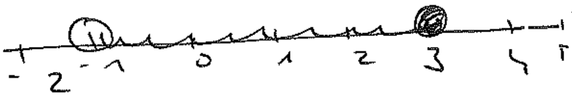
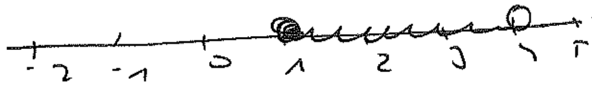
1) a) TEÓRICO

b) TEÓRICO

c) $(-1,3] = \{x \in \mathbb{R} \mid -1 < x \leq 3\}$



$[1,4) = \{x \in \mathbb{R} \mid 1 \leq x < 4\}$



$$(-1,3] \cup [1,4) = (-1,4) = E\left(\frac{3}{2}, \frac{5}{2}\right)$$

2) a) $$3\sqrt{3} - 24\sqrt{3} - \frac{8\sqrt{3}}{3} + 5\sqrt{3} =$$

$$= \frac{9\sqrt{3} - 72\sqrt{3} - 8\sqrt{3} + 15\sqrt{3}}{3} = \frac{-56\sqrt{3}}{3}$$

b) $$\frac{3\sqrt{2}}{\sqrt{3}-1} = \frac{3\sqrt{2}(\sqrt{3}+1)}{(\sqrt{3}-1)(\sqrt{3}+1)} = \frac{3\sqrt{6}+3\sqrt{2}}{2}$$

$$\frac{\sqrt{12}}{\sqrt{50}-3\sqrt{2}} = \frac{2\sqrt{3}}{5\sqrt{2}-3\sqrt{2}} = \frac{2\sqrt{3}}{2\sqrt{2}} = \frac{\sqrt{3}}{\sqrt{2}} = \frac{\sqrt{6}}{2}$$

$$\frac{3\sqrt{6}+3\sqrt{2}}{2} - \frac{\sqrt{6}}{2} = \frac{2\sqrt{6}+3\sqrt{2}}{2}$$

3) a)

$$\begin{array}{r|ccccc} & 2 & -m & 0 & -2 & m \\ -\frac{1}{2} & & -1 & \frac{m+1}{2} & \frac{-m-1}{4} & \frac{m+9}{8} \\ \hline & 2 & -m-1 & \frac{m+1}{2} & \frac{-m-9}{4} & m + \frac{m+9}{8} \end{array}$$

$m + \frac{m+9}{8} = 3$; $\quad 8m + m + 9 = 24$

$9m + 9 = 24$ $\quad 9m = 24 - 9 = 15$

$$m = \frac{15}{9} = \frac{5}{3} \;//$$

b)

$$
\begin{array}{r|rrrrr}
 & 3 & -7 & 5 & -7 & 2 \\
\frac{1}{3} & & 1 & -2 & 1 & -2 \\
\hline
 & 3 & -6 & 3 & -6 & \lfloor 0 \\
2 & & 6 & 0 & 0 & \\
\hline
 & 3 & 0 & 3 & \lfloor & \\
\end{array}
$$

$3x^2+3=0$ / **Raíces**: $\frac{1}{3}$ y 2

4) $p(x) = x\,(4x^5+7x^4-34x^3-64x^2-18x+9)$

$$
\begin{array}{r|rrrrrr}
 & 4 & 7 & -34 & -64 & -18 & 9 \\
-1 & & -4 & -3 & 37 & 27 & -9 \\
\hline
 & 4 & 3 & -37 & -27 & 9 & \lfloor 0 \\
-1 & & -4 & 4 & 36 & -9 & \\
\hline
 & 4 & -1 & -36 & 9 & \lfloor 0 & \\
3 & & 12 & 33 & -9 & & \\
\hline
 & 4 & 11 & -3 & \lfloor 0 & & \\
\end{array}
$$

$4x^2+11x-3=0$ / $x = \frac{-11 \pm \sqrt{121+48}}{8} =$

$= \frac{-11 \pm 13}{8} = \begin{cases} \frac{2}{8} = \frac{1}{4} \\ -3 \end{cases}$

$p(x) = 4\,x\,(x+1)^2(x-3)(x+3)(x-1/4)$

5) $x^4 - 4x^2 = x^2(x^2-4) = x^2(x+2)(x-2)$

	1	2	3	6
-2		-2	0	-6
	1	0	3	0

$x^3 + 2x^2 + 3x + 6 = (x+2)(x^2+3)$

$2x^4 + 2x^3 - 4x^2 = 2x^2(x^2+x-2) =$

$= 2x^2(x+2)(x-1)$

$x^3 - 4x^2 + 4x = x(x^2-4x+4) = x(x-2)^2$

$$\frac{x^2(x+2)(x-2)}{(x+2)(x^2+3)} : \frac{2x^2(x+2)(x-1)}{x(x-2)^2} =$$

$$= \frac{x^{\cancel{3}}\cancel{(x+2)}(x-2)^3}{2\cancel{x^2}(x+2)^{\cancel{2}}(x^2+3)(x-1)} = \frac{x(x-2)^3}{2(x+2)(x^2+3)(x-1)}$$

①

1) a) $\frac{2\sqrt{5}-1}{4\sqrt{2}-\sqrt{5}} + \frac{\sqrt{2}}{3\sqrt{5}-\sqrt{5}} = \frac{2\sqrt{5}-1}{4\sqrt{2}-\sqrt{5}} + \frac{\sqrt{2}}{2\sqrt{5}}$

$\frac{2\sqrt{5}-1}{4\sqrt{2}-\sqrt{5}} = \frac{(2\sqrt{5}-1)(4\sqrt{2}+\sqrt{5})}{(4\sqrt{2}-\sqrt{5})(4\sqrt{2}+\sqrt{5})} = \frac{8\sqrt{10}+10-4\sqrt{2}-\sqrt{5}}{32-5} =$

$\frac{8\sqrt{10}+10-4\sqrt{2}-\sqrt{5}}{27}$

$\frac{\sqrt{2}}{2\sqrt{5}} = \frac{\sqrt{2}\sqrt{5}}{2\sqrt{5}\sqrt{5}} = \frac{\sqrt{10}}{10}$

$\frac{8\sqrt{10}+10-4\sqrt{2}-\sqrt{5}}{27} + \frac{\sqrt{10}}{10} = \frac{80\sqrt{10}+100-40\sqrt{2}-10\sqrt{5}}{270} + \frac{27\sqrt{10}}{270} =$

$= \boxed{\frac{107\sqrt{10}+100-40\sqrt{2}-10\sqrt{5}}{270}}$

b) TEÓRICO

2) a)

	3	-7	$-m$	21	-6
1/3		1	-2	$-\frac{m}{3}-\frac{2}{3}$	$-\frac{m}{9}+\frac{61}{9}$
	3	-6	$-m-2$	$-\frac{m}{3}+\frac{61}{3}$	$-\frac{m}{9}+\frac{7}{9}$

$-\frac{m}{9}+\frac{7}{9}=0$; $-m+7=0 \rightarrow \underline{\underline{m=7}}$

b)

	3	-7	-7	21	-6
1/3		1	-2	-3	6
	3	-6	-9	18	0
2		6	0	-18	
	3	0	-9	0	

$3x^2-9=0$; $3x^2=9$; $x^2=\frac{9}{3}$; $x^2=3$ $\quad x=\pm\sqrt{3}$

c) TEÓRICO

3) a) $\left.\begin{array}{l} x^2+3y=1 \\ 3x-2y=8 \end{array}\right\} \longrightarrow y=\dfrac{1-x^2}{3}$

$3x-2\left(\dfrac{1-x^2}{3}\right)=8$; $3x-\dfrac{2-2x^2}{3}=8$

$9x-2+2x^2=24$; $2x^2+9x-26=0$

$x=\dfrac{-9\pm\sqrt{81+208}}{4}=\dfrac{-9\pm 17}{4}=\begin{cases}\frac{8}{4}=2\\ -\frac{13}{2}\end{cases}$

$x_1=2 \Rightarrow y_1=\dfrac{1-4}{3}=\dfrac{-3}{3}=-1 \longrightarrow (2,-1)$

$x_2=-\dfrac{13}{2} \Rightarrow y_2=\dfrac{1-\frac{169}{4}}{3}=\dfrac{-165}{12} \to \left(-\dfrac{13}{2},\dfrac{-55}{4}\right)$

$\downarrow \dfrac{-55}{4}$

b) $\sqrt{2x+3}=2x-3$

$2x+3=4x^2+9-12x$

$0=4x^2-14x+6$;

$x=\dfrac{14\pm\sqrt{196-96}}{8}=\dfrac{14\pm 10}{8}=\begin{cases}3 \to \text{sí}\\ \frac{1}{2} \to \underline{\text{no}}\end{cases}$

$3 \to \sqrt{6+3}=6-3$; $3=3 \to$

$\dfrac{1}{2} \to \sqrt{4}=1-3$; $2=1-3 \to$

4) Julia → x

Clara → y

Miguel → z

$$\left.\begin{array}{l} y = \dfrac{20(x+y+z)}{100} \\ z = x+100 \\ x+y = 850 \end{array}\right\}$$

$$\begin{array}{l} -20x + 80y - 20z = 0 \\ -x \qquad\quad + z = 100 \\ x + y \qquad\quad = 850 \end{array} \left\{ \begin{array}{l} -x + 4y - z = 0 \\ -x \qquad + z = 100 \\ x + y \qquad = 850 \end{array} \right\}$$

$$\left(\begin{array}{ccc|c} -1 & 4 & -1 & 0 \\ -1 & 0 & 1 & 100 \\ 1 & 1 & 0 & 850 \end{array}\right) \sim \left(\begin{array}{ccc|c} -1 & 4 & -1 & 0 \\ 0 & 4 & -2 & -100 \\ 0 & 5 & -1 & 850 \end{array}\right)$$

$$\cancel{\left(\begin{array}{ccc|c} -1 & 4 & -1 & 0 \\ 0 & 5 & -1 & 850 \\ 0 & 0 & -2 & -100 \end{array}\right)}$$

$\cancel{-2z = -100}$; $\cancel{z = 50}$

$\cancel{5y - 50 = 850}$; $\cancel{5y = 900}$; $\cancel{y = \frac{900}{5} = 180}$

$\cancel{-x + 720 - 50 = 0}$; $\cancel{x = 670}$

$\cancel{(670, 180, 50)}$

$$\left(\begin{array}{ccc|c} -1 & 4 & -1 & 0 \\ 0 & 4 & -2 & -100 \\ 0 & 0 & 6 & 3900 \end{array}\right)$$

$$\boxed{(550, 300, 650)}$$

$6z = 3900$

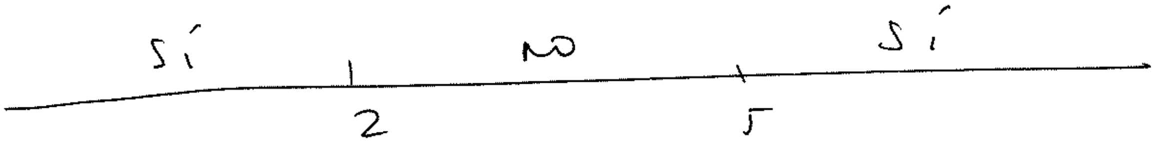
$z = \frac{3900}{6} = 650$; $4y - 1300 = -100$;

$4y = 1200$; $y = \frac{1200}{4} = 300$; $-x + 1200 - 650 = 0$

$x = 550$

5) $\frac{3x-9}{x-2} - 2 > 0$; $\frac{3x-9-2x+4}{x-2} > 0$

$\frac{x-5}{x-2} > 0$; $x-5=0 \rightarrow x=5$ // $x-2=0 \rightarrow x=2$ //

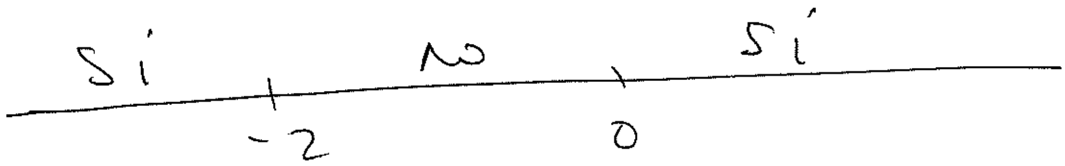


$S_1: (-\infty, 2) \cup (5, +\infty)$

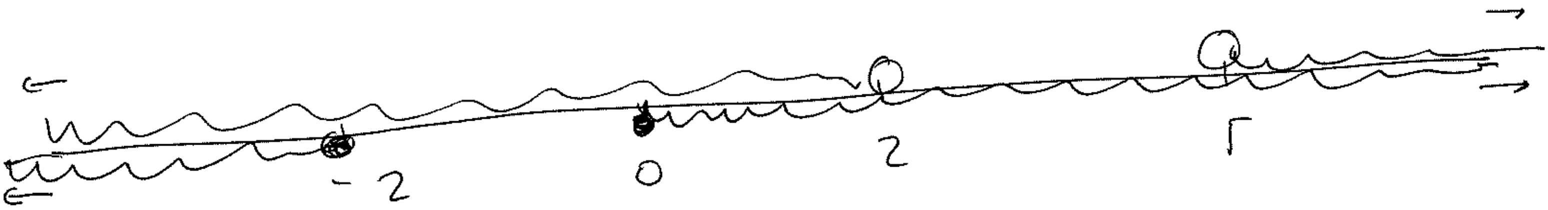
$x^2+9+6x-4x-5-4 \geq 0$

$x^2+2x \geq 0$; $x^2+2x=0$

$x(x+2)=0 \begin{cases} x=0 \\ x+2=0 \rightarrow x=-2 \end{cases}$



$S_2: (-\infty, -2] \cup [0, +\infty)$



$S_{\text{sistema}} = (-\infty, -2] \cup [0, 2) \cup (5, +\infty)$

1)

a) $\dfrac{5\sqrt{3}-\sqrt{2}}{3\sqrt{8}-\sqrt{3}} = \dfrac{5\sqrt{3}-\sqrt{2}}{6\sqrt{2}-\sqrt{3}} = \dfrac{(5\sqrt{3}-\sqrt{2})(6\sqrt{2}+\sqrt{3})}{(6\sqrt{2}-\sqrt{3})(6\sqrt{2}+\sqrt{3})} =$

