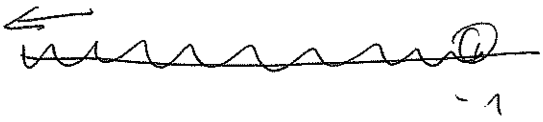
1) a) TEÓRICA

b) i) $\{x\in\mathbb{R} \mid x<-1\}$ 

ii) $[-1/2, 1/2]$

iii) $(-1'01, -0'99)$

iv) $E(-0'3, 0'4)$

2) a) $3\cdot 2\sqrt{10} - 10\sqrt{10} + (2\sqrt{3}+6\sqrt{2})^2 -$

$- (2\sqrt{6}-5\sqrt{6})^2 = 6\sqrt{10} - 10\sqrt{10} + 12 + 72 + 24\sqrt{6} -$

$- 54 = \underline{\underline{-4\sqrt{10} + 24\sqrt{6} + 30}}$

b) $$\frac{2\sqrt{3}}{2\sqrt{12}-\sqrt{3}-3} = \frac{2\sqrt{3}}{4\sqrt{3}-\sqrt{3}-3} = \frac{2\sqrt{3}}{3\sqrt{3}-3} = \frac{2\sqrt{3}(3\sqrt{3}+3)}{(3\sqrt{3}-3)(3\sqrt{3}+3)} =$$

$$= \frac{2\sqrt{3}(3\sqrt{3}+3)}{27-9} = \frac{2\sqrt{3}(3\sqrt{3}+3)}{18}$$

$$\frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{\sqrt{3}\sqrt{3}} = \frac{\sqrt{3}}{3}$$

$$\underline{\frac{2\sqrt{3}(3\sqrt{3}+3)}{18}} - \frac{\sqrt{3}}{3} = \frac{18+6\sqrt{3}-6\sqrt{3}}{18} = \frac{18}{18} = 1 //$$

3) a)
$$\begin{array}{c|ccccc}
 & 1 & -3 & 0 & 1 & m \\
\sqrt{2} & & \sqrt{2} & -3\sqrt{2}+2 & -6+2\sqrt{2} & -5\sqrt{2}+4 \\
\hline
 & 1 & -3+\sqrt{2} & -3\sqrt{2}+2 & -5+2\sqrt{2} & \boxed{-5\sqrt{2}+4+m}
\end{array}$$

$-5\sqrt{2}-4+m=0 \Rightarrow \underline{\underline{m=-4+5\sqrt{2}}}$

b) $P(-\frac{1}{2}) = \left(-\frac{1}{2}\right)^4 - 3\left(-\frac{1}{2}\right)^3 + \left(-\frac{1}{2}\right) + m =$

$= \frac{1}{16} - 3\left(-\frac{1}{8}\right) + \left(-\frac{1}{2}\right) + m =$

$= \frac{1}{16} + \frac{3}{8} - \frac{1}{2} + m = \frac{1+6-8+16m}{16} = 0$

$1+6-8+16m=0$; $16m = 1$; $\boxed{m = \frac{1}{16}}$

4) $P(x) = 3x^5 - 5x^4 - 16x^3 + 12x^2 = x^2(3x^3 - 5x^2 - 16x + 12)$

$= \boxed{3x^2(x-3)(x+2)(x-2/3)}$

$$\begin{array}{c|cccc}
 & 3 & -5 & -16 & 12 \\
3 & & 9 & 12 & -12 \\
\hline
 & 3 & 4 & -4 & \boxed{0}
\end{array}$$

$3x^2+4x-4=0$; $x = \frac{-4 \pm \sqrt{16+48}}{6} = \frac{-4 \pm 8}{6} =$ $\frac{4}{6} = \frac{2}{3}$, $\frac{-12}{6} = -2$

5) $\frac{5}{x^2-x} = \frac{5}{x(x-1)}$

$\frac{10}{x^2-x-2} = \frac{10}{(x+1)(x-2)}$

$\frac{5}{x^2-x} : \frac{10}{x^2-x-2} = \frac{5(x+1)(x-2)}{10x(x-1)}$

$\frac{x}{x-2} \cdot \left(\frac{5}{x^2-x} : \frac{10}{x^2-x-2}\right) = \frac{5\cancel{x}(x+1)\cancel{(x-2)}}{10\cancel{x}(x-1)\cancel{(x-2)}} =$

$= \boxed{\frac{(x+1)}{2(x-1)}}$

1) a) $\frac{3\sqrt{3}+1}{5\sqrt{3}-2\sqrt{3}} - \frac{2\sqrt{2}}{3\sqrt{2}-\sqrt{3}} = \frac{3\sqrt{3}+1}{3\sqrt{3}} - \frac{2\sqrt{2}}{3\sqrt{2}-\sqrt{3}}$

$$\frac{3\sqrt{3}+1}{3\sqrt{3}} = \frac{(3\sqrt{3}+1)\sqrt{3}}{3\sqrt{3}\sqrt{3}} = \frac{9+\sqrt{3}}{9}$$

$$\frac{2\sqrt{2}}{3\sqrt{2}-\sqrt{3}} = \frac{2\sqrt{2}(3\sqrt{2}+\sqrt{3})}{(3\sqrt{2}-\sqrt{3})(3\sqrt{2}+\sqrt{3})} = \frac{12+2\sqrt{6}}{18-3} = \frac{12+2\sqrt{6}}{15}$$

$$\frac{9+\sqrt{3}}{9} - \frac{12+2\sqrt{6}}{15} = \frac{45+5\sqrt{3}}{45} - \frac{36+6\sqrt{6}}{45} =$$

$$= \frac{45+5\sqrt{3}-36-6\sqrt{6}}{45} = \boxed{\frac{9+5\sqrt{3}-6\sqrt{6}}{45}}$$

b)

$$\begin{array}{c|cccc} & 1 & -7 & 15 & -9 \\ 1 & & 1 & -6 & 9 \\ \hline & 1 & -6 & 9 & \lfloor 0 \end{array}$$

$x^2-6x+9=0$; $x = \frac{6\pm\sqrt{36-36}}{2} = \frac{6\pm 0}{2} = \Big\langle \begin{matrix} 3 \\ 3 \end{matrix}$

$$\begin{array}{c|cccc} & 1 & -5 & 3 & 9 \\ -1 & & -1 & 6 & -9 \\ \hline & 1 & -6 & 9 & \lfloor 0 \end{array}$$

$x^2-6x+9=0$; $x = \Big\langle \begin{matrix} 3 \\ 3 \end{matrix}$

$$\frac{(x-1)\cancel{(x-3)^2}}{(x+1)\cancel{(x-3)^2}} = \boxed{\frac{x-1}{x+1}}$$

2) a)

$$\begin{array}{r|rrrrr} & 1 & 2 & -13 & -14 & 24 \\ 1 & & 1 & 3 & -10 & 24 \\ \hline & 1 & 3 & -10 & -24 & \lfloor 0 \\ 3 & & 3 & 18 & 24 & \\ \hline & 1 & 6 & 8 & \lfloor 0 & \end{array}$$

$x^2+6x+8=0$; $x = \frac{-6 \pm \sqrt{36-32}}{2} = \frac{-6 \pm 2}{2} = \left\langle \begin{array}{l} -2 \\ -4 \end{array} \right.$

$p(x) = (x-1)(x+2)(x-3)(x+4)$

b) i)

$$\begin{array}{r|rrrrr} & 2 & 0 & -m & 0 & 1 \\ -3 & & -6 & 18 & -54+3m & 162-9m \\ \hline & 2 & -6 & 18-m & -54+3m & \lfloor 163-9m \end{array}$$

$163-9m = 136$; $-9m = 136-163 = -27$

$m = \frac{-27}{-9} = 3$ //

ii) $2x^4 - 3x^2 + 1 = 0$; $x^2 = t$

$2t^2 - 3t + 1 = 0$; $t = \frac{3 \pm \sqrt{9-8}}{4} = \frac{3 \pm 1}{4} = \left\langle \begin{array}{l} 1 \\ \frac{1}{2} \end{array} \right.$

$x = \pm\sqrt{1}$; $x = \pm\sqrt{\frac{1}{2}}$; Sol: $1, -1, \sqrt{\frac{1}{2}}, -\sqrt{\frac{1}{2}}$

3) a) $\left.\begin{array}{r} x^2+2x-y=7 \\ 4x-y-7=0 \end{array}\right\} \rightarrow y = 4x-7$

$x^2+2x-(4x-7)=7$; $x^2+2x-4x+7-7=0$

$x^2-2x=0$; $x(x-2)=0 \left\langle \begin{array}{l} x=0 \\ x-2=0 \rightarrow x=2 \end{array} \right.$

$x_1 = 0 \Rightarrow y_1 = -7 \rightarrow (0,-7)$

$x_2 = 2 \Rightarrow y_2 = 8-7 = 1 \rightarrow (2,1)$

b) $\sqrt{3x+10} = 6-x$; $3x+10=(6-x)^2$

$3x+10 = 36+x^2-12x$

$0 = 36+x^2-12x-3x-10$; $0 = x^2-15x+26$

$$x = \frac{15 \pm \sqrt{225-104}}{2} = \frac{15 \pm 11}{2} = \begin{cases} \boxed{13} \rightarrow \text{no} \\ \boxed{2} \rightarrow \text{sí} \end{cases}$$

$13 \rightarrow \sqrt{49} = 6-13$ — no

$2 \rightarrow \sqrt{16} = 6-2$ — sí

4) Mayor → x
Mediano → y
Menor → z

$$\left.\begin{aligned} x+y+z&=15 \\ z+1&=(y+1)/2 \\ x-2&=2(y-2) \end{aligned}\right\}$$

$$\left.\begin{aligned} x+y+z&=15 \\ 2z+2&=y+1 \\ x-2&=2y-4 \end{aligned}\right\} \quad \left.\begin{aligned} x+y+z&=15 \\ -1y+2z&=-1 \\ x-2y\quad&=-2 \end{aligned}\right\}$$

$$\left(\begin{array}{ccc|c} 1 & 1 & 1 & 15 \\ 0 & -1 & 2 & -1 \\ 1 & -2 & 0 & -2 \end{array}\right) \xrightarrow[\rightarrow F3]{F3-F1} \left(\begin{array}{ccc|c} 1 & 1 & 1 & 15 \\ 0 & -1 & 2 & -1 \\ 0 & -3 & -1 & -17 \end{array}\right) \xrightarrow[\rightarrow F3]{-3F2+F3}$$

$$\left(\begin{array}{ccc|c} 1 & 1 & 1 & 15 \\ 0 & -1 & 2 & -1 \\ 0 & 0 & -7 & -14 \end{array}\right) \quad \left.\begin{aligned} x+y+z&=15 \\ -1y+2z&=-1 \\ -7z&=-14 \end{aligned}\right\} \rightarrow z = \frac{-14}{-7} = 2 \,//$$

