{48}{3} = \underline{\underline{16\ u^2}}$$