$= \dfrac{30\sqrt{6}+15-12-\sqrt{6}}{72-3} = \dfrac{3+29\sqrt{6}}{69}$

$\dfrac{\sqrt{2}}{\sqrt{27}-\sqrt{3}} = \dfrac{\sqrt{2}}{3\sqrt{3}-\sqrt{3}} = \dfrac{\sqrt{2}}{2\sqrt{3}} = \dfrac{\sqrt{2}\sqrt{3}}{2\sqrt{3}\sqrt{3}} = \dfrac{\sqrt{6}}{6}$

$\dfrac{3+29\sqrt{6}}{69} - \dfrac{\sqrt{6}}{6} = \dfrac{6+58\sqrt{6}}{138} - \dfrac{23\sqrt{6}}{138} =$

$= \boxed{\dfrac{6+35\sqrt{6}}{138}}$

b) TEÓRICO

2) a)

$$
\begin{array}{r|rrrrrr}
 & 3 & -1 & -21 & 7 & 36 & -12 \\
2 & & 6 & 10 & -22 & -30 & +12 \\
\hline
 & 3 & 5 & -11 & -15 & 6 & \lfloor 0 \\
-2 & & -6 & 2 & 18 & -6 & \\
\hline
 & 3 & -1 & -9 & 3 & \lfloor 0 & \\
1/3 & & 1 & 0 & -3 & & \\
\hline
 & 3 & 0 & -9 & \lfloor 0 & & \\
\end{array}
$$

$3x^2-9=0$; $3x^2=9$; $x^2=\frac{9}{3}=3$, $x=\pm\sqrt{3}$

$\boxed{P(x) = 3(x-2)(x+2)(x-1/3)(x-\sqrt{3})(x+\sqrt{3})}$

b) $q(-1) = (-1)^{80} - 6 \cdot (-1)^{40} - 4(-1) + 1 =$

$= 1 - 6 + 4 + \cancel{1} = 0$ //

Se utiliza el Tma del resto

3) a) $x^4 + 9 - 6x^2 - 4x^2 + 4 = 2x^4 - 43$

$0 = x^4 + 10x^2 - 56$, $x^2 = t$

$0 = t^2 + 10t - 56$, $t = \frac{-10 \pm \sqrt{100 + 224}}{2} =$

$= \frac{-10 \pm 18}{2} = \begin{cases} \frac{8}{2} = 4 \\ \frac{-28}{2} = -14 \end{cases}$

$x = \pm\sqrt{4} \begin{cases} \boxed{2} \\ \boxed{-2} \end{cases}$ $x = \pm\sqrt{-14} \rightarrow \nexists$

b) $\frac{(x+1)(x+1)}{x^2-1} - \frac{3x-7}{x^2-1} = \frac{7(x-1)}{x^2-1}$

$x^2 + 1 + 2x - 3x + 7 = 7x - 7$

$x^2 - 8x + 15 = 0$, $x = \frac{8 \pm \sqrt{64 - 60}}{2} =$

$= \frac{8 \pm 2}{2} = \begin{cases} \frac{10}{2} = 5 \text{ //} \\ \frac{6}{2} = 3 \text{ //} \end{cases}$

4) Películas (45 €) → x
Discos (72 €) → y
Libros (36 €) → z

$$\left.\begin{aligned} 45x+72y+36z &= 6300 \\ x &= \frac{x+y+z}{2} \\ 72y &= 45x+180 \end{aligned}\right\}$$

$$\left.\begin{aligned} 45x+72y+36z &= 6300 \\ x-y-z &= 0 \\ -45x+72y \quad &= 180 \end{aligned}\right\} \left(\begin{array}{ccc|c} 45 & 72 & 36 & 6300 \\ 1 & -1 & -1 & 0 \\ -45 & 72 & 0 & 180 \end{array}\right)$$

$$\left(\begin{array}{ccc|c} 45 & 72 & 36 & 6300 \\ 0 & 117 & 81 & 6300 \\ 0 & 144 & 36 & 6480 \end{array}\right) \quad \left(\begin{array}{ccc|c} 45 & 72 & 36 & 6300 \\ 0 & 117 & 81 & 6300 \\ 0 & 0 & -7452 & -149040 \end{array}\right)$$

$$\begin{aligned} 45x+72y+36z &= 6300 \\ 117y+81z &= 6300 \\ -7452z &= -149040 \end{aligned}$$

$\to x = 60$

$\to 117y + 1620 = 6300 \to y = \underline{\underline{40}}$

$\to z = \frac{-149040}{-7452} = \underline{\underline{20}}$

$$\underline{\underline{(60, 40, 20)}}$$

5) $\frac{x-1}{2} - \frac{3x-6}{4} \geq x$; $\frac{2x-2}{4} - \frac{3x-6}{4} \geq \frac{4x}{4}$

$2x-2-3x+6 \geq 4x$; $2x-3x-4x \geq 2-6$

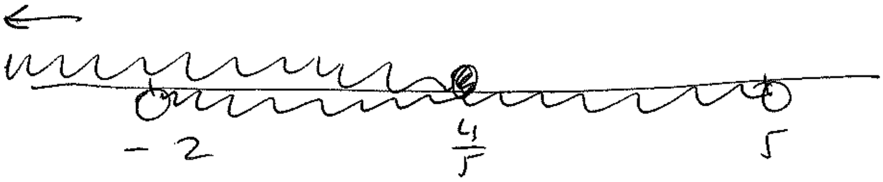
$-5x \geq -4$; $x \leq \frac{-4}{-5}$; $x \leq \frac{4}{5} \to \underline{\underline{S_1: (-\infty, \frac{4}{5}]}}$

$x^2+1-2x-\cancel{x}-3-8 < 0$; $x^2-3x-10<0$

$x^2-3x-10=0$; $x = \frac{3 \pm \sqrt{9+40}}{2} = \frac{3 \pm 7}{2} = \begin{cases} 5 \\ -2 \end{cases}$

NO	SÍ	NO
	-2	5

$\underline{\underline{S_2: (-2, 5)}}$



$\underline{\underline{\text{Solución}: (-2, \frac{4}{5}]}}$

1)

a) $x^2-4x=0 \begin{cases} x=0 \\ x=4 \end{cases}$ $\quad D_f = \mathbb{R} - \{0,4\}$

$x^2-1=0 \begin{cases} x=-1 \\ x=1 \end{cases}$ $\quad D_g = \mathbb{R} - \{-1,1\}$

$\dfrac{x+3}{x-2} \geq 0$

sí   no   sí

$-3 \qquad 2$

$D_h = (-\infty,-3] \cup (2,+\infty)$

b) $\dfrac{2x-4}{x^2-4x} = 2$; $\quad 2x-4 = 2x^2-8x$; $\quad 0 = 2x^2-10x+4$

$0 = x^2-5x+2$; $\quad x = \dfrac{5 \pm \sqrt{25-8}}{2}$ → SÍ PERTENECE

$\dfrac{2x^2+3}{x^2-1} = 2$; $\quad 2x^2+3 = 2x^2-2$; $\quad 0 = 5$ → NO PERTENECE

c) $2x-4=0 \rightarrow x=2$

$x^2-4x=0 \begin{cases} x=0 \\ x=4 \end{cases}$

$- \quad + \quad - \quad +$

$0 \quad 2 \quad 4$

$2x^2+3=0 \rightarrow \nexists$ sol

$x^2-1=0 \begin{cases} x=-1 \\ x=1 \end{cases}$

$+ \quad - \quad +$

$-1 \quad 1$

d) $f(-x) = \dfrac{2(-x)-4}{(-x)^2-4(-x)} = \dfrac{-2x-4}{x^2+4x}$ → NO TIENE

$g(-x) = \dfrac{2(-x)^2+3}{(-x)^2-1} = \dfrac{2x^2+3}{x^2-1} = g(x)$ → PAR respecto al eje OY

e) $\sqrt{\dfrac{x+3}{x-2}} = 2$; $\quad \dfrac{x+3}{x-2} = 4$; $\quad x+3 = 4x-8$;

$x-4x = -8-3$; $\quad -3x = -11$; $\quad x = \dfrac{11}{3}$

$I\left(\dfrac{11}{3}, 2\right)$

2) $f(x) = \begin{cases} x^2+4x & \text{si } x < -1 \\ -x^2+2x & \text{si } -1 \leq x < 2 \\ 2x-8 & \text{si } x \geq 2 \end{cases}$

x	-2	-1
y	-4	-3

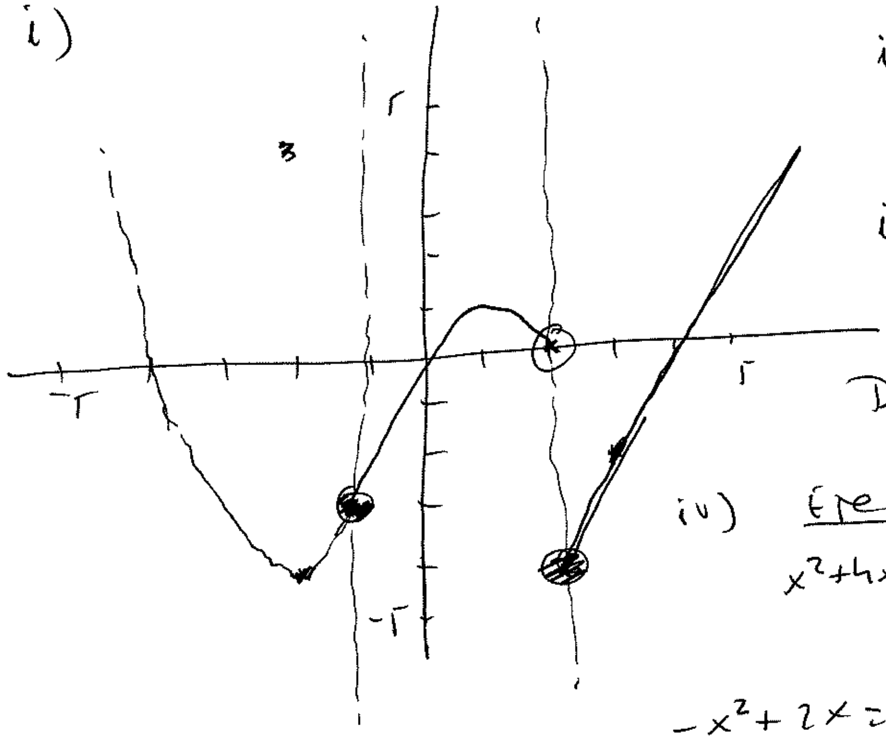
V(-2,-4) mín

x	-1	2
y	-3	0

V(1,1) máx

x	2	3
y	-4	-2

i)



ii) Discontinua en $x = 2$

iii)

Creciente: $(-2,1) \cup (2,+\infty)$

Decreciente: $(-\infty,-2) \cup (1,2)$

iv) Eje OX

$x^2+4x=0$ → $x=0$ → $\cancel{(0,0)}$; $x=-4$ → $\boxed{(-4,0)}$

$-x^2+2x=0$ → $x=0$ → $\boxed{(0,0)}$; $x=2$ → $\cancel{(2,0)}$

$2x-8=0 \rightarrow x=4 \rightarrow \boxed{(4,0)}$

v) Eje OY: $y(0) = -0^2+2\cdot 0 = 0 \rightarrow \boxed{(0,0)}$

3) a) $x^2-2x-3=0$; $x = \frac{2 \pm \sqrt{4+12}}{2} = \frac{2 \pm 4}{2} = \begin{cases} 3 \\ -1 \end{cases}$

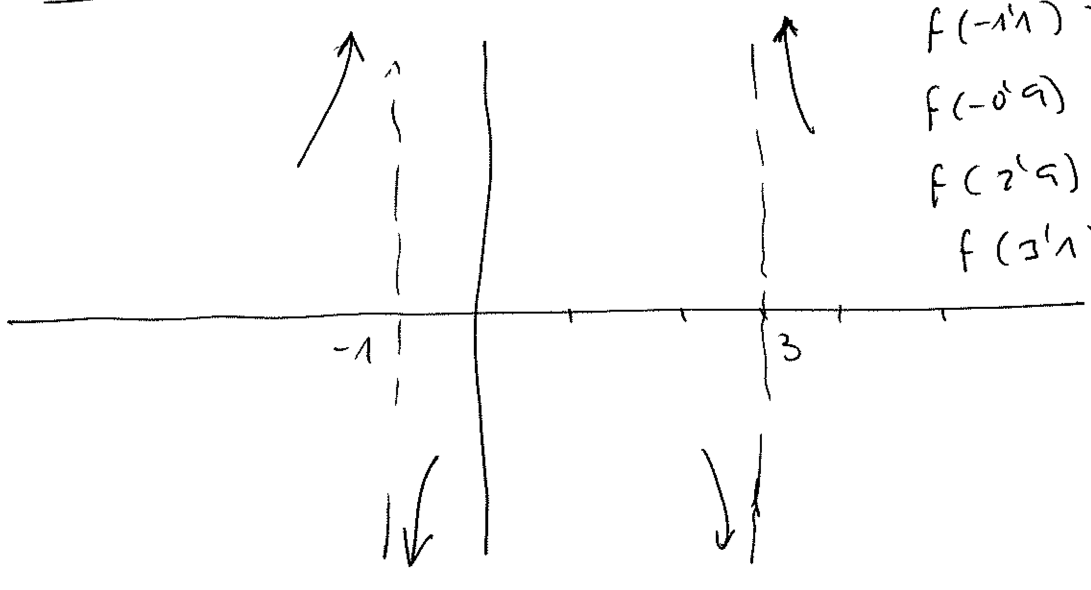
$\boxed{x=-1}$ A.V $\boxed{x=3}$ A.V

$f(-1'1) > 0$

$f(-0'9) < 0$

$f(2'9) < 0$

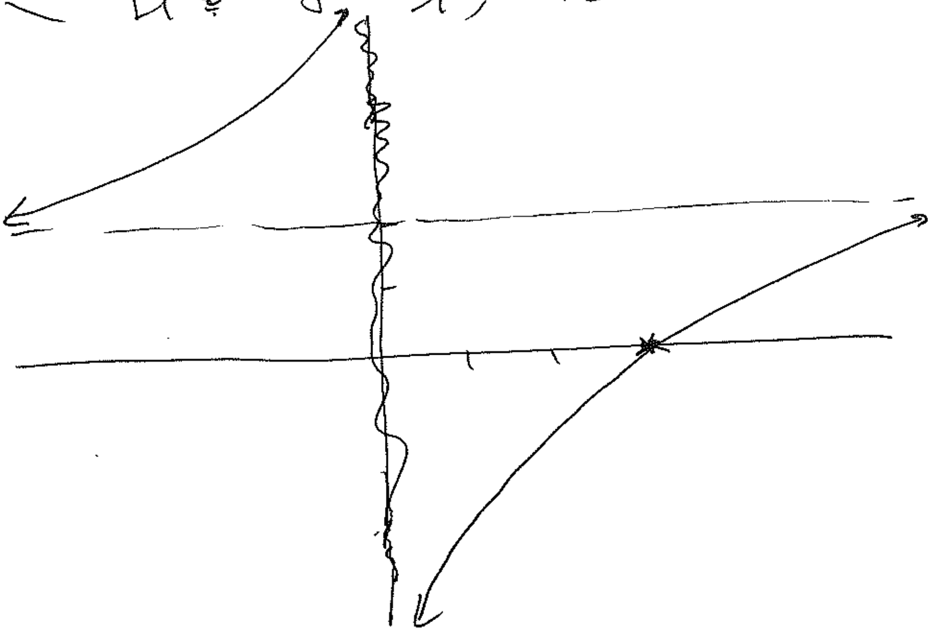
$f(3'1) > 0$



b)