$-3y+2=-1$; $-y+4=-1$. $y=5 \,//$

$x+5+2=15$; $x=15-5-2=8 \,//$

$$\boxed{(8,5,2)}$$

5) $\frac{2x-3}{x+2} - 1 \geq 0$; $\frac{2x-3}{x+2} - \frac{x+2}{x+2} \geq 0$

$\frac{2x-3-x-2}{x+2} \geq 0$; $\frac{x-5}{x+2} \geq 0$

$x-5=0 \rightarrow x=5$

$x+2=0 \rightarrow x=-2$

$-3 \longrightarrow \frac{-3-5}{-3+2} = \frac{-8}{-1} = 8 \geq 0$

$0 \longrightarrow \frac{0-5}{0+2} = -\frac{5}{2} \ngeq 0$

$6 \longrightarrow \frac{6-5}{6+2} = \frac{1}{8} \geq 0$

$S_1 = (-\infty, -2) \cup [5, +\infty)$

$\frac{2x-2}{3} - \frac{3x+3}{2} < 1 - 4x$

$\frac{4x-4}{6} - \frac{9x+9}{6} < \frac{6}{6} - \frac{24x}{6}$

$4x-4-9x-9 < 6-24x$

$4x-9x+24x < 6+4+9$

$19x < 19$; $x < \frac{19}{19}$; $x < 1$. $S_2 = (-\infty, 1)$

S. sistema $= (-\infty, -2)$

1) a) $\frac{2\sqrt2}{3\sqrt2-\sqrt3} = \frac{2\sqrt2(3\sqrt2+\sqrt3)}{(3\sqrt2-\sqrt3)(3\sqrt2+\sqrt3)} = \frac{12+2\sqrt6}{18-3} = \frac{12+2\sqrt6}{15}$

$$\frac{\sqrt3}{2\sqrt3+\sqrt2} = \frac{\sqrt3(2\sqrt3-\sqrt2)}{(2\sqrt3+\sqrt2)(2\sqrt3-\sqrt2)} = \frac{6-\sqrt6}{12-2} = \frac{6-\sqrt6}{10}$$

$$\frac{\sqrt2}{2\sqrt3} = \frac{\sqrt2\sqrt3}{2\sqrt3\sqrt3} = \frac{\sqrt6}{6}$$

$$\frac{12+2\sqrt6}{15} + \frac{6-\sqrt6}{10} - \frac{\sqrt6}{6} = \frac{24+4\sqrt6+18-3\sqrt6-5\sqrt6}{30} =$$

$$= \boxed{\frac{42-4\sqrt6}{30}} = \boxed{\frac{21-2\sqrt6}{15}}$$

b) $\left(\frac{x^2-x+x}{x-1}\right) : \left(\frac{x^2-x-x}{x-1}\right) - \frac{x}{x+2} =$

$$= \frac{x^2}{x-1} : \frac{x^2-2x}{x-1} - \frac{x}{x+2} =$$

$$= \frac{x^2}{x^2-2x} - \frac{x}{x+2} = \frac{x^{\cancel{2}}}{\cancel{x}(x-2)} - \frac{x}{x+2} =$$

$$= \frac{x}{x-2} - \frac{x}{x+2} = \frac{x(x+2)-x(x-2)}{(x-2)(x+2)} =$$

$$= \frac{\cancel{x^2}+2x-\cancel{x^2}+2x}{(x-2)(x+2)} = \boxed{\frac{4x}{(x+2)(x-2)}}$$

2) b)

$$\begin{array}{r|rrrrr}
 & 1 & -1 & -14 & 12 & 24 \\
-1 & & -1 & 2 & 12 & -24 \\
\hline
 & 1 & -2 & -12 & 24 & \boxed{0} \\
2 & & 2 & 0 & -24 & \\
\hline
 & 1 & 0 & -12 & \boxed{0} &
\end{array}$$

$x^2-12=0$, $x = \pm\sqrt{12} = \pm 2\sqrt3$

$$\boxed{p(x) = (x+1)(x-2)(x-\sqrt{12})(x+\sqrt{12})}$$

2) a)

$$\begin{array}{r|rrrrl} & 3 & 9 & -3 & m & -18 \\ -2 & & -6 & -6 & 18 & -2m-36 \\ \hline & 3 & 3 & -9 & m+18 & \boxed{-2m-54} \end{array}$$

$-2m-54=0$; $-2m=54$; $m=\frac{54}{-2}=-27$ //

$$\begin{array}{r|rrrrr} & 3 & 9 & -3 & -27 & -18 \\ -2 & & -6 & -6 & 18 & 18 \\ \hline & 3 & 3 & -9 & -9 & \boxed{0} \\ -1 & & -3 & 0 & 9 & \\ \hline & 3 & 0 & -9 & \boxed{0} & \end{array}$$

$3x^2-9=0$; $3x^2=9$; $x^2=\frac{9}{3}=3$; $x=\pm\sqrt{3}$

$$\underline{\underline{P(x)=3(x+1)(x+2)(x-\sqrt{3})(x+\sqrt{3})}}$$

3) a) $\sqrt{2x-3}=x-1$; $2x-3=(x-1)^2$

$2x-3=x^2+1-2x$; $0=x^2+1-2x-2x+3$

$0=x^2-4x+4$ $\boxed{x=2}$

$$x=\frac{4\pm\sqrt{16-16}}{2}=\frac{4\pm 0}{2}=\left\langle \begin{matrix} 2 \\ 2 \end{matrix}\right.$$

Comprob. $\sqrt{2.2-3}=2-1$

$\sqrt{4-3}=2-1$

$\sqrt{1}=1 \rightarrow$ Sí

b) $\left.\begin{array}{l} x^2+4x-y=2 \\ -x-y-2=0 \end{array}\right\} \to -x-2=y$

$x^2+4x-(-x-2)=2$; $x^2+4x+x+2=2$

$x^2+4x+x+2-2=0$

$x^2+5x=0$; $x(x+5)=0 \begin{array}{l} \to x=0 // \\ \to x+5=0 \to x=-5 // \end{array}$

$x_1=0 \longrightarrow y_1=-2 \longrightarrow \boxed{(0,-2)}$

$x_2=-5 \longrightarrow y_2=5-2=3 \longrightarrow \boxed{(-5,3)}$

4) Mayor → x
Mediano → y
Pequeño → z

$\left.\begin{array}{l} x+y+z=50 \\ 2y=x+5 \\ \frac{\cancel{z}}{2}=\frac{x}{5} \end{array}\right\}$

$\left.\begin{array}{l} x+y+z=50 \\ -x+2y=5 \\ -2x+5z=0 \end{array}\right\}$ $\left(\begin{array}{ccc|c} 1 & 1 & 1 & 50 \\ -1 & 2 & 0 & 5 \\ -2 & 0 & 5 & 0 \end{array}\right)$ $\begin{array}{l} F1+F2 \to F2 \\ 2F1+F3 \to F3 \end{array}$

$\left(\begin{array}{ccc|c} 1 & 1 & 1 & 50 \\ 0 & 3 & 1 & 55 \\ 0 & 2 & 7 & 100 \end{array}\right) \begin{array}{l} -2F2+3F3 \\ \to F3 \end{array} \left(\begin{array}{ccc|c} 1 & 1 & 1 & 50 \\ 0 & 3 & 1 & 55 \\ 0 & 0 & 19 & 190 \end{array}\right)$

$x+15+10=50 \longrightarrow x=50-15-10=25$

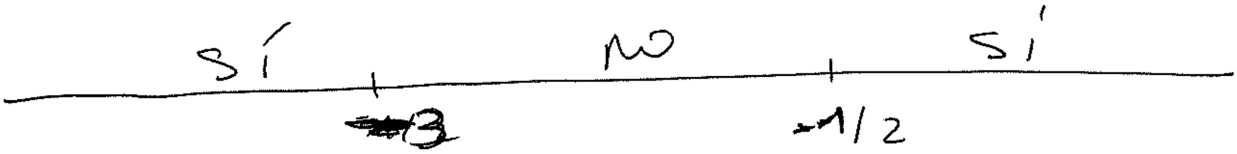
$\left.\begin{array}{l} x+y+z=50 \\ 3y+z=55 \\ 19z=190 \end{array}\right\} \begin{array}{l} \to 3y+10=55 \to 3y=45; \ y=\frac{45}{3}=15 \\ \\ \to z=\frac{190}{19}=10 \end{array}$

$\boxed{(25,15,10)}$

5) $\frac{3x-1}{2x+1} - 2 < 0$; $\frac{3x-1-4x-2}{2x+1} < 0$

$\frac{-x-3}{2x+1} < 0$ $-x-3=0$; $x = -3$ //

$2x+1=0$; $x = -\frac{1}{2}$ //



$-4 \longrightarrow \frac{4-3}{-8+1} = \frac{1}{-7} < 0$

$-1 \longrightarrow \frac{1-3}{-2+1} = \frac{-2}{-1} \not< 0$

$0 \longrightarrow \frac{-0-3}{0+1} = \frac{-3}{1} < 0$

$S_1: (-\infty, -3) \cup (-1/2, +\infty)$

$x^2 - 3x \geq 0$; $x(x-3) \geq 0$; $x(x-3) = 0 \begin{cases} x=0 \\ x=3 \end{cases}$

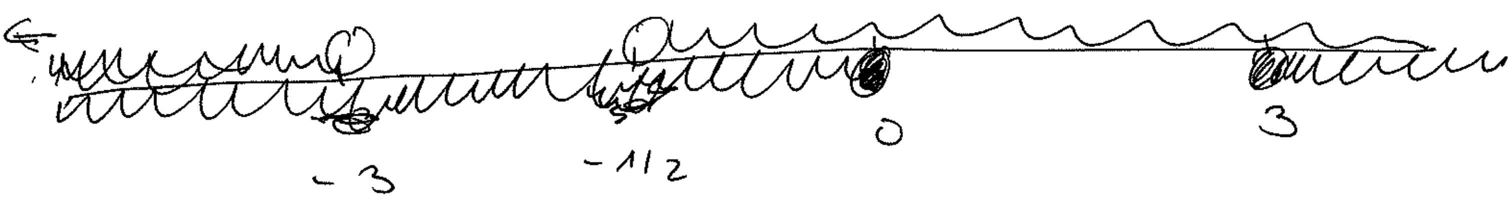
$-1 \longrightarrow 1+3=4 \geq 0$

$1 \longrightarrow 1-3=-2 \ngeq 0$

$4 \longrightarrow 16-12=4 \geq 0$

$S_2: (-\infty, 0] \cup [3, +\infty)$

Solución global: $\boxed{(-\infty,-3) \cup (-1/2, 0] \cup [3, +\infty)}$



1)

a) $D_f = \mathbb{R} \setminus \{-4\}$

$D_g = \mathbb{R}$

$D_h = \mathbb{R} \setminus \{-1\}$

b) $\frac{x^3-x}{x+4} = -1$; $x^3 - x = -x-4$; $x^3 = -4$

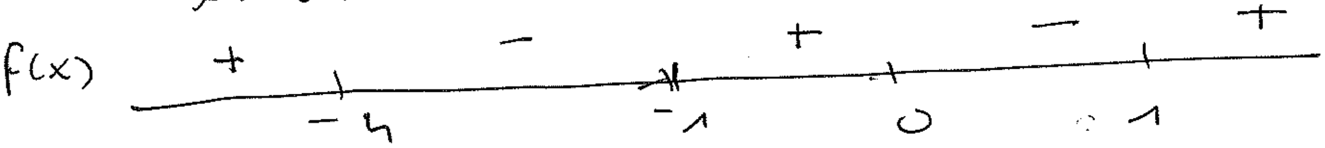
Sí pertenece

$\sqrt[3]{\frac{3-x}{x+1}} = -1$; $\frac{3-x}{x+1} = -1$; $3-x = -x-1$

No pertenece

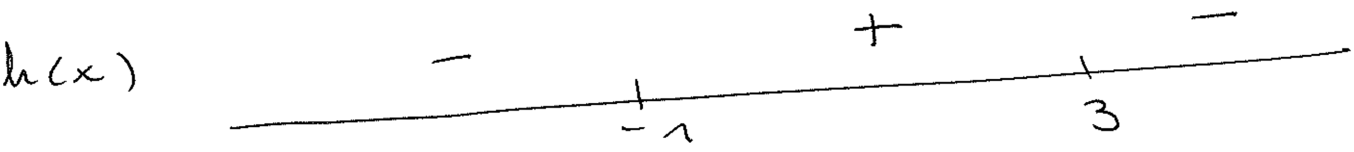
c) $x^3 - x = 0 \rightarrow x(x^2-1) = 0$ $\rightarrow x = 0$, $x = -1$, $x = 1$