Corte ejes $\begin{cases} OX: \; 0 = \dfrac{2x-6}{x}; \; 2x-6=0 \rightarrow x=3 \rightarrow (3,0) \\ OY: \; g(0) = \dfrac{-6}{0} \rightarrow \nexists \text{ no corta} \end{cases}$

Asíntotas $\begin{cases} V: \boxed{x=0} \\ H: \; y = \dfrac{2}{1}; \; \boxed{y=2} \end{cases}$



c) i) $P_1(-2,-6)$ $\quad m = \dfrac{0-(-6)}{1-(-2)} = \dfrac{6}{3} = 2$

$P_2(1,0)$

$y = 2(x-1)+0;$ $\quad \boxed{y = 2x-2}$

$y(-1) = 2\cdot(-1) - 2 = -2-2 = \underline{\underline{-4}}$

ii) $P_1(1,0)$ $\quad m = \dfrac{1-0}{3-1} = \dfrac{1}{2}$

$P_2(3,1)$

$y = \dfrac{1}{2}(x-1)+0;$ $\quad \boxed{y = \dfrac{1}{2}x - \dfrac{1}{2}}$

$y(2) = \dfrac{1}{2}\cdot 2 - \dfrac{1}{2} = 1 - \dfrac{1}{2} = \underline{\underline{\dfrac{1}{2}}}$

iii) $\boxed{y = \dfrac{1}{2}x - \dfrac{1}{2}}$ $\quad y(4) = \dfrac{1}{2}\cdot 4 - \dfrac{1}{2} = 2 - \dfrac{1}{2} = \underline{\underline{\dfrac{3}{2}}}$

1) a) $D_f = \mathbb{R} \setminus \{0\}$

$D_g = \mathbb{R} \setminus \{1\}$

$D_h = \mathbb{R} - \{-2, 0\}$

b) $\boxed{f(x)}$ $x^2 + x - 2 = 0 \begin{cases} x = -2 \\ x = 1 \end{cases}$

−		+		−		+
	-2		0		1	

$x = 0$

$\boxed{g(x)}$ $x - 1 = 0$

−		+
	1	

c) $\boxed{f(x)}$ $OX: x^2 + x - 2 = 0 \begin{cases} x = -2 \rightarrow (-2, 0) \\ x = 1 \rightarrow (1, 0) \end{cases}$

OY: NO CORTA

$\boxed{g(x)}$ $OX: \dfrac{1}{x-1} = 0 \rightarrow$ NO CORTA

$OY: \sqrt[3]{\dfrac{1}{0-1}} = \sqrt[3]{-1} = -1 \rightarrow (0, -1)$

$\boxed{h(x)}$ $OX: x - 5 = 0 \rightarrow x = 5 \rightarrow (5, 0)$

OY: NO CORTA

d) $\boxed{f(x)}$ $\dfrac{x^2 + x - 2}{x} = 1;\ x^2 + x - 2 = x;\ x^2 - 2 = 0$

$x = \pm\sqrt{2} \rightarrow \underline{\underline{\text{sí}}}$

$\boxed{g(x)}$ $\sqrt[3]{\dfrac{1}{x-1}} = 1;\ \dfrac{1}{x-1} = 1;\ 1 = x - 1;\ x = 2 \rightarrow \underline{\underline{\text{sí}}}$

$\boxed{h(x)}$ $\dfrac{x - 5}{x^2 + 2x} = 1;\ x - 5 = x^2 + 2x \quad 0 = x^2 + x + 5$

$x = \dfrac{-1 \pm \sqrt{1 - 20}}{2} \rightarrow \nexists$ $\quad \underline{\underline{\text{NO}}}$

2) $f(x) = \begin{cases} x^2+4x & \text{si } x < -1 \\ -x^2+2x-1 & \text{si } -1 \leq x < 3 \\ 2x-8 & \text{si } x \geq 3 \end{cases}$

x	-2	-1
y	-4	-3

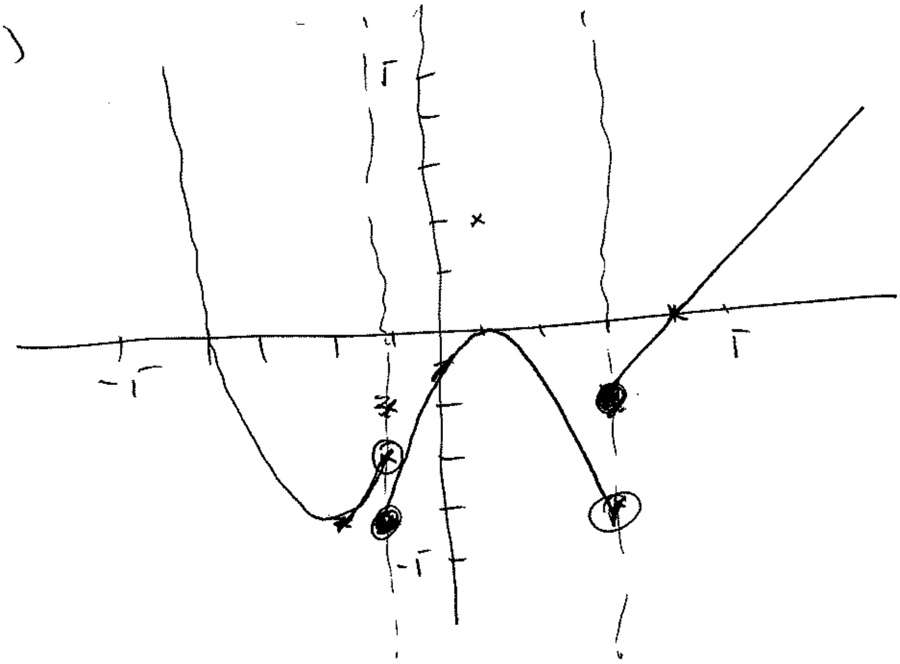
$V(-2,-4)$ mín.

x	-1	3
y	-4	-4

$V(1,0)$ máx

x	3	4
y	-2	0

i)



ii) Creciente: $(-2,-1) \cup (-1,1) \cup (3,+\infty)$

Decreciente: $(-\infty,-2) \cup (1,3)$

iii) Eje OX

$x^2+4x=0$; $x(x+4)=0$ → $x=0 \rightarrow \cancel{(0,0)}$; $x=-4 \rightarrow \boxed{(-4,0)}$

$-x^2+2x-1=0$; $x=+1 \rightarrow \boxed{(1,0)}$

$2x-8=0$ $\quad x=4 \rightarrow \boxed{(4,0)}$

Eje OY

$-0^2+2\cdot 0-1=-1 \longrightarrow \boxed{(0,-1)}$

iv) Discontinua en $x=-1$ y $x=3$

3) a) $\frac{3^x}{3^2} - 2\cdot 3^x\cdot 3 + 4\cdot 3^x + 17 = 0$

$\boxed{3^x = t}$ $\quad \frac{t}{9} - 6t + 4t + 17 = 0$

$t - 54t + 36t + 153 = 0$

$-17t = -153;\quad t = \frac{-153}{-17} = 9$

$3^x = 9 \Rightarrow \underline{\underline{x = 2}}$

b) $\frac{5^x}{5} - 10\cdot \frac{5^{2x}}{5^3} + 25^x = 24$

$\boxed{5^x = t}$ $\quad \frac{t}{5} - \frac{10t^2}{125} + t^2 = 24$

$\frac{25t}{125} - \frac{10t^2}{125} + \frac{125t^2}{125} = \frac{3000}{125}$

$25t - 10t^2 + 125t^2 = 3000$

$115t^2 + 25t - 3000 = 0$

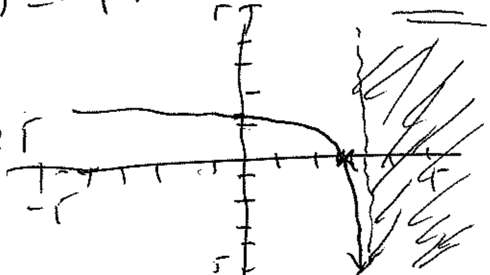
$t = \frac{-25 \pm \sqrt{625 + 1380000}}{230} = \frac{-25 \pm 1175}{230} = \begin{cases} 5 \\ -5'21 \end{cases}$

$5^x = 5 \Rightarrow \boxed{x = 1}$ $\qquad 5^x = -5'21 \Rightarrow \underline{\underline{\nexists \text{ sol}}}$

c) ii) Corte ejes
- OX: $\log(13-4x) = 0;\ 13-4x = 1;\ 12 = 4x;\ x = 3$ $\quad \underline{\underline{(3, 0)}}$
- OY: $y(0) = \log(13-4\cdot 0) = \log 13 = 1'11 \rightarrow \underline{\underline{(0, 1'11)}}$

iii) A.V: $13 - 4x = 0;\ 13 = 4x;\ x = \frac{13}{4} = 3'25$

i) $13 - 4x > 0,\ -4x > -13;\ x < \frac{13}{4}$ $\qquad D = (-\infty, \frac{13}{4})$



4) a) i) $(3x-1)^{-2}=\frac{1}{4}$; $\frac{1}{(3x-1)^2}=\frac{1}{4}$

$(3x-1)^2=4$; $9x^2+1-6x=4$

$9x^2-6x-3=0$; $x=\frac{6\pm\sqrt{36+108}}{18}=\frac{6\pm 12}{18}=$ 1 ; $\cancel{-\frac{1}{3}}$

ii) $27^{1/3}=x^2+2x$; $\sqrt[3]{27}=x^2+2x$

$3=x^2+2x$; $0=x^2+2x-3$; $x=\frac{-2\pm\sqrt{4+12}}{2}=$

$=\frac{-2\pm 4}{2}=$ 1 ; -3

iii) $2^{\frac{2x+1}{3}}=12$; $\log 2^{\frac{2x+1}{3}}=\log 12$

$\frac{2x+1}{3}\log 2=\log 12$; $\frac{2x+1}{3}=\frac{\log 12}{\log 2}$

$\frac{2x+1}{3}=3'58$; $2x+1=10'75$; $2x=9'75$

$x=\frac{9'75}{2}$; $\underline{\underline{x=4'87}}$

b) $100=250\cdot(1-0'12)^t$; $100=250\cdot 0'88^t$

$\frac{100}{250}=0'88t$; $0'4=0'88^t$; $\log 0'4=\log 0'88^t$

$\log 0'4=t\cdot\log 0'88$; $t=\frac{\log 0'4}{\log 0'88}=$ $\underline{\underline{7'16 \text{ años}}}$

c) $3\cdot 2\cdot\log x-5\cdot\frac{1}{2}\log x+8\cdot\frac{1}{4}\log x=10$

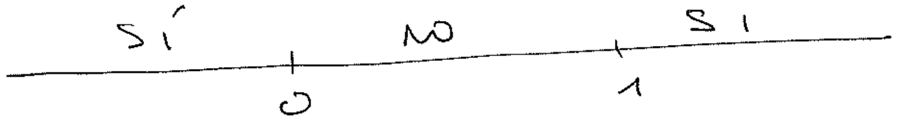
$6\log x-\frac{5}{2}\log x+2\log x=10$

$12\log x-5\log x+4\log x=20$

$11\log x=20$; $\log x=\frac{20}{11}=1'81$; $x=10^{1'81}=\underline{\underline{64'56}}$

1) a) i) $x^2 - x = 0$; $x(x-1) = 0$ $\begin{cases} x=0 \\ x=1 \end{cases}$ $D_f = \mathbb{R} \setminus \{0, 1\}$

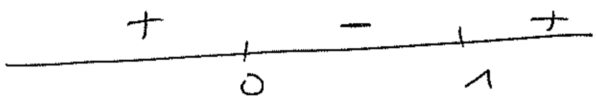
$\dfrac{x}{x^3 - 1} \geq 0$ $\quad x = 0$; $x^3 - 1 = 0 \to x^3 = 1$; $x = \sqrt[3]{1} = 1$



$D_g = (-\infty, 0] \cup (1, +\infty)$

$x + 4 = 0 \to x = -4$; $\quad D_h = \mathbb{R} \setminus \{-4\}$

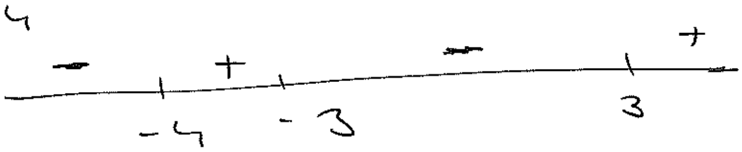
ii) ~~f(x)~~ → Corte eje OX: NO CORTA



$x^2 - x = 0 \to \begin{cases} x = 0 \\ x = 1 \end{cases}$

$g(x) \to$ Corte eje OX $\to x^2 - 9 = 0 \begin{cases} x = -3 \\ x = 3 \end{cases}$

$x + 4 = 0 \to x = -4$



iii) $f(x)$ $\begin{cases} \text{Eje OX: NO CORTA} \\ \text{Eje OY: } f(0) = \nexists \text{ NO CORTA} \end{cases}$

$g(x)$ $\begin{cases} \text{Eje OX: } x = 0 \to (0,0) \\ \text{Eje OY: } g(0) = 0 \to (0,0) \end{cases}$

$h(x)$ $\begin{cases} \text{Eje OX: } x^2 - 9 = 0 \to x = \begin{cases} -3 \to (-3, 0) \\ 3 \to (3, 0) \end{cases} \\ \text{Eje OY: } h(0) = -\frac{9}{4} \to \left(0, -\frac{9}{4}\right) \end{cases}$

b) TEÓRICO

2)

$$f(x) = \begin{cases} x^2+6x & \text{si } x \le -2 \\ -x^2-1 & \text{si } -2 < x \le 2 \\ -x-3 & \text{si } x > 2 \end{cases}$$

x	-3	-2
y	-9	-8

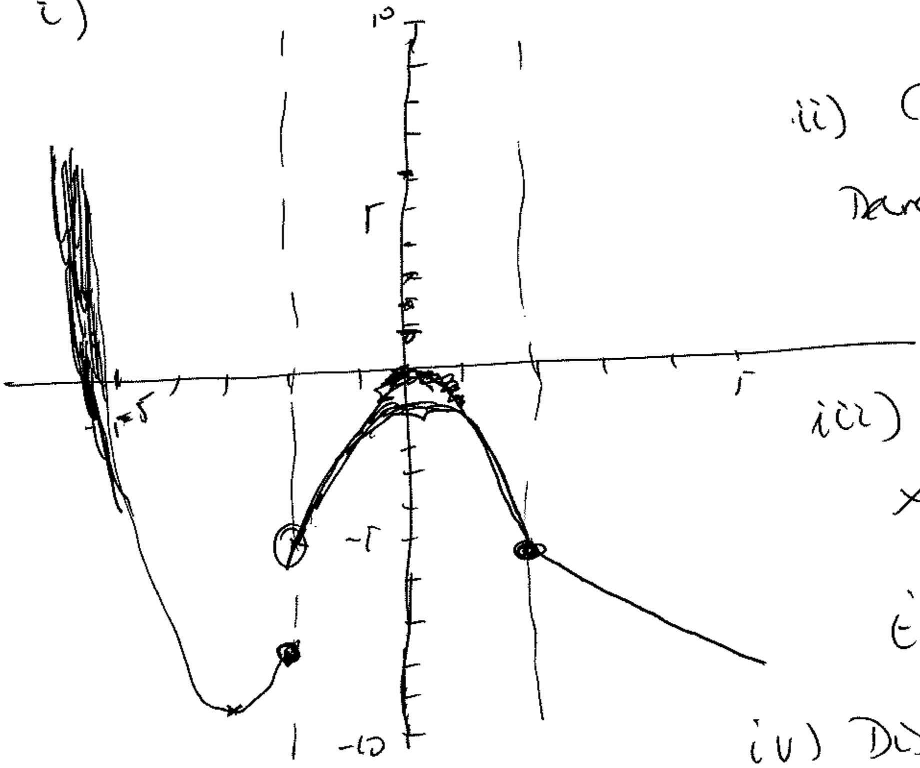
$V(-3,-9)$ mínimo

x	-2	2
y	-5	-5

$V(0,-1)$ máximo

x	2	3
y	-5	-6

i)



ii) Creciente: $(-3,-2)\cup(-2,0)$

Decreciente: $(-\infty,-3)\cup(0,2)\cup(2,+\infty)$

iii) Eje OX:

$x^2+6x=0$; $x(x+6)=0$ $\begin{cases} x=0 \\ x=-6 \end{cases}$ $(-6,0)$

Eje OY: $(0,-1)$

iv) Discontinua en $x=-2$

3) a) $\dfrac{2^1}{2^x} + 3\cdot 2^x = \dfrac{25}{2}$ $\boxed{2^x = t}$

$\dfrac{2}{t} + 3t = \dfrac{25}{2}$; $4 + 6t^2 = 25t$

$6t^2 - 25t + 4 = 0$; $t = \dfrac{25 \pm \sqrt{625-96}}{12} =$

$= \dfrac{25 \pm 23}{12} = \begin{cases} \dfrac{2}{12} = \dfrac{1}{6} \\ \dfrac{48}{12} = 4 \end{cases}$

$2^x = 4 \Rightarrow \underline{\underline{x = 2}}$

$2^x = \dfrac{1}{6} \Rightarrow \log 2^x = \log \dfrac{1}{6}$; $x \log 2 = \log \dfrac{1}{6}$

$x = \dfrac{\log \frac{1}{6}}{\log 2} = \underline{\underline{-2'56}}$

b) $4\cdot 3\cdot 3^x - (3^2)^x + \frac{3^x}{3} = \frac{34}{3}$ $\boxed{3^x = t}$

$12t - t^2 + \frac{t}{3} = \frac{34}{3}$; $36t - 3t^2 + t = 34$

$0 = 3t^2 - 37t + 34$; $t = \frac{37 \pm \sqrt{1369 - 408}}{6} =$

$= \frac{37 \pm 31}{6} = \begin{cases} \frac{68}{6} = \frac{34}{3} \\ \frac{6}{6} = 1 \end{cases}$

$3^x = 1 \Rightarrow x = 0$

$3^x = \frac{34}{3}$; $\lg 3^x = \lg \frac{34}{3}$; $x \lg 3 = \lg \frac{34}{3}$

$x = \frac{\lg 34/3}{\lg 3} = \underline{\underline{2'2}}$

c)

x	1	2
$f(x)$	1	5/2

$m = \frac{\frac{5}{2} - 1}{2 - 1} = \frac{\frac{3}{2}}{1} = \frac{3}{2}$

$y = \frac{3}{2}(x-1) + 1$; $y = \frac{3}{2}x - \frac{3}{2} + 1$

$\underline{\underline{y = \frac{3}{2}x - \frac{1}{2}}}$ $y(1'8) = \frac{3}{2}\cdot 1'8 - \frac{1}{2} = \underline{\underline{2'2}}$

4) a) $(x^2-2)^2 = 49$; $x^4 + 4 - 4x^2 = 49$

$x^4 - 4x^2 - 45 = 0$; $\boxed{x^2 = t}$ $t^2 - 4t - 45 = 0$

$t = \frac{4 \pm \sqrt{16 + 180}}{2} = \frac{4 \pm 14}{2} = \begin{cases} \frac{18}{2} = 9 \\ \frac{-10}{2} = -5 \end{cases}$

$x^2 = 9 \Rightarrow x = \begin{cases} -3 \\ 3 \end{cases}$

$x^2 = -5 \Rightarrow \nexists$ sol

ii) $\cancel{\log}$ $\left(\frac{1}{32}\right)^{1/5} = x^2 - \frac{1}{2}$ $\quad \frac{1}{2} = x^2 - \frac{1}{2}$

$1 = 2x^2 - 1; \quad 2x^2 = 2; \quad x^2 = \frac{2}{2} = 1; \quad x = \begin{cases} -1 \\ 1 \end{cases}$

iii) $\cancel{\log}$ $3^{\left(\frac{3x-5}{2}\right)} = 7; \quad \log 3^{\left(\frac{3x-5}{2}\right)} = \log 7$

$\frac{3x-5}{2} \log 3 = \log 7; \quad \frac{3x-5}{2} = \frac{\log 7}{\log 3} = 1'77$

$3x-5 = 3'54; \quad 3x = 8'54; \quad x = \frac{8'54}{3} = \underline{\underline{2'84}}$

b) $7.500 = 6.000(1+x)^8; \quad \frac{7500}{6000} = (1+x)^8$

$1'25 = (1+x)^8; \quad 1+x = \sqrt[8]{1'25} = 1'028$

$x = 1'028 - 1 = 0'028 \longrightarrow \underline{\underline{2'8\%}}$

c) $500.000 = C_0 (1-0'03)^{20}$

$500.000 = C_0 \; 0'97^{20}$

$C_0 = \frac{500.000}{0'97^{20}} = \underline{\underline{919.465 \text{ hab}}}$

a) $\lim\limits_{x \to +\infty} \left(\sqrt{4x^2-3x} - \sqrt{4x^2+2x-1} \right) =$

$= \lim\limits_{x \to +\infty} \dfrac{\left(\sqrt{4x^2-3x} - \sqrt{4x^2+2x-1} \right)\left(\sqrt{4x^2-3x} + \sqrt{4x^2+2x-1} \right)}{\sqrt{4x^2-3x} + \sqrt{4x^2+2x-1}} =$

$= \lim\limits_{x \to +\infty} \dfrac{(4x^2-3x)-(4x^2+2x-1)}{\sqrt{4x^2-3x} + \sqrt{4x^2+2x-1}} =$

$= \lim\limits_{x \to +\infty} \dfrac{\cancel{4x^2} - 3x - \cancel{4x^2} - 2x + 1}{\sqrt{4x^2-3x} + \sqrt{4x^2+2x-1}} =$

$= \lim\limits_{x \to +\infty} \dfrac{-5x+1}{\sqrt{4x^2-3x} + \sqrt{4x^2+2x-1}} =$

$= \lim\limits_{x \to +\infty} \dfrac{-5x+1}{2x+2x} = \lim\limits_{x \to +\infty} \dfrac{-5x+1}{4x} = -\dfrac{5}{4}$ //

b) $\lim\limits_{x \to \infty} \left(\dfrac{(2x^4-3x^2)(x^2+2) - (2x^4+x^2)(x^2-4)}{(x^2-4)(x^2+2)} \right) =$

$= \lim\limits_{x \to \infty} \dfrac{\cancel{2x^6} + 4x^4 - 3x^4 - 6x^2 - \cancel{2x^6} + 8x^4 - x^4 + 4x^2}{x^4 + 2x^2 - 4x^2 - 8} =$

$= \lim\limits_{x \to \infty} \dfrac{8x^4 - 2x^2}{x^4 - 2x^2 - 8} = 8$ //

c) $\lim\limits_{x \to +\infty} \left(\dfrac{x^4 - 2x^2 + x}{4x^4 - x^2} \right)^{\frac{x^2-3x}{x-1}} = \left(\dfrac{1}{4} \right)^{+\infty} = 0$

d) $$\lim_{x \to 2} \frac{(x^2-2x)(x^2+1)}{(x^2-x-2)(x+1)} = \lim_{x \to 2} \frac{x\cancel{(x-2)}(x^2+1)}{(x+1)\cancel{(x-2)}(x+1)} =$$

$$= \lim_{x \to 2} \frac{x(x^2+1)}{(x+1)^2} = \frac{2 \cdot (4+1)}{3^2} = \frac{10}{9} \; //$$

e) $$\lim_{x \to -1} \frac{(\sqrt{x^2-3x-3}+x)(\sqrt{x^2-3x-3}-x)}{(2x^2+x-1)(\sqrt{x^2-3x-3}-x)} =$$

$$= \lim_{x \to -1} \frac{(x^2-3x-3)-x^2}{(2x^2+x-1)(\sqrt{x^2-3x-3}-x)} =$$

$$= \lim_{x \to -1} \frac{\cancel{x^2}-3x-3}{(2x^2+x-1)(\sqrt{x^2-3x-3}-x)} =$$

$$= \lim_{x \to -1} \frac{-3\cancel{(x+1)}}{2\cancel{(x+1)}(x-\frac{1}{2})(\sqrt{x^2-3x-3}-x)} =$$

$$= \lim_{x \to -1} \frac{-3}{2(x-\frac{1}{2})(\sqrt{x^2-3x-3}-x)} =$$

$$= \frac{-3}{2 \cdot (-1-\frac{1}{2})(1+1)} = \frac{-3}{-6} = \frac{1}{2} \; //$$

2)

a) $\lim\limits_{x \to 1^-} f(x) = \lim\limits_{x \to 1^-} \dfrac{\sqrt{x+3}-2}{x-1} =$

$= \lim\limits_{x \to 1^-} \dfrac{(\sqrt{x+3}-2)(\sqrt{x+3}+2)}{(x-1)(\sqrt{x+3}+2)} =$

$= \lim\limits_{x \to 1^-} \dfrac{x+3-4}{(x-1)(\sqrt{x+3}+2)} = \lim\limits_{x \to 1^-} \dfrac{\cancel{(x-1)}}{\cancel{(x-1)}(\sqrt{x+3}+2)}$

$= \lim\limits_{x \to 1^-} \dfrac{1}{\sqrt{x+3}+2} = \dfrac{1}{2+2} = \dfrac{1}{4}$

$\lim\limits_{x \to 1^+} f(x) = \dfrac{1}{4}$

$k-1 = \dfrac{1}{4}$

$k = \dfrac{1}{4}+1;$ $\boxed{k = \dfrac{5}{4}}$

$f(1) = k-1;$

b) TEÓRICO

3) FUNCIONES

$x = 0$ D.I.S.I

CONEXIONES

$x = -1$ D.I.S.F

$x = 2$ D.E

$\lim\limits_{x \to 0^-} f(x) = +\infty$

$\lim\limits_{x \to 0^+} f(x) = -\infty$

$f(0) = \nexists$

$\lim\limits_{x \to -1^-} f(x) = \lim\limits_{x \to -1^-} \dfrac{x^2+x}{x^2-1} =$