$x+4 = 0 \rightarrow x = -4$



$3-x = 0 \rightarrow x = 3$

$x+1 = 0 \rightarrow x = -1$



d) $f(-x) = \frac{(-x^3)-(-x)}{(-x)+4} = \frac{-x^3+x}{-x+4}$ No tiene

$g(-x) = (-x)^2 - 3 = x^2 - 3 \rightarrow$ PAR

e) $\frac{x^3-x}{x+4} = x^2-3$; $x^3-x = (x^2-3)(x+4)$

$x^3-x = x^3+4x^2-3x-12$; $0 = 4x^2-2x-12$

$$x = \frac{2 \pm \sqrt{4+192}}{8} = \frac{2 \pm 14}{8} = \begin{cases} \frac{16}{8} = 2 \\ -\frac{12}{8} = -\frac{3}{2} \end{cases}$$

$I_1(2, 1)$ $I_2\left(-\frac{3}{2}, -\frac{3}{4}\right)$

2) i)

$$f(x) = \begin{cases} 2x+8 & \text{si } x \leq -3 \\ -x^2+4 & \text{si } -3 < x < 1 \\ x^2-x-2 & \text{~~si } x \geq 1\text{~~} \end{cases}$$

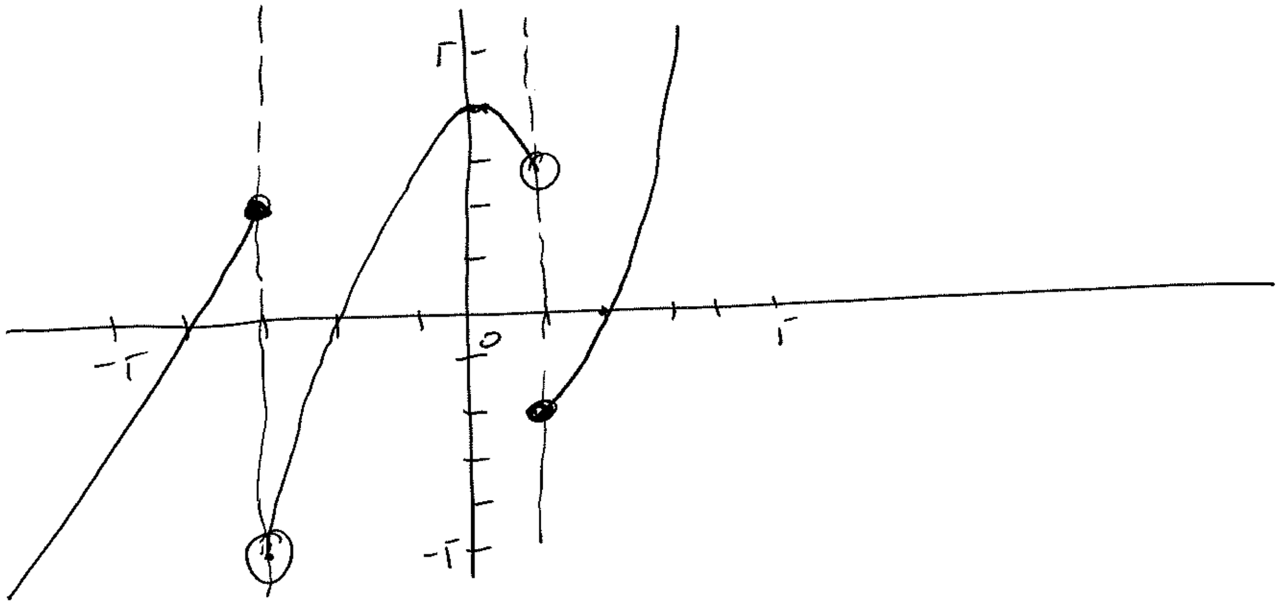
x	-4	-3
y	0	2

x	-3	1
y	-5	3

V(0, 4) Máx

x	1	2
y	-2	0

V(0'5, -2'25)



ii) Discontinua en $x = -3$ y $x = 1$

iii) Creciente: $(-\infty, -3) \cup (-3, 0) \cup (1, +\infty)$

Decreciente: $(0, 1)$

iv) Corte eje OX: $2x+8=0 \to x=-4$; $(-4, 0)$

$-x^2+4=0 \to x = \begin{cases} -2 \to (-2, 0) \\ \cancel{2} \end{cases}$

Corte eje OY: $(0, 4)$ $x^2-x-2=0 \to x = \begin{cases} \cancel{-1} \\ 2 \to (2, 0) \end{cases}$

②

3) a) $x^2-4x=0$; $x(x-4)=0$ $\begin{cases} \to x=0 \text{ //} \\ \to x=4 \text{ //} \end{cases}$

	+		−		+
		0		4	

$$f(x)=\begin{cases} x^2-4x & \text{si } x<0 \\ -x^2+4x & \text{si } 0\leq x\leq 4 \\ x^2-4x & \text{si } 4<x \end{cases}$$

x	-1	0
y	5	0

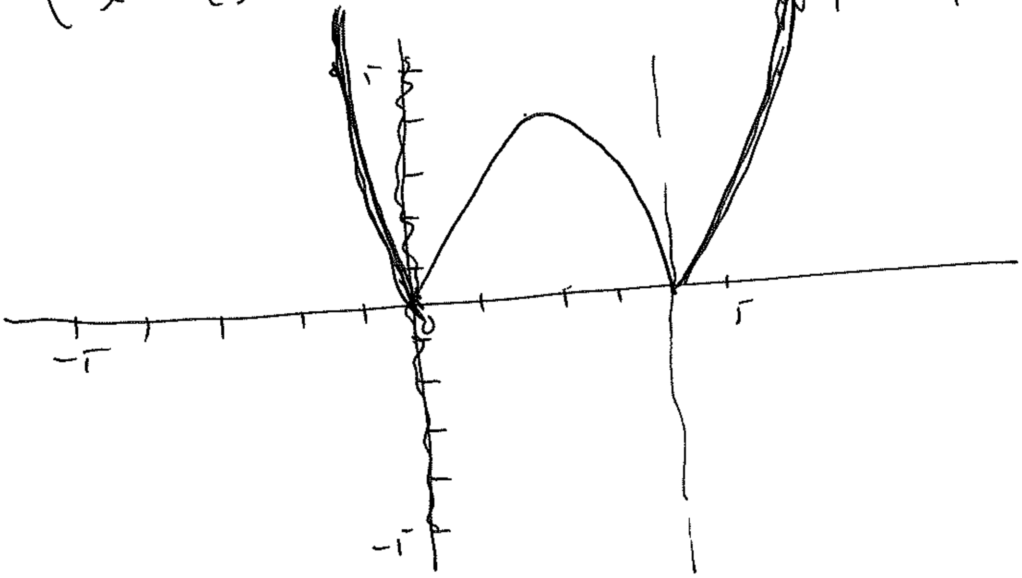
$V(2,-4)$ mín

x	0	4
y	0	0

$V(2,4)$ máx

x	4	5
y	0	5

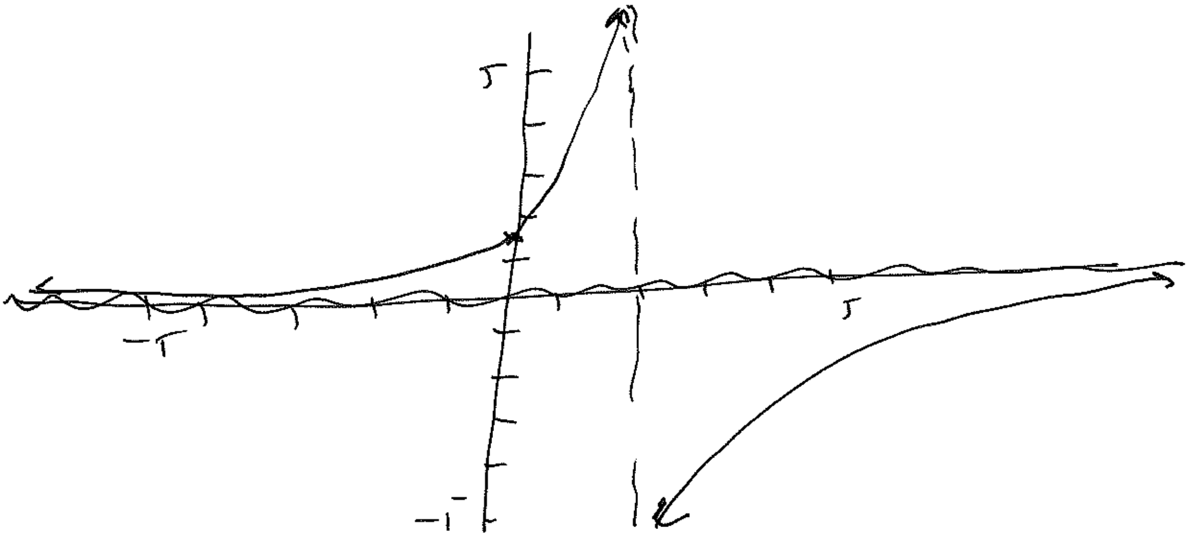
$V(2,-4)$



b) $g(x)=\dfrac{3}{2-x}$

Corte ejes $\begin{cases} OX: 3=0 \to \text{No corta} \\ OY: (0, 3/2) \end{cases}$