$= \lim\limits_{x \to -1^-} \dfrac{x\cancel{(x+1)}}{\cancel{(x+1)}(x-1)} =$

$= \lim\limits_{x \to -1^-} \dfrac{x}{x-1} = \dfrac{-1}{-2} = \dfrac{1}{2}$

$\lim\limits_{x \to -1^+} f(x) = \dfrac{-3-2}{-2} = \dfrac{5}{2}$

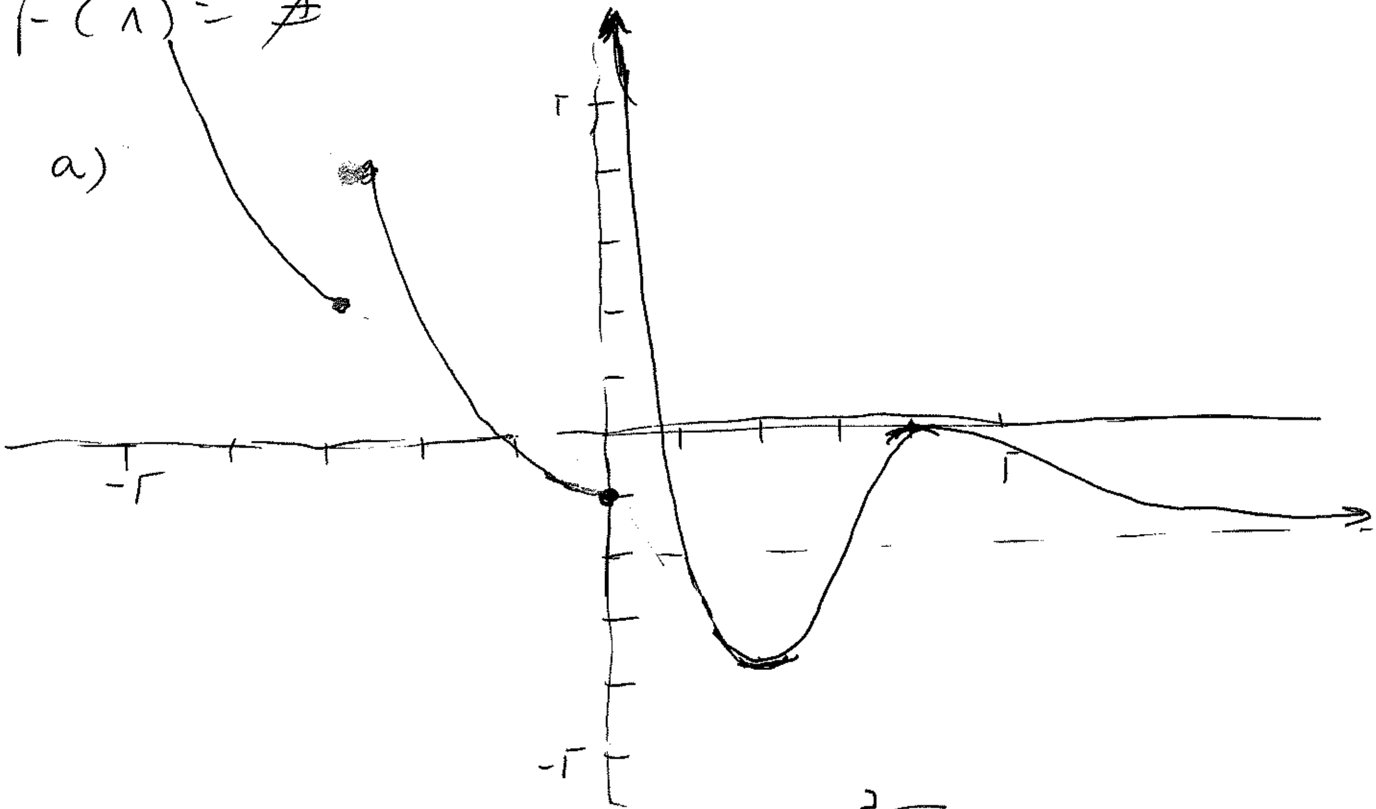
$f(-1) = \dfrac{5}{2}$

$$\lim_{x\to 2^-} f(x) = \frac{3\cdot 2 - 2}{2\cdot 2} = \frac{6-2}{4} = \frac{4}{4} = 1$$

$$\lim_{x\to 2^+} f(x) = \frac{4\cdot 2}{2+6} = \frac{8}{8} = 1$$

$f(1) = \nexists$

4) a)

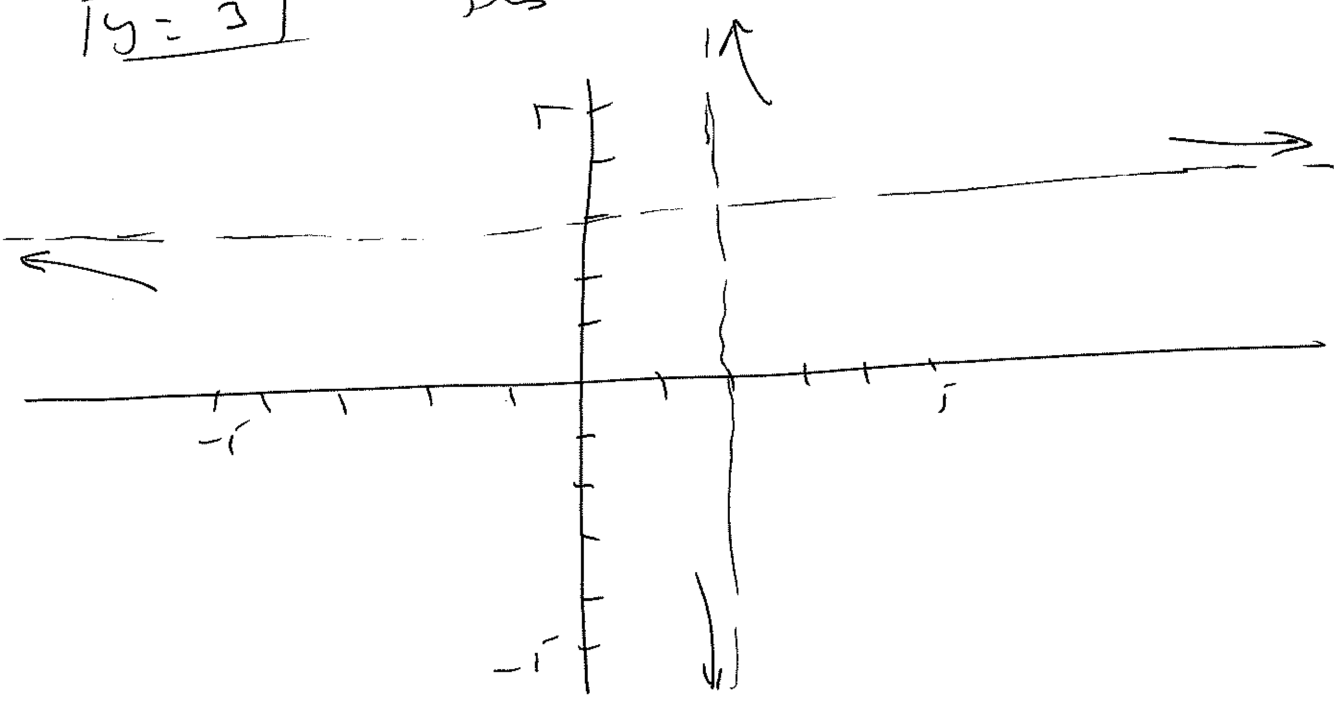


b)

A.V: $x^3-8=0$; $x^3=8$; $x=\sqrt[3]{8}=2$; $\underline{x=2}$

A.H: $\lim_{x\to+\infty}\frac{3x^3-2}{x^3-8}=3$; $\lim_{x\to-\infty}\frac{3x^3-2}{x^3-8}=3$

$\boxed{y=3}$



1) a)

FUNCIONES

$x=-1$ D.I.S.I

$x=1$ D.E.

CONEXIONES

$x=0$ Continua

$x=2$ D.E

$$\lim_{x\to -1^-} f(x) = +\infty$$

$$\lim_{x\to -1^+} f(x) = +\infty$$

$$f(-1) = \nexists$$

$$\lim_{x\to 1^-} f(x) = \lim_{x\to 1^-} \frac{(\cancel{x-1})}{(x+1)(\cancel{x-1})} = \lim_{x\to 1^-} \frac{1}{x+1} = \frac{1}{2}$$

$$\lim_{x\to 1^+} f(x) = \frac{1}{2}$$

$$f(1) = \nexists$$

$$\lim_{x\to 0^-} f(x) = 1$$

$$\lim_{x\to 0^+} f(x) = 1$$

$$f(0) = 1$$

$$\lim_{x\to 2^-} f(x) = \frac{2-1}{4-1} = \frac{1}{3}$$

$$\lim_{x\to 2^+} f(x) = \frac{2+1}{8+1} = \frac{3}{9} = \frac{1}{3}$$

$$f(2) = \nexists$$

b)

$$\lim_{x\to 2^-} \frac{\sqrt{x^2+5}-3}{x-2} = \lim_{x\to 2^-} \frac{(\sqrt{x^2+5}-3)(\sqrt{x^2+5}+3)}{(x-2)(\sqrt{x^2+5}+3)} =$$

$$= \lim_{x\to 2^-} \frac{x^2+5-9}{(x-2)(\sqrt{x^2+5}+3)} = \lim_{x\to 2^-} \frac{(x+2)(\cancel{x-2})}{(\cancel{x-2})(\sqrt{x^2+5}+3)} =$$

$$= \lim_{x\to 2^-} \frac{x+2}{\sqrt{x^2+5}+3} = \frac{4}{6} = \frac{2}{3}$$

$$\lim_{x\to 2^+} \frac{x^2-4}{x-2} = \lim_{x\to 2^+} \frac{(x+2)(\cancel{x-2})}{(\cancel{x-2})} = 4$$

$$f(2) = \nexists$$

D.I.S.F

2) i) $\lim_{x \to 1} \frac{(\sqrt{x^2+3} - 2)(\sqrt{x^2+3} + 2)}{(x^2+3x-4)(\sqrt{x^2+3}+2)} =$

$= \lim_{x \to 1} \frac{x^2+3-4}{(x^2+3x-4)(\sqrt{x^2+3}+2)} = \lim_{x \to 1} \frac{(x+1)\cancel{(x-1)}}{\cancel{(x-1)}(x+4)(\sqrt{x^2+3}+2)} =$

$= \lim_{x \to 1} \frac{x+1}{(x+4)(\sqrt{x^2+3}+2)} = \frac{1+1}{(1+4)(2+2)} = \frac{2}{20} = \frac{1}{10}$ //

ii) $\lim_{x \to 2} \frac{x^2 \cancel{(x-2)}}{3\cancel{(x-2)}(x+2/3)} =$ $\qquad 3x^2-4x-4=0$, $x = \frac{4 \pm \sqrt{16+48}}{6} = \frac{4 \pm 8}{6} = \begin{cases} 2 \\ -\frac{2}{3} \end{cases}$

$= \lim_{x \to 2} \frac{x^2}{3x+2} = \frac{4}{8} = \frac{1}{2}$ //

iii) $\lim_{x \to +\infty} \left(\frac{(2x^2+x-3)(x-1) - (x^2+3)(2x+1)}{(2x+1)(x-1)} \right) =$

$= \lim_{x \to +\infty} \frac{\cancel{2x^3} - 2x^2 + x^2 - x - 3x + \cancel{3} - \cancel{2x^3} - x^2 - 6x - \cancel{3}}{2x^2 - 2x + x - 1} =$

$= \lim_{x \to +\infty} \frac{-2x^2 - 10x}{2x^2 - x - 1} = -1$ //

iv) $2^{+\infty} = +\infty$ //

3) a) $f'(x) = -\frac{3}{x^2} - \frac{1}{2\sqrt{x+3}}$; $f(1) = \frac{3}{1} - \sqrt{1+3} = 3-2=1$

$P(1, 1)$ $\qquad f'(1) = -\frac{3}{1} - \frac{1}{2 \cdot 2} = -3 - \frac{1}{4} = -\frac{13}{4}$

$y = -\frac{13}{4}(x-1) + 1$; $y = -\frac{13}{4}x + \frac{13}{4} + 1$

$\boxed{y = -\frac{13}{4}x + \frac{17}{4}}$

b) $f'(x) = 4x^3 - 4x = 0$

$4x(x^2-1) = 0$ ⟨ $x = 0$ // ; $x^2 - 1 = 0$ ⟨ $x = 1$ // ; $x = -1$ //

$E_1(0, 3)$ Máx relat $E_2(1, 2)$ Min relt $E_3(-1, 2)$ Min relt

$f''(x) = 12x^2 - 4$

$f''(0) = -4 < 0$; $f''(1) = 8 > 0$; $f''(-1) = 8 > 0$

$f''(x) = 0$; $12x^2 - 4 = 0$; $12x^2 = 4$; $x^2 = \frac{4}{12} = \frac{1}{3}$

$x = \pm\sqrt{\frac{1}{3}}$ $I_1\left(\sqrt{\frac{1}{3}}, \frac{22}{9}\right)$ Inflex $I_2\left(-\sqrt{\frac{1}{3}}, \frac{22}{9}\right)$ Inflex

$f\left(\sqrt{\frac{1}{3}}\right) = \frac{1}{9} - \frac{2}{3} + 3 = \frac{1 - 6 + 27}{9} = \frac{22}{9}$

$f\left(-\sqrt{\frac{1}{3}}\right) = \frac{22}{9}$

4) i) $D = \mathbb{R}$

ii) Corte ejes ⟨ OX : $(0, 0)$; OY : $(0, 0)$

iii) Asíntotas

Hor; $\lim_{x \to \pm\infty} \frac{x}{x^2+4} = 0$; $\boxed{y = 0}$ Dos lados.

Vert; $x^2 + 4 = 0$ $\nexists$ sol. No hay

iv) Extr relat

$$f'(x) = \frac{1(x^2+4) - 2x \cdot x}{(x^2+4)^2} = \frac{x^2+4-2x^2}{(x^2+4)^2} = \frac{-x^2+4}{(x^2+4)^2}$$

$$-x^2+4=0 \; ; \quad x = \begin{cases} -2 \\ 2 \end{cases}$$

$f'<0$		$f'>0$		$f'<0$
Decrec	-2	Crec	2	Decrec
	↓		↓	
	mín relat		máx relat	

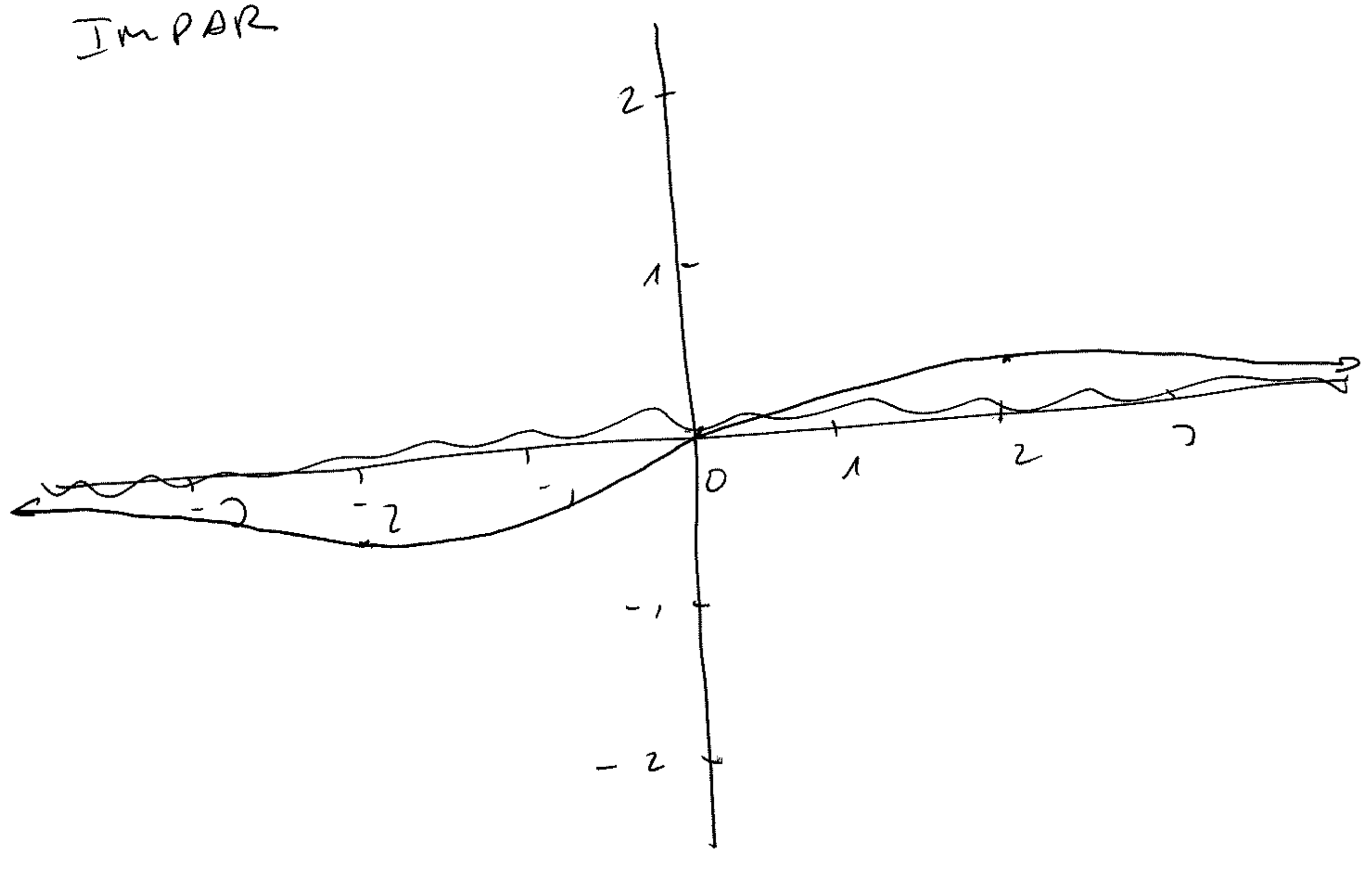
$E_2\left(2, \frac{1}{4}\right)$ máx relat

$E_1\left(-2, -\frac{1}{4}\right)$ mín relat

v) limites

$$f(-x) = \frac{(-x)}{(-x)^2+4} = \frac{-x}{x^2+4} = -f(x)$$

IMPAR



1) i) $\lim_{x\to -1^-} f(x) = \frac{-1+2}{(-1)^2-1-2} = \frac{1}{-2}$

$\lim_{x\to -1^+} f(x) = (-1)^2 - a(-1) = 1+a$

$f(-1) = -\frac{1}{2}$; $1+a = -\frac{1}{2}$; $2+2a=-1$ $\boxed{a=-\frac{3}{2}}$

ii) FUNCIONES | CONEXIONES

$x=-2 \to$ D.E | $x=-1$ D.I.S.F

$x=2$ D.I.S.I

$\lim_{x\to -2^-} \frac{(\cancel{x+2})}{(\cancel{x+2})(x-1)} = -\frac{1}{3}$

$\lim_{x\to -2^+} \frac{(x+2)}{(x+2)(x-1)} = -\frac{1}{3}$

$f(-2) = \nexists$

$\lim_{x\to -1^-} f(x) = \frac{1}{-2}$

$\lim_{x\to -1^+} f(x) = 4$

$f(-1) = -\frac{1}{2}$

$\lim_{x\to 2^-} f(x) = -2$

$\lim_{x\to 2^+} f(x) = +\infty$

$f(2) = \nexists$

2) i) $\lim_{x\to -1} \frac{(\sqrt{x+5}+2x)(\sqrt{x+5}-2x)}{(x^2+2x+1)(\sqrt{x+5}-2x)} =$

$= \lim_{x\to -1} \frac{x+5-4x^2}{(x+1)^2(\sqrt{x+5}-2x)} = \lim_{x\to -1} \frac{-4(x+\cancel{1})(x-\frac{5}{4})}{(x+1)^2(\sqrt{x+5}-2x)}$

$= \infty$

ii) $\lim\limits_{x \to 1} \dfrac{(x-\cancel{1})(x^2+x+1)}{x(x+1)(\cancel{x-1})} =$

$$\begin{array}{c|cccc} & 1 & 0 & 0 & -1 \\ 1 & & 1 & 1 & 1 \\ \hline & 1 & 1 & 1 & 0 \end{array}$$

$= \lim\limits_{x \to 1} \dfrac{x^2+x+1}{x(x+1)} = \dfrac{1+1+1}{1\cdot(1+1)} = \dfrac{3}{2}$ //

iii) $\lim\limits_{x \to +\infty} \dfrac{\left(\sqrt{9x^2-3x+2} - \sqrt{9x^2-x-5}\right)\left(\sqrt{9x^2-3x+2} + \sqrt{9x^2-x-5}\right)}{\left(\sqrt{9x^2-3x+2} + \sqrt{9x^2-x-5}\right)} =$

$= \lim\limits_{x \to +\infty} \dfrac{(9x^2-3x+2)-(9x^2-x-5)}{\sqrt{9x^2-3x+2} + \sqrt{9x^2-x-5}} =$

$= \lim\limits_{x \to +\infty} \dfrac{\cancel{9x^2}-3x+2-\cancel{9x^2}+x+5}{\sqrt{9x^2-3x+2} + \sqrt{9x^2-x-5}} =$

$= \lim\limits_{x \to +\infty} \dfrac{-2x+7}{3x+3x} = \lim\limits_{x \to +\infty} \dfrac{-2x+7}{6x} = -\dfrac{1}{3}$ //

3) a) $f(-1) = (-2)^3 \cdot 1^2 = -8 \qquad P(-1,-8)$

$f'(x) = 3(x-1)^2(x+2)^2 + (x-1)^3 \cdot 2 \cdot (x+2)$

$f'(-1) = 3\cdot(-2)^2 \cdot 1^2 + (-2)^3 \cdot 2 \cdot 1 =$

$= 12 - 16 = -4$

$y = -4(x+1) - 8 \; ; \; \underline{\underline{y = -4x - 12}}$

b) $f(2) = -6$ $\qquad f'(x) = 3ax^2 + b$

$f'(2) = 0$

$a \cdot 8 + 2b + 10 = -6$ $\qquad \left\{ \begin{array}{l} 8a + 2b = -16 \\ 12a + b = 0 \end{array} \right\}$

$12a + b = 0$

$8a - 24a = -16$; $-16a = -16 \Rightarrow \underline{\underline{a = 1}}$. $\underline{\underline{b = -12}}$

$f(x) = x^3 - 12x + 10$

$f'(x) = 3x^2 - 12 = 0$; $3(x^2 - 4) = 0 \to x = -2,\ x = 2$

$E_1(-2, 26)$ Máx relat

$E_2(2, -6)$ Mín relat

$f''(-2) = -12 < 0$

$f''(x) = 6x$; $\qquad f''(2) = 12 > 0$

$I(0, 10)$ Inflexión

$6x = 0 \Rightarrow x = 0$

4) i) $D = \mathbb{R} - \{-2, 2\}$

ii) Corte ejes
- OX: NO CORTA
- OY: $(0, -\frac{3}{4})$

iii) Asíntotas

Hor $\lim_{x \to \pm\infty} \frac{3}{x^2-4} = 0$ $\boxed{y = 0}$ Dos lados

Vert $x^2 - 4 = 0$ $\boxed{x = -2}$ $\boxed{x = 2}$

$\lim_{x \to -2^-} f(x) = +\infty$ / $\lim_{x \to 2^-} f(x) = -\infty$

$\lim_{x \to -2^+} f(x) = -\infty$ / $\lim_{x \to 2^+} f(x) = +\infty$

iv) Extrem. Relat

$$f'(x) = \frac{0(x^2-4) - 2x \cdot 3}{(x^2-4)^2} = \frac{-6x}{(x^2-4)} = 0 \to \underline{x=0}$$

$f'>0$ Crec | -2 | $f'>0$ Crec | 0 | $f'<0$ Decrec | 2 | $f'<0$ Decrec

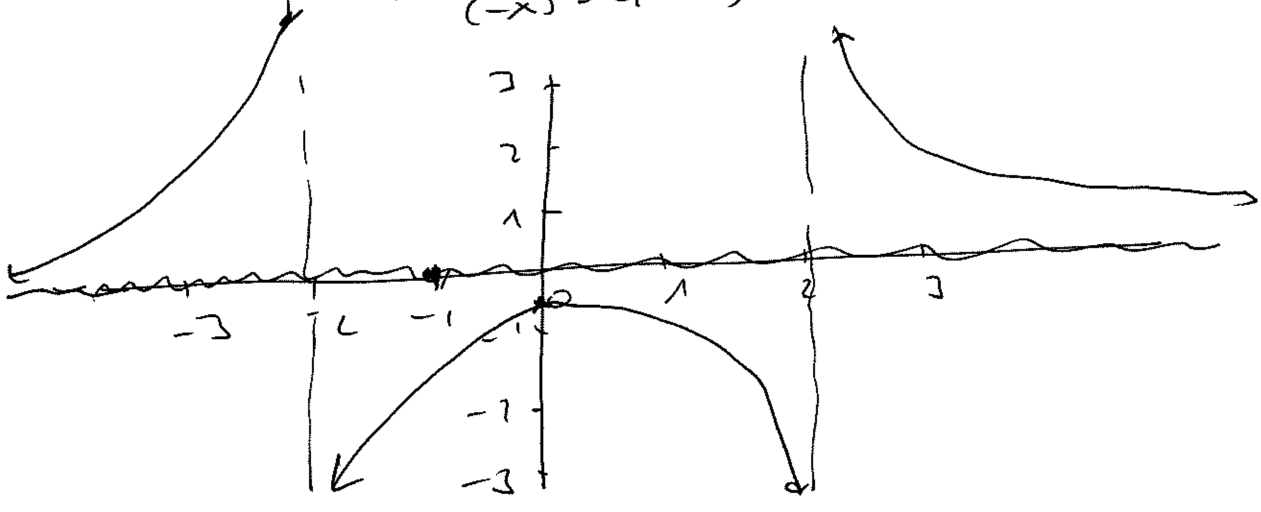
$-2 \downarrow$ A.V

$0 \downarrow$ Máx Relat

$2 \downarrow$ A.V

$(0, -\frac{3}{4})$ Máx Relat

v) Simetrías: $f(-x) = \frac{3}{(-x)^2-4} = \frac{3}{x^2-4} = f(x)$ PAR



1)

x	y	f	xf	yf	xxf	yyf	xyf
0	1	4	0	4	0	4	0
1	0	2	2	0	2	0	0
1	1	5	5	5	5	5	5
1	2	3	3	6	3	12	6
2	0	1	2	0	4	0	0
2	1	3	6	3	12	3	6
3	3	2	6	6	18	18	18
		20	24	24	44	42	35

Med x	1,2			Cov	0,31
Med y	1,2			r	0,43771
Var x	0,76			Fiab	19,1587
Var y	0,66				
		Y (x)	0,40789	0,71053	
Dt x	0,87178				
		X(y)	0,4697	0,63636	
Dt y	0,8124				

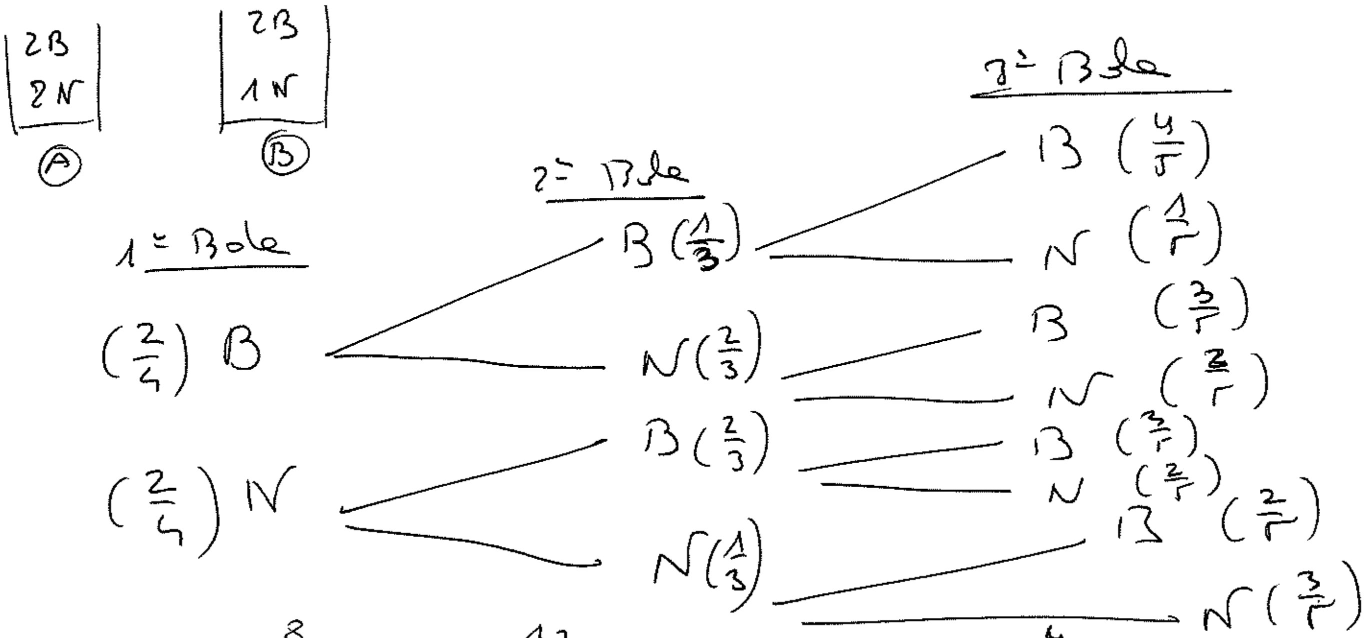
b) $y(4) = 0'4 \times 4 + 0'71 = 1'6 + 0'71 = \underline{\underline{2'31\ h}}$