Asíntotas $\begin{cases} V: \boxed{\text{Denom}=0}\ 2-x=0; \quad \underline{\underline{x=2}} \\ H: y=\frac{0}{2}; \quad \underline{\underline{y=0}} \end{cases}$



c) $(-3,-5)$ y $(-1,3)$

$$m = \frac{3-(-5)}{-1-(-3)} = \frac{3+5}{-1+3} = \frac{8}{2} = 4 //$$

$y = 4x + n;$ $\quad 3 = 4\cdot(-1) + n;$ $3 = -4+n;$ $n = 7$

$$\underline{\underline{y = 4x+7}}$$

$f(-4) = 4\cdot(-4)+7 = -16+7 = -9 //$

$(-1,3)$ y $(4,5)$

$$m = \frac{5-3}{4-(-1)} = \frac{2}{5}$$

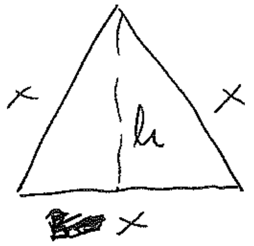
$y = \frac{2}{5}x + n;$ $\quad 5 = \frac{2}{5}\cdot 4 + n;$ $5 = \frac{8}{5} + n$

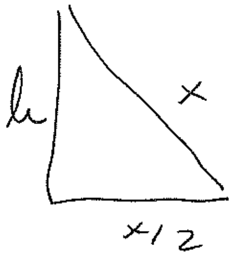
$n = 5 - \frac{8}{5} = \frac{17}{5};$ $\quad \underline{\underline{y = \frac{2}{5}x + \frac{17}{5}}}$

$f(2) = \frac{2}{5}\cdot 2 + \frac{17}{5} = \frac{4}{5} + \frac{17}{5} = \frac{21}{5} //$

$f(5) = \frac{2}{5}\cdot 5 + \frac{17}{5} = \frac{10}{5} + \frac{17}{5} = \frac{27}{5} //$

1) a)





$h^2+\left(\frac{x}{2}\right)^2=x^2$; $h^2+\frac{x^2}{4}=x^2$; $h^2=x^2-\frac{x^2}{4}=\frac{4x^2-x^2}{4}$

$h^2=\frac{3x^2}{4}$; $h=\sqrt{\frac{3x^2}{4}}=\frac{\sqrt{3}\,x}{2}$

$$A=\frac{B\cdot h}{2}=\frac{x\cdot\frac{\sqrt{3}x}{2}}{2}=\frac{\sqrt{3}\,x^2}{4} \;//$$

b) i) $x^3-x=0$; $x(x^2-1)=0$ $\begin{cases} x=0 \\ x^2-1=0 \begin{cases} x=-1 \\ x=1 \end{cases} \end{cases}$

$Df=\mathbb{R}\setminus\{-1,0,1\}$

$1=0 \rightarrow \nexists$ sol

$\frac{1}{1-x^4}\geq 0$; $1-x^4=0$ $x^4=1$; $x=\pm\sqrt[4]{1}=\begin{cases} 1 \\ -1 \end{cases}$

no | sí | no
--- -1 --- 1 ---

$Dg=(-1,1)$

ii) $f(-x)=\frac{1}{(-x)^3-(-x)}=\frac{1}{-x^3+x}=-f(x)$. IMPAR

$g(-x)=\sqrt{\frac{1}{1-(-x)^4}}=\sqrt{\frac{1}{1-x^4}}=g(x)$ PAR

iii) $OX: x^2-1=0 \Rightarrow x = \begin{cases} -1 \longrightarrow (1,0) \\ 1 \longrightarrow (-1,0) \end{cases}$

$OY: x=0 \Rightarrow$ NO CORTA

iv) $x^3+3x=0;\quad x(x^2+3)=0 \begin{cases} \rightarrow x=0 // \\ \rightarrow x^2+3=0 \rightarrow \nexists\ sol \end{cases}$

$x-1=0 \longrightarrow x=1//$

$$\begin{array}{ccccc} + & & - & & + \\ \hline & 0 & & 1 & \end{array}$$

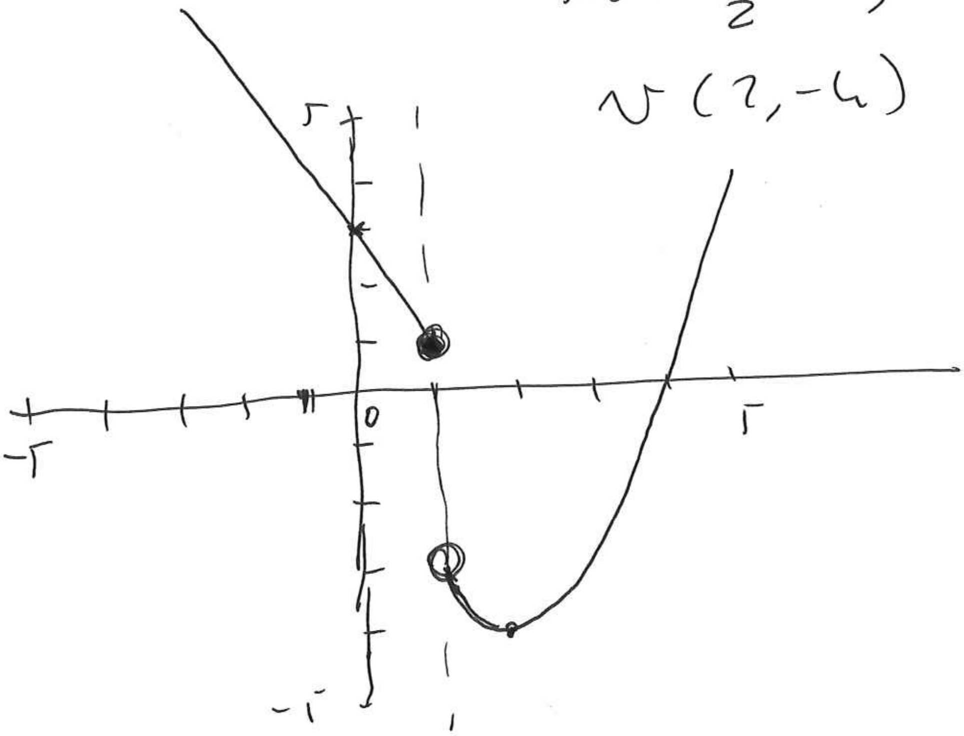
2) a) i) $f(x) = \begin{cases} -2x+3 & si\ x \leq 1 \\ x^2-4x & si\ x > 1 \end{cases}$

x	0	1
y	3	1

x	1	2
y	-3	-4

$x_v = \frac{4}{2} = 2;\quad y_v = 2^2-4\cdot2 = 4-8 = -4$

$V(2,-4)$ Mínimo



ii) Creciente: $(2,+\infty)$

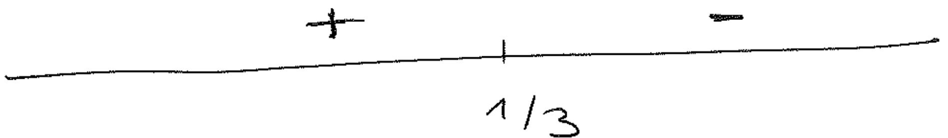
Decreciente: $(-\infty,1) \cup (1,2) \rightarrow (-\infty, 2)$

iii) $OX: x^2-4x=0;\quad x(x-4)=0 \begin{cases} ~~x=0~~ \\ x=4 \longrightarrow (4,0) \end{cases}$

$OY: -2x+3=0;\quad -2\cdot0+3=3 \rightarrow (0,3)$

iv) Discontinua en $x=1$

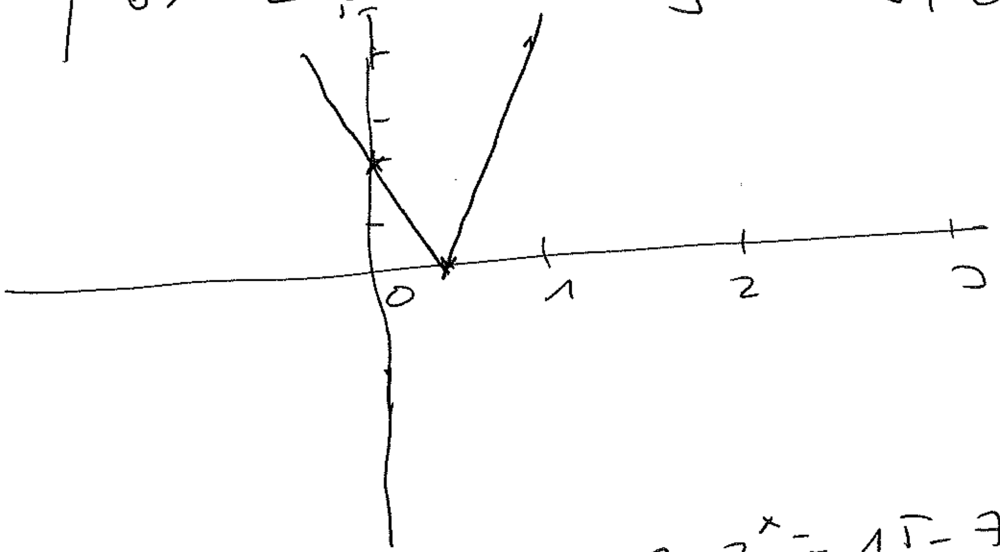
b) $2-6x=0$; $-6x=-2$; $x=\frac{-2}{-6}=\frac{1}{3}$



$$g(x)=\begin{cases} 2-6x & \text{si } x<\frac{1}{3} \\ 6x-2 & \text{si } x\geq\frac{1}{3} \end{cases}$$

x	0	1/3
y	2	0

x	1/3	1
y	0	4



3) a) i) $3\cdot 2^x+7=15$; $3\cdot 2^x=15-7$

$3\cdot 2^x=8$; $2^x=\frac{8}{3}$; $\log 2^x=\log 8/3$

$x\log 2=\log 8/3$; $x=\frac{\log 8/3}{\log 2}=1'41$ //

ii) $2^{2x}-2^x\cdot 2=8$; $t^2-2t=8$ $\boxed{2^x=t}$

$t^2-2t-8=0$; $t=\frac{2\pm\sqrt{4+32}}{2}=\frac{2\pm 6}{2}=\begin{cases}4\\-2\end{cases}$

$2^x=4\Rightarrow \underline{\underline{x=2}}$; $2^x=-2\Rightarrow \nexists$ sol.

iii) $\boxed{3^x=t}$; $t^2-10t+9=0$

$t=\frac{10\pm\sqrt{100-36}}{2}=\frac{10\pm 8}{2}=\begin{cases}9 \rightarrow 3^x=9\Rightarrow \underline{\underline{x=2}}\\ 1\rightarrow 3^x=1\Rightarrow \underline{\underline{x=0}}\end{cases}$

b)

x	1	2
y	2	7

$y(1) = 3^1 - 1 = 3 - 1 = 2$

$y(2) = 3^2 - 2 = 9 - 2 = 7$

$(1,2)$ y $(2,7)$ $\quad m = \frac{7-2}{2-1} = \frac{5}{1} = 5$

$y = 5x + n$; $\quad 2 = 5 \cdot 1 + n$; $\quad n = -3$

$\underline{\underline{y = 5x - 3}}$ $\quad y(1'4) = 5 \cdot 1'4 - 3 = 7 - 3 = \underline{\underline{4}}$

4) a) i) $\log_7 11 = x - 1$; $\quad 7^{x-1} = 11$

$(x-1)\log 7 = \log 11$; $\quad x - 1 = \frac{\log 11}{\log 7} = 1'23$

$\underline{\underline{x = 2'23}}$

ii) $\log_{\sqrt{2}} x = -4$; $\quad x = (\sqrt{2})^{-4} = \frac{1}{(\sqrt{2})^4} = \frac{1}{4}$ //

iii) $\log_{x^2-9} 4 = \frac{1}{2}$; $\quad (x^2-9)^{1/2} = 4$; $\quad \sqrt{x^2-9} = 4$

$x^2 - 9 = 16$; $\quad x^2 = 16 + 9 = 25$; $\quad x = \pm\sqrt{25}$; $\quad \underline{\underline{x = \pm 5}}$

b) $1300 = 1000\left(1 + \frac{x}{100}\right)^1$; $\quad \frac{1300}{1000} = 1 + \frac{x}{100}$

$1'3 = 1 + \frac{x}{100}$; $\quad 130 = 100 + x$; $\quad x = 130 - 100 = 30\%$.

$C_F = 1000\left(1 + \frac{30}{100}\right)^{12} = 1000(1'3)^{12} = \underline{\underline{23.298'085}}$

1) a) TEÓRICO

b) i) $D_f = \mathbb{R} \setminus \{1\}$

$D_g = \mathbb{R} \setminus \{-2, 1\}$

c) ii) $\sqrt[3]{\frac{x}{1-x}} = 1$; $\frac{x}{1-x} = 1$; $x = 1 - x$

$2x = 1$; $x = \frac{1}{2} \rightarrow$ <u>sí</u>

$\frac{x+4}{x^2+x-2} = 1$; $x + 4 = x^2 + x - 2$; $0 = x^2 - 6$

$x = \pm\sqrt{6} \rightarrow$ <u>SÍ</u>

iii) $x = 0$ $\boxed{f(x)}$

−		+		−
	0		1	

$1 - x = 0 \rightarrow x = 1$

$x + 4 = 0 \rightarrow x = -4$ $\boxed{g(x)}$

−		+		−		+
	−4		−2		1	

$x^2 + x - 2 = 0 \rightarrow x = -2, 1$

2) $f(x) = \begin{cases} -x^2+4 & \text{si } x < 0 \\ 4-3x & \text{si } 0 \le x \le 3 \\ x^2-2x-8 & \text{si } x > 3 \end{cases}$

x	−1	0
y	3	4

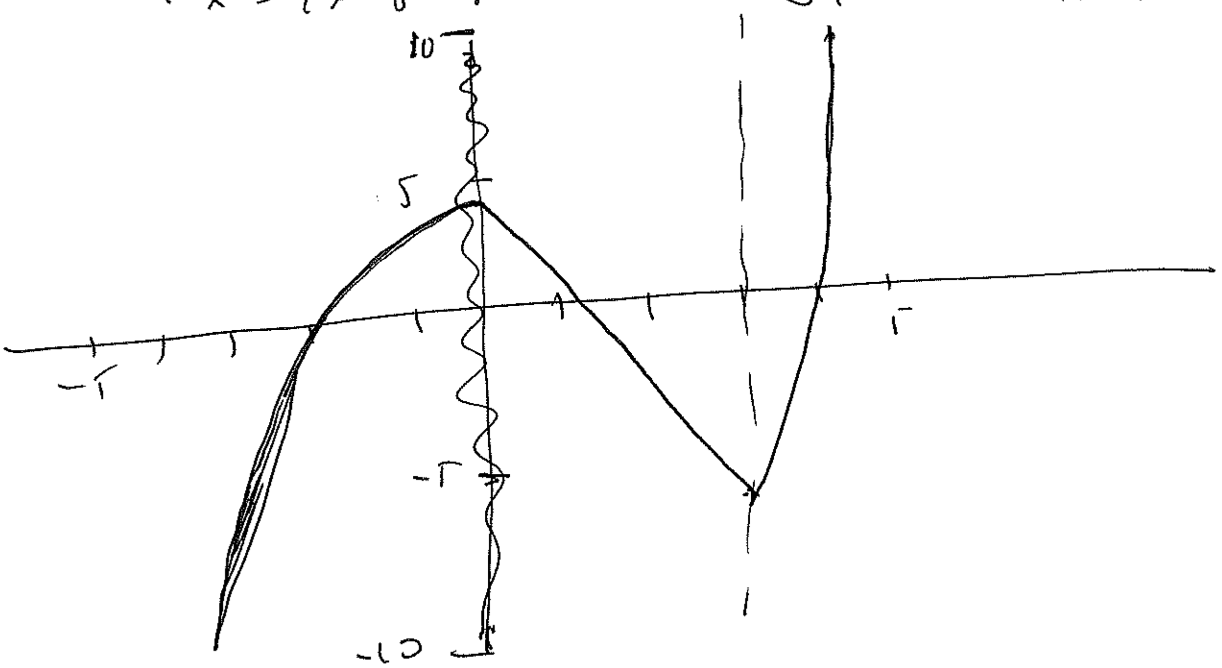
$x_v = 0$
$y_v = 4$ $V(0,4)$ máximo

x	0	3
y	4	−5

x	3	4
y	−5	0

$x_v = 1$
$y_v = -9$
$V(1,-9)$ mínimo

i)



ii) Eje OX: $(-2,0)$, $(4/3,0)$ y $(4,0)$

Eje OY: $(0,4)$

iii) Creciente: $(-\infty, 0) \cup (3,+\infty)$

Decreciente: $(0, 3)$

b) $3 \log x - 2 \log x + \cancel{3}\, 3 \cdot \frac{1}{3} \log x = 2$

$3 \log x - 2 \log x + \log x = 2$

$2 \log x = 2$; $\log x = \frac{2}{2} = 1$; $\boxed{x = 10}$

3) a) - Dominio $8-2x > 0$; $-2x > -8$

$x < \frac{-8}{-2}$; $x < 4$; $D = (-\infty, 4)$

- Corte ejes

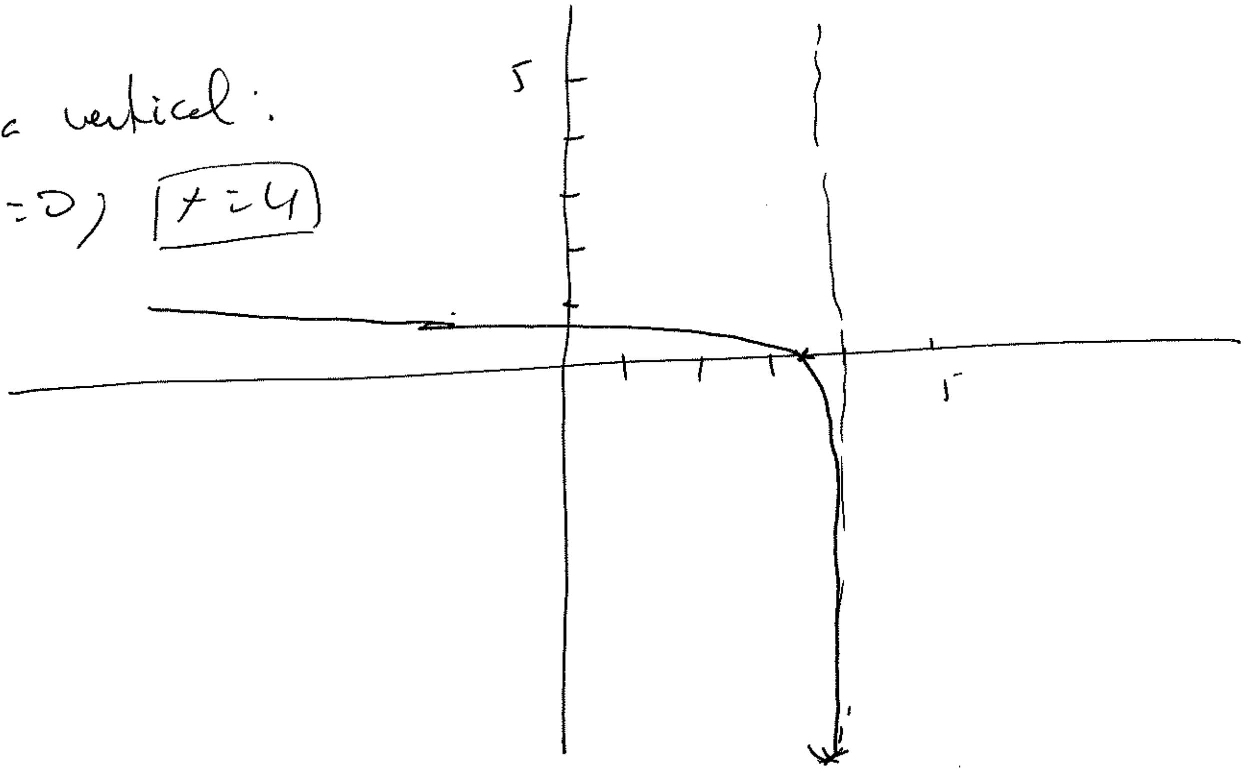
OX: $0 = \log(8-2x)$; $8-2x=1$; $x = \frac{7}{2}$

$(\frac{7}{2}, 0)$

OY: $y = \log(8-2\cdot 0) = \log 8$; $(0, \log 8)$

- Asíntota vertical:

$8-2x=0$; $\boxed{x=4}$



b) i) $3 \cdot 3^x - 4 \cdot \frac{3^x}{3} + 2 \cdot \frac{3^x}{3^2} = 17$

$3 \cdot 3^x - \frac{4 \cdot 3^x}{3} + \frac{2 \cdot 3^x}{9} = 17$ $\boxed{3^x = t}$

$3t - \frac{4t}{3} + \frac{2t}{9} = 17$; $27t - 12t + 2t = 153$

$17t = 153$; $t = \frac{153}{17} = 9$; $3^x = 9 \Rightarrow \underline{\underline{x = 2}}$

ii) $5 \cdot 5^x - 4 \cdot 5^{2x} + 5^2 \cdot 5^x = 26$ $\boxed{5^x = t}$

$5 \cdot 5^x - 4 \cdot 5^{2x} + 25 \cdot 5^x = 26$

$5t - 4t^2 + 25t = 26$; $0 = 4t^2 - 30t + 26$

$$\begin{array}{r} 26 \\ \times 16 \\ \hline 156 \\ 26 \\ \hline 416 \end{array}$$

$$t = \frac{30 \pm \sqrt{900 - 416}}{8} = \frac{30 \pm 22}{8} = \begin{cases} \frac{52}{8} = 6'5 \\ 1 \end{cases}$$

$5^x = 6'5$; $\log 5^x = \log 6'5$; $x \log 5 = \log 6'5$

$x = \frac{\log 6'5}{\log 5}$; $\boxed{x = 1'16}$

$5^x = 1 \Rightarrow \boxed{x = 0}$

4) a) i) $(\sqrt{x-1})^4 = 81$; $(x-1)^2 = 81$

$x^2 + 1 - 2x = 81$; $x^2 - 2x - 80 = 0$

$$x = \frac{2 \pm \sqrt{4 + 320}}{2} = \frac{2 \pm 18}{2} = \begin{cases} \boxed{10} \\ \cancel{-8} \end{cases}$$

ii) $x^4-8=2^3$; $x^4-8=8$; $x^4=16$

$x=\pm\sqrt[4]{16}=\pm 2$; $\boxed{x=\pm 2}$

iii) $5^{\frac{x-1}{3}}=7$; $\log 5^{\frac{x-1}{3}}=\log 7$

$\frac{x-1}{3}\cdot\log 5=\log 7$; $\frac{x-1}{3}=\frac{\log 7}{\log 5}=1'209$

$\frac{x-1}{3}=1'209$; $x-1=3'627$; $\boxed{x=4'627}$

b) $60.000=50.000(1+x)^{13}$;