2) $P(A) = \frac{3}{5} \cdot \frac{2}{4} = \frac{6}{20}$; $P(B) = \frac{3}{5} \cdot \frac{2}{4} = \frac{6}{20}$

$P(C_1') = P(P,I) + P(I,P) = \frac{2}{5} \cdot \frac{3}{4} + \frac{3}{5} \cdot \frac{2}{4} = \frac{12}{20}$

$P(A \cap B) = \frac{6}{20}$; $P(A \cup B) = \frac{6}{20} + \frac{6}{20} - \frac{6}{20} = \frac{6}{20}$

3)



a) $p(B) = \overbrace{\frac{4}{5}\cdot\frac{1}{3}\cdot\frac{2}{4}}^{8} + \overbrace{\frac{3}{5}\cdot\frac{2}{3}\cdot\frac{2}{4}}^{12} + \overbrace{\frac{3}{5}\cdot\frac{2}{3}\cdot\frac{2}{4}}^{12} + \overbrace{\frac{2}{5}\cdot\frac{1}{3}\cdot\frac{2}{4}}^{4} = \frac{36}{60}$

b) $p(\neq \text{color}) = p(B,N) + p(N,B) = \frac{2}{4}\cdot\frac{2}{3} + \frac{2}{4}\cdot\frac{2}{3} = \frac{8}{12}$

c) $p(3 = \text{color}) = \overbrace{\frac{4}{5}\cdot\frac{1}{3}\cdot\frac{2}{4}}^{8} + \overbrace{\frac{3}{5}\cdot\frac{1}{3}\cdot\frac{2}{4}}^{6} = \frac{14}{60}$

4) $n = 9$; $p = 0'85$; $q = 0'15$

a) $P(X=8) + P(X=9) = \binom{9}{8}(0'85)^8(0'15)^1 + \binom{9}{9}(0'85)^9 =$

$= 0'367 + 0'231 = \underline{0'598}$

b) $1 - [P(X=0) + P(X=1) + P(X=2)] =$

$1 - [P(X=0)+P(X=1)+P(X=2)] \ldots = 1 - [\ldots + \binom{9}{2}(0'85)^2(0'15)^7] =$

$= 1 - [\binom{9}{0}(0'85)^0(0'15)^9 + \binom{9}{1}(0'85)^1(0'15)^8 + \ldots$

$= 1 - [0'00\ldots + 0'\text{—} + 0'0004] \simeq 0'9999$

c) $1 - [P(X=9)] = 1 - \binom{9}{9}(0'85)^9 = \underline{0'768}$

5) a) $P(X>60) = P(Z \geq 0'62) = 1 - P(Z \leq 0'62) = 1 - 0'7324 = \underline{0'2676}$

$= 1 - P(Z \leq 1'75) = 1 - 0'8944 = \underline{0'1056}$

b) $P(X<30) = P(Z \leq -1'75) = 1 - P(Z \leq 1'75)$

c) $P(35 \leq X \leq 45) = P(-0'94 \leq X \leq -0'31) = P(Z \leq -0'31) - P(Z \leq -0'94) =$

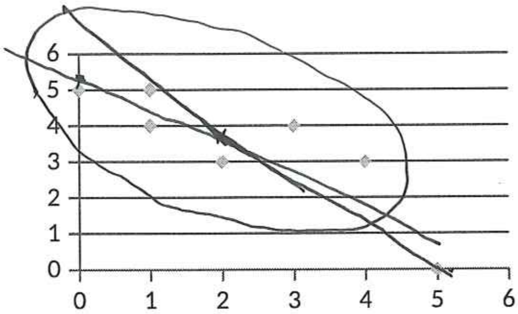
$= (1 - P(Z \leq 0'31)) - (1 - P(Z \leq 0'94)) = (1 - 0'6217) - (1 - 0'8264) = \underline{0'2047}$

$0'2047 \times 200 \simeq$ <u>41 servicios</u>

1) a)

x	y	f	xf	yf	xxf	yyf	xyf
0	5	2	0	10	0	50	0
1	4	3	3	12	3	48	12
1	5	3	3	15	3	75	15
2	3	5	10	15	20	45	30
3	4	5	15	20	45	80	60
4	3	1	4	3	16	9	12
5	0	1	5	0	25	0	0
		20	40	75	112	307	129

Med x	2		Cov	-1,05
Med y	3,75		r	-0,731
Var x	1,6		Fiab	53,51
Var y	1,2875			
		Y (x)	-0,6563	5,062
Dt x	1,26491			
		X(y)	-0,8155	5,058
Dt y	1,13468			



b) TEÓRICO

2)

	1	2	3	4	5	6
1	0	1	2	3	4	5
2	1	0	1	2	3	4
3	2	1	0	1	2	3
4	3	2	1	0	1	2
5	4	3	2	1	0	1
6	5	4	3	2	1	0

a) $p(A) = \frac{12}{36} = \frac{1}{3}$

$p(B) = \frac{12}{36} = \frac{1}{3}$

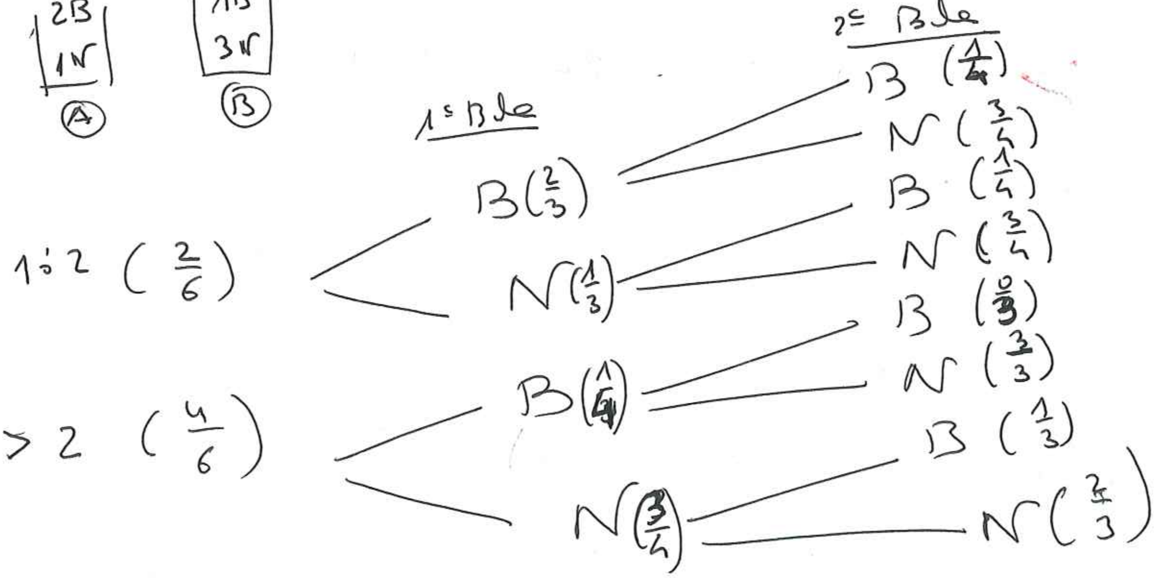
$p(A \cap B) = \frac{6}{36} = \frac{1}{6}$

$p(A \cup B) = \frac{18}{36} = \frac{1}{2}$

b) TEÓRICO

c) TEÓRICO

3) 2B 1N (A) 1B 3N (B)



a) $P(B,B) = \frac{1}{4} \cdot \frac{2}{3} \cdot \frac{2}{6} + \frac{0}{3} \cdot \frac{1}{4} \cdot \frac{4}{6} = \frac{4}{72}$

b) $P(\neq \text{color}) = \overbrace{\frac{3}{4} \cdot \frac{2}{3} \cdot \frac{2}{6}}^{12} + \overbrace{\frac{1}{4} \cdot \frac{1}{3} \cdot \frac{2}{6}}^{2} + \overbrace{\frac{3}{3} \cdot \frac{1}{4} \cdot \frac{4}{6}}^{12} + \overbrace{\frac{1}{3} \cdot \frac{3}{4} \cdot \frac{4}{6}}^{12} =$

$= \frac{38}{72}$

c) $P(\text{Par y mismo color}) = \overbrace{\frac{1}{6} \cdot \frac{2}{3} \cdot \frac{1}{4}}^{2} + \overbrace{\frac{1}{6} \cdot \frac{1}{3} \cdot \frac{3}{4}}^{3} +$

$+ \overbrace{\frac{2}{6} \cdot \frac{1}{4} \cdot \frac{0}{3}}^{0} + \overbrace{\frac{2}{6} \cdot \frac{3}{4} \cdot \frac{2}{3}}^{12} = \frac{17}{72}$

4) $n = 12$ $\quad p = \frac{30}{20} = 0'6$ (mujer)

$q = 0'4$ (hombre)

a) $1 - \left[\binom{12}{10}(0'6)^{10}(0'4)^2 + \binom{12}{11}(0'6)^{11}(0'4)^1 + \binom{12}{12}(0'6)^{12}(0'4)^0\right] =$

$= 1 - [0'063 + 0'017 + 0'002] = 1 - 0'082 = \underline{0'918}$

b) $\binom{12}{7}(0'6)^7(0'4)^3 + \binom{12}{6}(0'6)^6(0'4)^6 + \binom{12}{5}(0'6)^5(0'4)^7 =$

$= 0'227 + 0'176 + 0'100 = \underline{0'503}$

c) $1 - \left[\underbrace{\binom{12}{0}(0'6)^0(0'4)^{12}}_{0'0001} + \underbrace{\binom{12}{1}(0'6)^1(0'4)^{11}}_{0'0003}\right] = \underline{0'9993}$

5)
a) $P(X \geq 9) = P(Z \geq 2'12) = 1 - P(Z \leq 2'12) =$

$1 - 0'9830 = \underline{0'017}$

b) $P(X \leq 4) = P(Z \leq -1'88) = 1 - P(Z \leq 1'88) =$

$= 1 - 0'9699 = \underline{0'0301}$

c) $P(X \geq 5) = P(Z \geq -1'08) = P(Z \leq 1'08) = \underline{0'8599}$

$0'8599 \times 70 = 60'19 \simeq \underline{60 \text{ alumnos}}$

① x: nº de alumnos matriculados en 1º INF.

y: " " " " " 2º INF

z: " " " " " 3º INF

$$\left.\begin{aligned} x+y+z &= 180 \\ \frac{x}{z} &= \frac{5}{6} \\ x+y &= 2z \end{aligned}\right\} \longrightarrow \left(\begin{array}{ccc|c} 1 & 1 & 1 & 180 \\ 6 & 0 & -5 & 0 \\ 1 & 1 & -2 & 0 \end{array}\right) \overset{E_1-E_3}{\sim}$$

$$\sim \left(\begin{array}{ccc|c} 0 & 0 & 3 & 180 \\ 6 & 0 & -5 & 0 \\ 1 & 1 & -2 & 0 \end{array}\right) \overset{E_3/3}{\sim} \left(\begin{array}{ccc|c} 0 & 0 & 1 & 60 \\ 6 & 0 & -5 & 0 \\ 1 & 1 & -2 & 0 \end{array}\right) \overset{E_2+5E_1}{\sim}$$

$$\sim \left(\begin{array}{ccc|c} 0 & 0 & 1 & 60 \\ 6 & 0 & 0 & 300 \\ 1 & 1 & -2 & 0 \end{array}\right) \overset{E_2/6}{\sim} \left(\begin{array}{ccc|c} 0 & 0 & 1 & 60 \\ 1 & 0 & 0 & 50 \\ 1 & 1 & -2 & 0 \end{array}\right) \quad \boxed{\begin{aligned} z &= 60 \\ x &= 50 \end{aligned}}$$

$50 + y - 120 = 0$

$\boxed{y = 70}$

Hay 50 alumnos en 1º INF, 70 en 2º INF y 60 en 3º INF.

② a) $\dfrac{3x}{x-1} + \dfrac{x+1}{x} = \dfrac{3}{x^2-x} \Rightarrow \dfrac{3x^2+x^2-1}{x^2-x} = \dfrac{3}{x^2-x} \Rightarrow$

$\Rightarrow 4x^2 - 1 - 3 = 0 \Rightarrow 4(x^2-1) = 0 \rightarrow \cancel{x=1}$ Anula denominadores

$\rightarrow \boxed{x = -1}$

b) $P(x) = 3x^4 - 7x^3 - x^2 + 7x - 2 = \boxed{(3x-1)(x+1)(x-1)(x-2)}$

$$\begin{array}{r|rrrrr} & 3 & -7 & -1 & 7 & -2 \\ 1 & & 3 & -4 & -5 & 2 \\ \hline & 3 & -4 & -5 & 2 & \lfloor 0 \\ -1 & & -3 & 7 & -2 & \\ \hline & 3 & -7 & 2 & \lfloor 0 & \end{array} \qquad \begin{array}{r|rrr} & 3 & -7 & 2 \\ 2 & & 6 & -2 \\ \hline & 3 & -1 & \lfloor 0 \end{array}$$

③ $f(x) = \begin{cases} \dfrac{x+1}{x^2+x} & \text{si} \quad x < -1 \\ \dfrac{2x+3}{x} & \text{si} \quad \cancel{-}1 < x \leq 1 \\ ax+2 & \text{si} \quad x \geq 1 \end{cases}$

a) $\left.\begin{array}{l} f(1) = a+2 = \lim\limits_{x \to 1^+} f(x) \\ \lim\limits_{x \to 1^-} f(x) = 5 \end{array}\right\} \Rightarrow a+2 = 5 \Rightarrow \boxed{a = 3}$

b) * $\boxed{\text{En } x = -1}$ $\quad \nexists f(-1)$; $\lim\limits_{x \to -1^-} f(x) = \lim\limits_{x \to -1} \dfrac{x+1}{x^2+x} = \dfrac{0}{0}$ (IND) $=$

$= \lim\limits_{x \to -1} \dfrac{\cancel{x+1}}{x(\cancel{x+1})} = -1$; $\lim\limits_{x \to -1^+} f(x) = \lim\limits_{x \to -1} \dfrac{2x+3}{x} = \dfrac{1}{-1} = -1 \Rightarrow$