$\frac{60.000}{50.000}=(1+x)^{13}$; $1'2=(1+x)^{13}$

$1+x=\sqrt[13]{1'2}=1'014$; $x=1'014-1=0'14$

$\boxed{14\%}$

$48.000=40.000(1+x)^{10}$

$\frac{48.000}{40.000}=(1+x)^{10}$; $1'2=(1+x)^{10}$

$1+x=\sqrt[10]{1'2}=1'018$; $x=1'018-1=0'18$

$\boxed{18\%}$

El segundo me ofrece mejor rédito

1) a) $$\lim_{x\to+\infty} \frac{\left(\sqrt{x^2-x}-\sqrt{x^2+3x}\right)\left(\sqrt{x^2-x}+\sqrt{x^2+3x}\right)}{\left(\sqrt{x^2-x}+\sqrt{x^2+3x}\right)} =$$

$$= \lim_{x\to+\infty} \frac{(x^2-x)-(x^2+3x)}{\sqrt{x^2-x}+\sqrt{x^2+3x}} = \lim_{x\to+\infty} \frac{x^2-x-x^2-3x}{\sqrt{x^2-x}+\sqrt{x^2+3x}} =$$

$$= \lim_{x\to+\infty} \frac{-4x}{x+x} = \lim_{x\to+\infty} \frac{-4x}{2x} = -2 //$$

b) $$\lim_{x\to-1} \left(\frac{x+2}{(x+1)(x-1)} - \frac{x-2}{x(x+1)} \right) =$$

$$= \lim_{x\to-1} \frac{(x+2)x-(x-2)(x-1)}{x(x+1)(x-1)} =$$

$$= \lim_{x\to-1} \frac{\cancel{x^2}+2x-\cancel{x^2}+x+2x-2}{x(x+1)(x-1)} =$$

$$= \lim_{x\to-1} \frac{5x-2}{x(x+1)(x-1)} = \frac{-7}{0} = \infty //$$

c) $$\left(\frac{2}{3}\right)^{-\infty} = \left(\frac{3}{2}\right)^{+\infty} = +\infty //$$

d) $$\lim_{x\to1} \frac{(x+1)\cancel{(x-1)}\cancel{x}(x+1)}{3\cancel{x}\cancel{(x-1)}(x-2)} =$$

$$= \lim_{x\to1} \frac{(x+1)^2}{3(x-2)} = \frac{4}{-3} = \frac{-4}{3} //$$

e) $$\lim_{x\to3} \frac{\left(\sqrt{x^2-x+3}-3\right)\left(\sqrt{x^2-x+3}+3\right)}{(x^2+2x-15)\left(\sqrt{x^2-x+3}+3\right)} =$$

$$= \lim_{x\to3} \frac{(x^2-x+3)-9}{(x^2+2x-15)\left(\sqrt{x^2-x+3}+3\right)} = \lim_{x\to3} \frac{x^2-x-6}{(x^2+2x-15)\left(\sqrt{x^2-x+3}+3\right)} =$$

$$= \lim_{x\to3} \frac{\cancel{(x-3)}(x+2)}{\cancel{(x-3)}(x+5)\left(\sqrt{x^2-x+3}+3\right)} = \frac{5}{8\cdot6} = \frac{5}{48} //$$

2) a) $\lim_{x \to 1^-} f(x) = a - 2$

$\lim_{x \to 1^+} f(x) = 4 - 2a$

$f(1) = a - 2$

$a - 2 = 4 - 2a; \quad 3a = 6; \quad a = \frac{6}{3} = 2$ //

b) $\lim_{x \to 3^-} f(x) = 2^{3-2} = 2^1 = 2$

$\lim_{x \to 3^+} f(x) = \sqrt{3+k}$

$f(3) = 2$

$\sqrt{3+k} = 2; \quad 3+k = 4; \quad k = 4-3; \quad k = 1$ //

Sí es continua en todo $\mathbb{R}$ porque las funciones lo son en su dominio.

3) FUNCIONES — CONEXIONES

$x = 2 \to$ D.I.S.I

$x = 0 \to$ DISF

$x = 3 \to$ CONTINUA

$\lim_{x \to 2^-} f(x) = +\infty$

$\lim_{x \to 2^+} f(x) = -\infty$

$f(2) = \nexists$

$\lim_{x \to 0^-} f(x) = -5$

$\lim_{x \to 0^+} f(x) = 3$

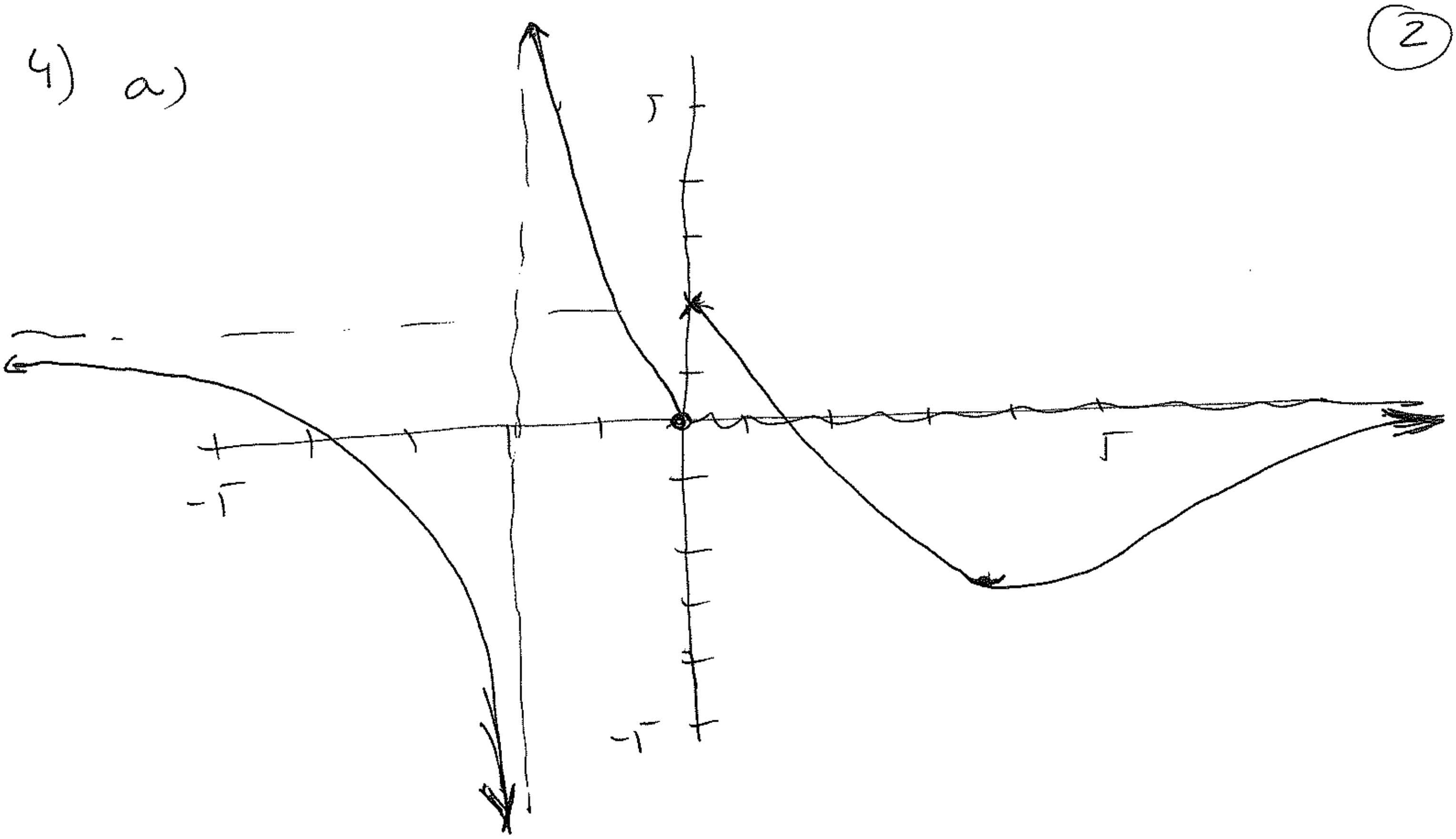
$f(0) = -5$

$\lim_{x \to 3^-} f(x) = -3$

$\lim_{x \to 3^+} f(x) = -3$

$f(3) = -3$

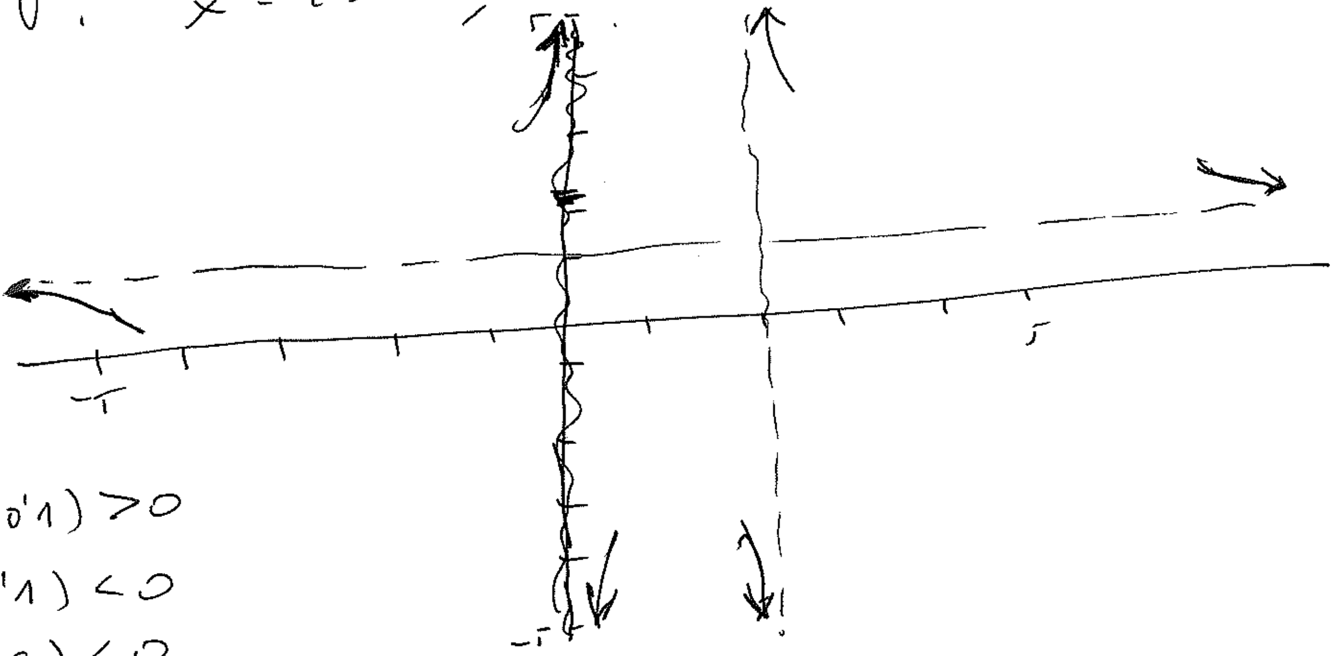
4) a)



b) A.H: $\lim_{x \to +\infty} \frac{x^2+1}{x^2-2x} = 1$ $\boxed{y=1}$ dos lados.

$\lim_{x \to -\infty} \frac{x^2+1}{x^2-2x} = 1$

A.V: $x^2-2x=0$; $x(x-2)=0$; $\boxed{x=0}$ $\boxed{x=2}$



$f(-0'1)>0$
$f(0'1)<0$
$f(1'9)<0$
$f(2'1)>0$

$f(-100)=1'1$

$f(100)=$

e) TEÓRICO

1) FUNCIONES

$x = -3 \rightarrow$ D.I.S.I

CONEXIONES

$x = -1 \rightarrow$ CONTINUA

$x = 1 \rightarrow$ D.E

$$\lim_{x\to -1^-} f(x) = \frac{-1+a}{-4}$$

$$\lim_{x\to -1^+} f(x) = \frac{-1+3}{1+1} = \frac{2}{2} = 1$$

$$f(-1) = 1$$

$$\frac{-1+a}{-4} = 1$$

$$-1+a = -4$$

$$\boxed{a = -3}$$

$$\lim_{x\to -3^-} f(x) = -\infty$$

$$\lim_{x\to -3^+} f(x) = +\infty$$

$$f(-3) = \nexists$$

$$\lim_{x\to -1^-} f(x) = \frac{-1-3}{1-2-3} = \frac{-4}{-4} = 1$$

$$\lim_{x\to -1^+} f(x) = \frac{-1+3}{1+1} = \frac{2}{2} = 1$$

$$f(1) = 1$$

$$\lim_{x\to 1^+} f(x) = \lim_{x\to 1^-} \frac{x^2-1}{x-1} = \lim_{x\to 1^-} \frac{(x+1)\cancel{(x-1)}}{\cancel{(x-1)}} = 2$$

$$\lim_{x\to 1^+} f(x) = \frac{1+3}{1+1} = \frac{4}{2} = 2$$

$$f(1) = \nexists$$

2) i) $$\lim_{x \to -2} \frac{(\sqrt{2x+5} - 1)(\sqrt{2x+5} + 1)}{(x^2 - x - 6)(\sqrt{2x+5} + 1)} =$$

$$= \lim_{x \to -2} \frac{(2x+5) - 1}{(x+2)(x-3)(\sqrt{2x+5} + 1)} =$$

$$= \lim_{x \to -2} \frac{2(x + \cancel{2})}{\cancel{(x+2)}(x-3)(\sqrt{2x+5} + 1)} = \frac{2}{(-5) \cdot 2} = -\frac{1}{5} //$$

ii) $$\lim_{x \to -\infty} \frac{(\sqrt{9x^2-2x+3} - \sqrt{9x^2-2})(\sqrt{9x^2-2x+3} + \sqrt{9x^2-2})}{(\sqrt{9x^2-2x+3} + \sqrt{9x^2-2})} =$$

$$= \lim_{x \to -\infty} \frac{(9x^2 - 2x + 3) - (9x^2 - 2)}{(\sqrt{9x^2-2x+3} + \sqrt{9x^2 2})} =$$

$$= \lim_{x \to -\infty} \frac{9\cancel{x^2} - 2x + 3 - \cancel{9x^2} + 2}{(\sqrt{9x^2 - \cancel{2x+3}} + \sqrt{9x^2 - \cancel{2}})} =$$

$$= \lim_{x \to -\infty} \frac{-2x + 5}{3x + 3x} = \lim_{x \to -\infty} \frac{-2x+5}{6x} = \frac{-1}{3} //$$

iii) $$\lim_{x \to +\infty} \frac{(x^3+1) - (x^2 - 3x + 2)(x+1)}{(x^2 - 1)} =$$

$$= \lim_{x \to +\infty} \frac{(x^3+1) - (x^3 + x^2 - 3x^2 - 3x + 2x + 2)}{x^2 - 1} =$$

$$= \lim_{x \to +\infty} \frac{\cancel{x^3} + 1 - \cancel{x^3} - x^2 + 3x^2 + 3x - 2x - 2}{x^2 - 1} =$$

$$= \lim_{x \to +\infty} \frac{2x^2 + x - 1}{x^2 - 1} = 2 //$$

iv) $\lim_{x \to 2} \frac{x^2\cancel{(x-2)}}{(x-1)\cancel{(x-2)}} = \frac{2^2}{2-1} = \frac{4}{1} = 4$ //

3) a) $f(x) = 2 \quad 3 \cdot x^{-1} + 4x^{-1/2}$

$f'(x) = 2 + 3 \cdot x^{-2} + 4 \cdot -\frac{1}{2} x^{-3/2} =$

$= 2 + \frac{3}{x^2} - \frac{2}{\sqrt{x^3}}$

$f(1) = 2.1 - \frac{3}{1} + \frac{4}{\sqrt{1}} = 2 - 3 + 4 = 3$

$P(1, 3)$

$f'(1) = 2 + \frac{3}{1^2} - \frac{2}{\sqrt{1^3}} = 2 + 3 - 2 = \cancel{2}\, 3$

$m_t = \cancel{2}\, 3$

$y = \cancel{2}\, 3 \cdot (x-1) + 3$ ✓

$\boxed{y = 3x}$

b) $f'(x) = \frac{1}{2\sqrt{\frac{x^3}{x^2-1}}} \cdot \frac{3x^2(x^2-1) - 2x \cdot x^3}{(x^2-1)^2} =$

$= \frac{1}{2\sqrt{\frac{x^3}{x^2-1}}} \cdot \frac{3x^4 - 3x^2 - 2x^4}{(x^2-1)^2} =$

$= \frac{x^4 - 3x^2}{2\sqrt{\frac{x^3}{x^2-1}}\,(x^2-1)^2}$.

$$g'(x) = 3(x^2+x)^2 \cdot (2x+1)(x^3-1)^2 +$$

$$+ \; 2(x^3-1) \cdot 3x^2 \cdot (x^2+x)^3 =$$

$$= 3(x^2+x)^2(x^3-1)\left[(2x+1)(x^3-1) + 2x^2(x^2+x)\right] =$$

$$= 3(x^2+x)^2(x^3-1)\left[2x^4 - 2x + x^3 - 1 + 2x^4 + 2x^2\right] =$$

$$= 3(x^2+x)^2(x^3-1)(4x^4 + x^3 + 2x^2 - 2x - 1)$$

c) $f(x) = ax^3 - bx^2 + cx + 2$

$f'(x) = 3ax^2 - 2bx + c$

$f''(x) = 6ax - 2b$

$f(4) = 64a - 16b + 4c + 2 = -30$

$\boxed{64a - 16b + 4c = -32}$

$f'(4) = \boxed{48a - 8b + c = 0}$

$f''(2) = \boxed{12a - 2b = 0}$

$$\left.\begin{array}{l} 16a - 4b + c = -8 \\ 48a - 8b + c = 0 \\ 6a - b = 0 \end{array}\right\} \qquad \left.\begin{array}{l} 32a - 4b = 8 \\ 6a - b = 0 \end{array}\right\}$$

$32a - 24a = 8$

$8a = 8;\ \boxed{\begin{array}{l} a = 1 \\ b = 6 \\ c = 0 \end{array}}$

4) i) $D = \mathbb{R}$

ii) Corte ejes
- OX : $(-1, 0)$
- OY : $(0, 1/3)$

iii) Asíntotas

Hor. $\lim\limits_{x \to \pm\infty} \dfrac{x+1}{x^2+3} = 0$ $\quad \boxed{y = 0}$ Dos lados

Vert. no tiene

iv) Extremos relativos

$$f'(x) = \frac{1 \cdot (x^2+3) - 2x(x+1)}{(x^2+3)^2} = \frac{x^2+3-2x^2-2x}{(x^2+3)^2} =$$

$$= \frac{-x^2-2x+3}{(x^2+3)^2}$$

$$-x^2-2x+3=0; \quad x^2+2x-3=0 \begin{array}{l} \nearrow -3 \\ \searrow 1 \end{array}$$

$$f(-3) = \frac{-3+1}{9+3} = \frac{-2}{12} = \frac{-1}{6} \qquad E_1\left(-3, \frac{-1}{6}\right)$$

$$f(1) = \frac{1+1}{1+3} = \frac{2}{4} = \frac{1}{2} \qquad E_2\left(1, \frac{1}{2}\right)$$

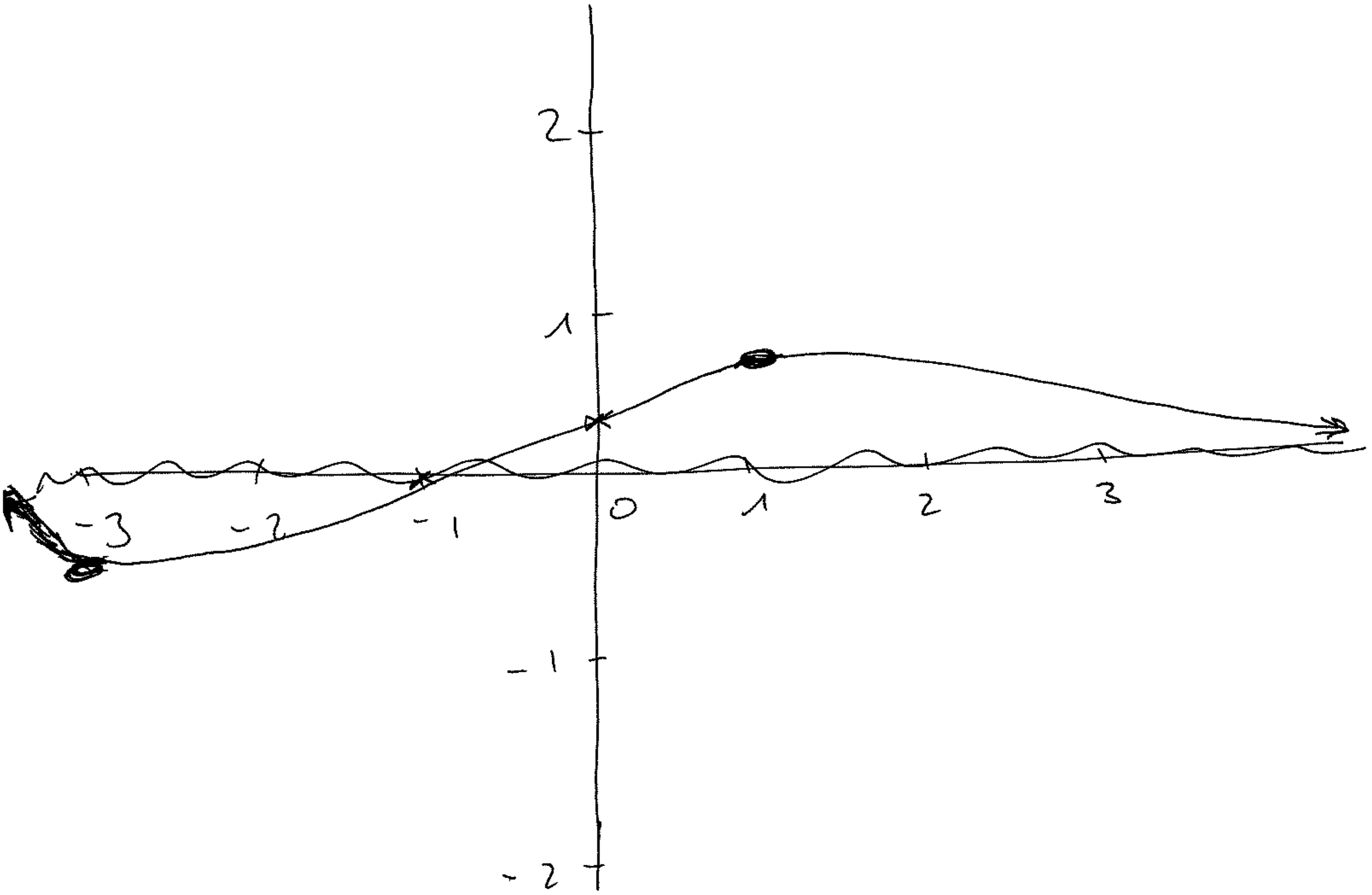
$f'<0$		$f'>0$		$f'<0$
Decrec	-3 → Mínimo relat	Crec.	1 → Máx relat	Decrec