$\left.\begin{array}{l} \Rightarrow \exists \lim\limits_{x \to -1} f(x) = -1 \\ \nexists f(-1) \end{array}\right\} \Rightarrow$ $\boxed{\text{Discontinuidad evitable en } x = -1}$

* En $x = 0$: $\nexists f(0)$; $\lim\limits_{x \to 0^{\pm}} f(x) = \dfrac{3}{0^{\pm}} = \pm\infty$

$\boxed{\text{Discontinuidad inevitable de salto infinito.}}$

* En $x = 1$ $\quad f(1) = \lim\limits_{x \to 1^+} f(x) = 2 \neq \lim\limits_{x \to 1^-} f(x) = 5 \Rightarrow$

$\Rightarrow$ Discontinuidad inevitable de salto finito $=$

$|5-2| = \boxed{3}$

$\boxed{f(x) \text{ es continua } \forall x \in \mathbb{R} \setminus \{-1, 0, 1\}}$

④ $f(x) = ax^3 - bx^2 + 4$ con extremo relativo en $P(-1,3)$

a) $f(-1) = 3 \Rightarrow 3 = -a - b + 4 \Rightarrow a + b = 1$

$f'(-1) = 0 \Rightarrow 3a(-1)^2 - 2b(-1) = 0 \Rightarrow 3a + 2b = 0$

$\Rightarrow a = 1 - b \Rightarrow 3 - 3b + 2b = 0 \Rightarrow \boxed{b = 3 \Rightarrow a = -2}$

b) Para $a = 1 \wedge b = 3$ $\quad f(x) = x^3 - 3x^2 + 4$

$f'(x) = 3x^2 - 6x = 3x(x-2) \Rightarrow f'(x) = 0 \Rightarrow \boxed{\begin{array}{c} x = 0 \\ x = 2 \end{array}}$

	$-\infty$ a 0	0 a 2	2 a $+\infty$
$f'(x)$	+	−	+
	↗	↘	↗

$f(x)$ creciente en $(-\infty, 0) \cup (2, +\infty)$

$f(x)$ decreciente en $(0,2)$

$f''(x) = 6x - 6 \Rightarrow \begin{cases} f''(0) = -6 < 0 \text{ (MÁXIMO RELATIVO)} \\ f''(2) = 12 - 6 = 6 > 0 \text{ (MÍNIMO RELATIVO)} \end{cases}$

⑤ a) $\lim\limits_{x \to \infty} \left(\sqrt{x^2+5x+1} - \sqrt{x^2+4x}\right) = \infty - \infty$ (IND) $=$

$= \lim\limits_{x \to \infty} \dfrac{\left(\sqrt{x^2+5x+1} - \sqrt{x^2+4x}\right)\left(\sqrt{x^2+5x+1} + \sqrt{x^2+4x}\right)}{\sqrt{x^2+5x+1} + \sqrt{x^2+4x}} = \lim\limits_{x \to \infty} \dfrac{\cancel{x^2} + 5x + 1 - \cancel{x^2} - 4x}{\sqrt{x^2+5x+1} + \sqrt{x^2+4x}} =$

$= \lim\limits_{x \to \infty} \dfrac{\cancel{x}/\cancel{x} \overset{\nearrow 1}{} + \overset{\nearrow 0}{1/x}}{\underbrace{\sqrt{x^2/x^2 + 5x/x^2 + 1/x^2}}_{\sqrt{1}} + \underbrace{\sqrt{x^2/x^2 + 4x/x^2}}_{\sqrt{1}}} = \boxed{\dfrac{1}{2}}$

b) $\lim\limits_{x \to -\infty} \left(\dfrac{x^4 - 3x^2}{2x^4 + x + 3}\right)^{\frac{x^3+1}{7x}} = \left(\dfrac{1}{2}\right)^{+\infty} = \boxed{0}$

c) $\lim\limits_{x \to 2} \dfrac{\sqrt{x+2} - x}{x^2 - 5x + 6} = \dfrac{0}{0}$ IND $= \lim\limits_{x \to 2} \dfrac{(x+2) - x^2}{(x-2)(x-3)(\sqrt{x+2} + x)} =$

$= \lim\limits_{x \to 2} \dfrac{-(x+1)\cancel{(x-2)}}{\cancel{(x-2)}(x-3)(\sqrt{x+2} + x)} = \dfrac{-3}{(-1)(4)} = \boxed{\dfrac{3}{4}}$

6) $f(x) = \dfrac{x}{x^2-4}$; $Dom(f) = \mathbb{R} \setminus \{-2, 2\}$

$(x^2-4 = 0 \Rightarrow x = \pm 2)$

Cortes con ejes de coordenadas:

* Con eje de ordenadas $(x=0) \Rightarrow f(0)=0$
* Con eje de abscisas $(y=0) \Rightarrow x=0$

$\Rightarrow$ Con $O=(0,0)$ (origen de coordenadas)

Simetría: $f(-x) = \dfrac{(-x)}{(-x)^2-4} = -\dfrac{x}{x^2-4} = -f(x) \Rightarrow$

$\Rightarrow f(x)$ es una función impar (simétrica respecto al origen de coordenadas)

Signo de $f(x)$:

	$-\infty$ a -2	-2 a 0	0 a 2	2 a $+\infty$
$f(x)$	$-$	$+$	$-$	$+$

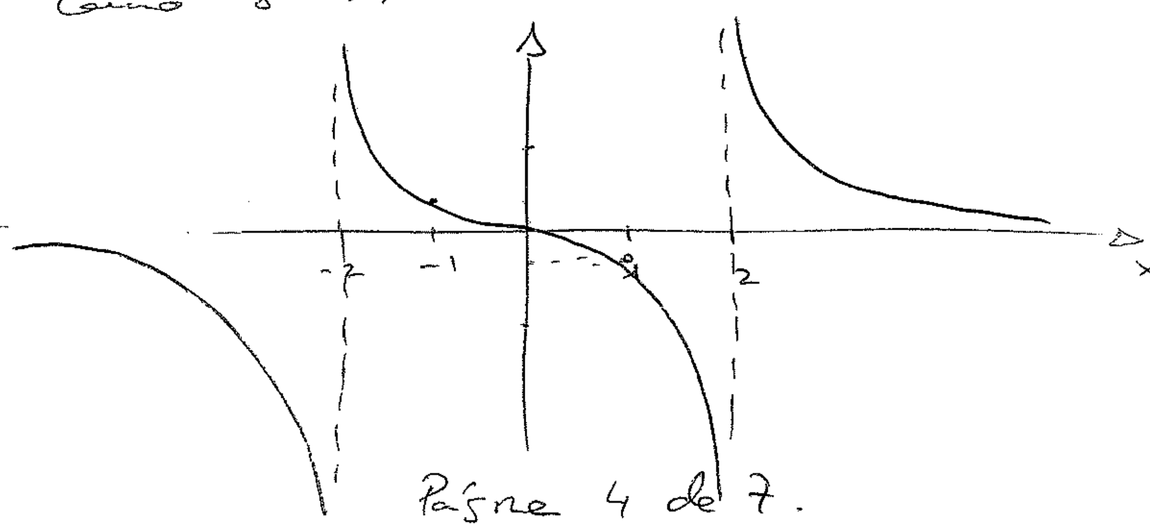
Asíntotas: Verticales:

* $\lim_{x \to -2^{\pm}} f(x) = \dfrac{-2}{0^{\mp}} = \pm\infty \Rightarrow \boxed{x=-2}$
* $\lim_{x \to 2^{\pm}} f(x) = \pm\infty \longrightarrow \boxed{x=2}$

A. Horizontal: $\lim_{x \to \pm\infty} f(x) = 0^{\pm} \Rightarrow \boxed{y=0}$

Crecimiento: $f'(x) = \dfrac{x^2-4-x(2x)}{(x^2-4)^2} = -\dfrac{x^2+4}{(x^2-4)^2} < 0$

$f(x)$ es decreciente en todo su dominio.
Como $f'(x) \neq 0 \Rightarrow \nexists$ extremos relativos



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x_i	y_j	f_{ij}	$f_{ij}x_i$	$f_{ij}y_j$	$f_{ij}x_i^2$	$f_{ij}y_j^2$	$f_{ij}x_i y_j$
1	9	1	1	9	1	81	9
2	9	4	8	36	16	324	72
3	8	1	3	8	9	64	24
5	5	2	10	10	50	50	50
5	7	2	10	14	50	98	70
6	6	4	24	24	144	144	144
7	5	1	7	5	49	25	35
9	1	2	18	2	162	2	18
10	2	3	30	6	300	12	60
Σ		20	111	114	781	800	482

a) $\bar{x} = \frac{111}{20} = 5{,}55$

$\bar{y} = \frac{114}{20} = 5{,}7$

$\sigma_x^2 = \frac{781}{20} - (5{,}55)^2 = 8{,}2475$

$\boxed{\sigma_x = 2{,}872}$

$\sigma_y^2 = \frac{800}{20} - 5{,}7^2 = 7{,}51$

$\sigma_y = \sqrt{7{,}51} = 2{,}74$

$\sigma_{xy} = \frac{482}{20} - 5{,}55 \cdot 5{,}7 = -7{,}535$

$\boxed{r =} \frac{-7{,}535}{2{,}872 \cdot 2{,}74} = \boxed{-0{,}957}$

b) Recta regresión de y sobre x:

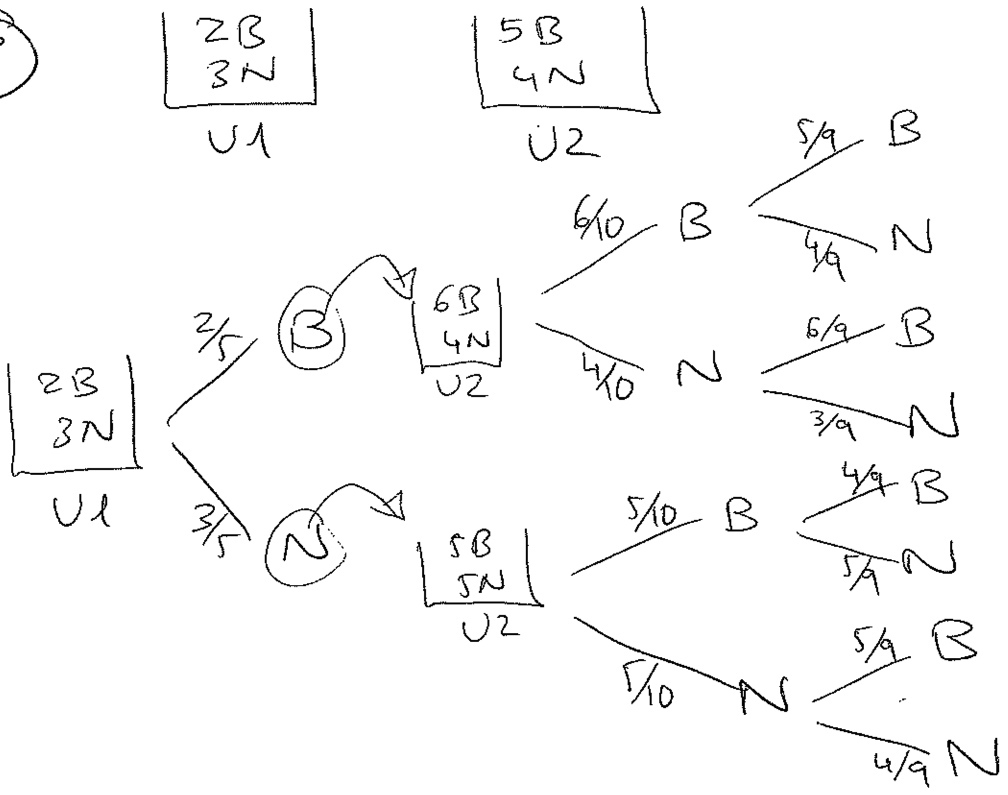
$y = 5{,}7 - \frac{7{,}535}{8{,}2475}(x - 5{,}55) = 5{,}7 - 0{,}9136x + 5{,}07$

$= 0{,}9136 \Longrightarrow \boxed{y = 10{,}77 - 0{,}9136x}$

$\hat{y}(x=4) = 10{,}77 - 0{,}9136(4) = \boxed{7{,}116}$

$r^2 \cdot 100 =$ 91,67% de fiabilidad de las estimaciones

8



a) P("Dos blancas") $= \frac{2}{5} \cdot \frac{6}{10} \cdot \frac{5}{9} + \frac{3}{5} \cdot \frac{5}{10} \cdot \frac{4}{9} =$

$= \frac{60}{450} + \frac{60}{450} = \frac{120}{450} = \frac{12}{45} = \boxed{\frac{4}{15}}$

b) P("Distinto color") = 1 − P("Mismo color") =

$= 1 - \left(\frac{4}{15} + \underbrace{\frac{2}{5} \cdot \frac{4}{10} \cdot \frac{3}{9}}_{\frac{24}{450}} + \underbrace{\frac{3}{5} \cdot \frac{5}{10} \cdot \frac{4}{9}}_{\frac{60}{450}} \right) =$

$= 1 - \frac{204}{450} = \frac{450-204}{450} = \frac{246}{450} = \frac{123}{225} = \boxed{\frac{41}{75}}$

(9) $B(n,p) = B(4;\ 0{,}05)$

a) $P(X=4) = \binom{4}{4}(0{,}05)^4 \cdot (0{,}95)^0 = \boxed{6{,}25 \cdot 10^{-6}}$

b) $P(X \geq 1) = 1 - P(X=0) = 1 - \binom{4}{0} \cdot (0{,}05)^0 \cdot (0{,}95)^4 =$

$= \boxed{0{,}1855}$

(10) $N(\mu,\sigma) = N(20;\ 2{,}5)$

a) $P(X > 25) = P\left(Z > \frac{25-20}{2{,}5}\right) = P(Z > 2) =$

$= 1 - P(Z \leq 2) = 1 - 0{,}9772 = \boxed{0{,}0228}$

b) $P(18 < X < 23) = P(-0{,}8 < Z < 1{,}2) =$

$= P(Z \leq 0{,}8) + P(Z \leq 1{,}2) - 1 =$

$= 0{,}7881 + 0{,}8849 - 1 = \boxed{0{,}673}$