$f'(-4) < 0$; $f'(0) > 0$; $f'(2) < 0$

v) Simetrías

$$f(-x) = \frac{(-x)+1}{(-x)^2+3} = \frac{-x+1}{x^2+3} = \begin{cases} f(x) - \text{no} \\ -f(x) \to \text{no} \end{cases}$$

No tiene



1) a)

FUNCIONES

$x = -2 \rightarrow$ D.I.S.I

$x = 2 \rightarrow$ D.I.S.I

CONEXIONES

$x = -1 \rightarrow$ D.I.S.F

$x = 3 \rightarrow$ D.E

$$x^2+3x+2=0 \begin{cases} \cancel{x=-1} \\ x=-2 \end{cases}$$

$$x^2-4=0 \begin{cases} \cancel{x=-2} \\ x=2 \end{cases}$$

$$2x-2=0 \rightarrow \cancel{x=1}$$

$$\lim_{x\to -2^-} \frac{x^2+x}{x^2+3x+2} = +\infty \, ; \quad \lim_{x\to -2^+} \frac{x^2+x}{x^2+3x+2} = -\infty \, ; \quad f(-2) = \nexists$$

$$\lim_{x\to 2^-} \frac{2x-1}{x^2-4} = -\infty \, ; \quad \lim_{x\to 2^+} \frac{2x-1}{x^2-4} = +\infty \, ; \quad f(2) = \nexists$$

$$\lim_{x\to -1^-} \frac{x^2+x}{x^2+3x+2} = \lim_{x\to -1^-} \frac{x\cancel{(x+1)}}{\cancel{(x+1)}(x+2)} = \lim_{x\to -1^-} \frac{x}{x+2} = -1$$

$$\lim_{x\to -1^+} \frac{2x-1}{x^2-4} = \frac{-3}{-3} = 1 \, ; \quad f(-1) = 1$$

$$\lim_{x\to 3^-} \frac{2x-1}{x^2-4} = \frac{6-1}{9-4} = \frac{5}{5} = 1 \qquad f(3) = \nexists$$

$$\lim_{x\to 3^+} \frac{x+1}{2x-2} = \frac{3+1}{6-2} = \frac{4}{4} = 1$$

b)
$$\lim_{x\to 3^-} \frac{\sqrt{x+1}-2}{x-3} = \lim_{x\to 3^-} \frac{(\sqrt{x+1}-2)(\sqrt{x+1}+2)}{(x-3)(\sqrt{x+1}+2)} =$$

$$= \lim_{x\to 3^-} \frac{x+1-4}{(x-3)(\sqrt{x+1}+2)} = \lim_{x\to -3^-} \frac{\cancel{(x-3)}}{\cancel{(x-3)}(\sqrt{x+1}+2)} =$$

$$= \lim_{x\to -3^-} \frac{1}{\sqrt{x+1}+2} = \frac{1}{2+2} = \frac{1}{4}$$

$\lim\limits_{x\to 3^+} f(x) = \frac{1}{4}$; $f(3) = k+1$

$k+1 = \frac{1}{4}$; $k = \frac{1}{4} - 1 = -\frac{3}{4}$ //

2) i) $\lim\limits_{x\to 2} \frac{(\sqrt{3-x}-1)(\sqrt{3-x}+1)}{(x^2+2x-8)(\sqrt{3-x}+1)} =$

$= \lim\limits_{x\to 2} \frac{3-x-1}{(x-2)(x+4)(\sqrt{3-x}+1)} = \lim\limits_{x\to 2} \frac{-x+2}{(x-2)(x+4)(\sqrt{3-x}+1)} =$

$= \lim\limits_{x\to 2} \frac{-(x-\cancel{2})}{(\cancel{x-2})(x+4)(\sqrt{3-x}+1)} = \lim\limits_{x\to 2} \frac{-1}{(x+4)(\sqrt{3-x}+1)} =$

$= \lim\limits_{x\to 2} \frac{-1}{6 \cdot 2} = -\frac{1}{12}$ //

ii) $\lim\limits_{x\to -1} \frac{x(x\cancel{+1})(x-1)}{2(\cancel{x}+1)(x+\frac{1}{2})} = \lim\limits_{x\to -1} \frac{x(x-1)}{2(x+\frac{1}{2})} =$

$= \frac{(-1)(-2)}{2(-\frac{1}{2})} = \frac{2}{-1} = -2$ //

iii) $\lim\limits_{x\to +\infty} \left(\frac{(x^2+x-1)(x+2) - (x^3-1)}{x^2-4} \right) =$

$= \lim\limits_{x\to +\infty} \frac{\cancel{x^3} + 2x^2 + x^2 + 2x - x - 2 - \cancel{x^3} + 1}{x^2-4} =$

$= \lim\limits_{x\to +\infty} \frac{3x^2 + x - 1}{x^2-4} = 3$ //

iv) $\lim\limits_{x\to -\infty} \left(\frac{3}{2}\right)^{-\infty} = \left(\frac{2}{3}\right)^{+\infty} = 0$ //

3) a) $P(3, 0)$

$$f(3) = \frac{3}{3} - \frac{4}{2} + 1 = 1 - 2 + 1 = 0$$

$$f(x) = 3 \cdot x^{-1} - 4(x+1)^{1/2} + 1$$

$$m_{tg} = f'(x) = -1 \cdot 3 \cdot x^{-2} - 4 \cdot \frac{1}{2}(x+1)^{-1/2} =$$

$$= \frac{-3}{x^2} - \frac{2}{\sqrt{x+1}}$$

$$f'(3) = \frac{-3}{9} - \frac{2}{2} = -\frac{1}{3} - 1 = -\frac{4}{3}$$

$$y = -\frac{4}{3}(x-3) ; \quad \underline{\underline{y = -\frac{4}{3}x + 4}}$$

b) $P(0, 2)$; $f(0) = \sqrt{\frac{0+4}{0+1}} = \sqrt{4} = 2$

$$m_{tg} = f'(x) = \frac{1}{2\sqrt{\frac{x^2+4}{x+1}}} \cdot \frac{2x(x+1) - 1(x+4)}{(x+1)^2} =$$

$$= \frac{1}{2\sqrt{\frac{x^2+4}{x+1}}} \cdot \frac{2x^2+2x-x-4}{(x+1)^2} =$$

$$= \frac{1}{2\sqrt{\frac{x^2+4}{x+1}}} \cdot \frac{2x^2+x-4}{(x+1)^2}$$

$$f'(0) = \frac{1}{2 \cdot 2} \cdot \frac{-4}{1} = \frac{-4}{4} = -1$$

$$y = -1(x-0) + 2 ; \quad \underline{\underline{y = -x + 2}}$$

c) $f(x) = x^4 - 2x^2$

$f'(x) = 4x^3 - 4x = 0$; $4\underline{x}\,\underline{(x^2-1)} = 0$

$\underline{\underline{x=0}}$; $x^2-1=0$; $\underline{\underline{x=-1}}$; $\underline{\underline{x=1}}$

$E_1(0,0)$ Máx relat $E_2(-1,-1)$ Mín relat $E_3(1,-1)$ Mín relat

$f''(x) = 12x^2 - 4$

$f''(0) = -4 < 0$; $f''(-1) = 8 > 0$; $f''(1) = 8 > 0$

$12x^2 - 4 = 0$; $12x^2 = 4$; $x^2 = \frac{4}{12} = \frac{1}{3}$; $x = \pm\sqrt{\frac{1}{3}}$

$E_4\left(\sqrt{\frac{1}{3}}, \frac{-5}{9}\right)$ Inflex $E_5\left(-\sqrt{\frac{1}{3}}, \frac{-5}{9}\right)$ Inflex

$f\left(\sqrt{\frac{1}{3}}\right) = \frac{1}{9} - \frac{2}{3} = \frac{1-6}{9} = \frac{-5}{9}$

$f\left(-\sqrt{\frac{1}{3}}\right) = \frac{1}{9} - \frac{2}{3} = \frac{1-6}{9} = \frac{-5}{9}$

4) $f(x) = \frac{x^2+1}{x^2-4}$;

i) $x^2 - 4 = 0 \begin{cases} x = -2 \\ x = 2 \end{cases}$; $D = \mathbb{R} \setminus \{-2, 2\}$

ii) Corte ejes $\begin{cases} OX : 0 = \frac{x^2+1}{x^2-4} ; \; 0 = x^2+1 ; \text{ NO CORTA} \\ OY ; \; f(0) = (0, -\frac{1}{4}) \end{cases}$

iii) Asíntotas:

Horizontal $\lim_{x \to \pm\infty} \frac{x^2+1}{x^2-4} = 1$; $\boxed{y=1}$

Vertical ; $x^2 - 4 = 0 \begin{cases} x = -2 \\ x = 2 \end{cases}$

③

$\lim_{x \to -2^-} f(x) = +\infty$ $\lim_{x \to -2^+} f(x) = -\infty$

$\lim_{x \to 2^-} f(x) = -\infty$ $\lim_{x \to 2^+} f(x) = +\infty$

iv) Extremos relativos.

$$f'(x) = \frac{2x(x^2-4) - 2x(x^2+1)}{(x^2-4)^2} = \frac{\cancel{2x^3} - 8x - \cancel{2x^3} - 2x}{(x^2-4)^2} =$$

$$= \frac{-10x}{(x^2-4)^2} = 0 \ ; \ -10x = 0 \Rightarrow x = 0$$

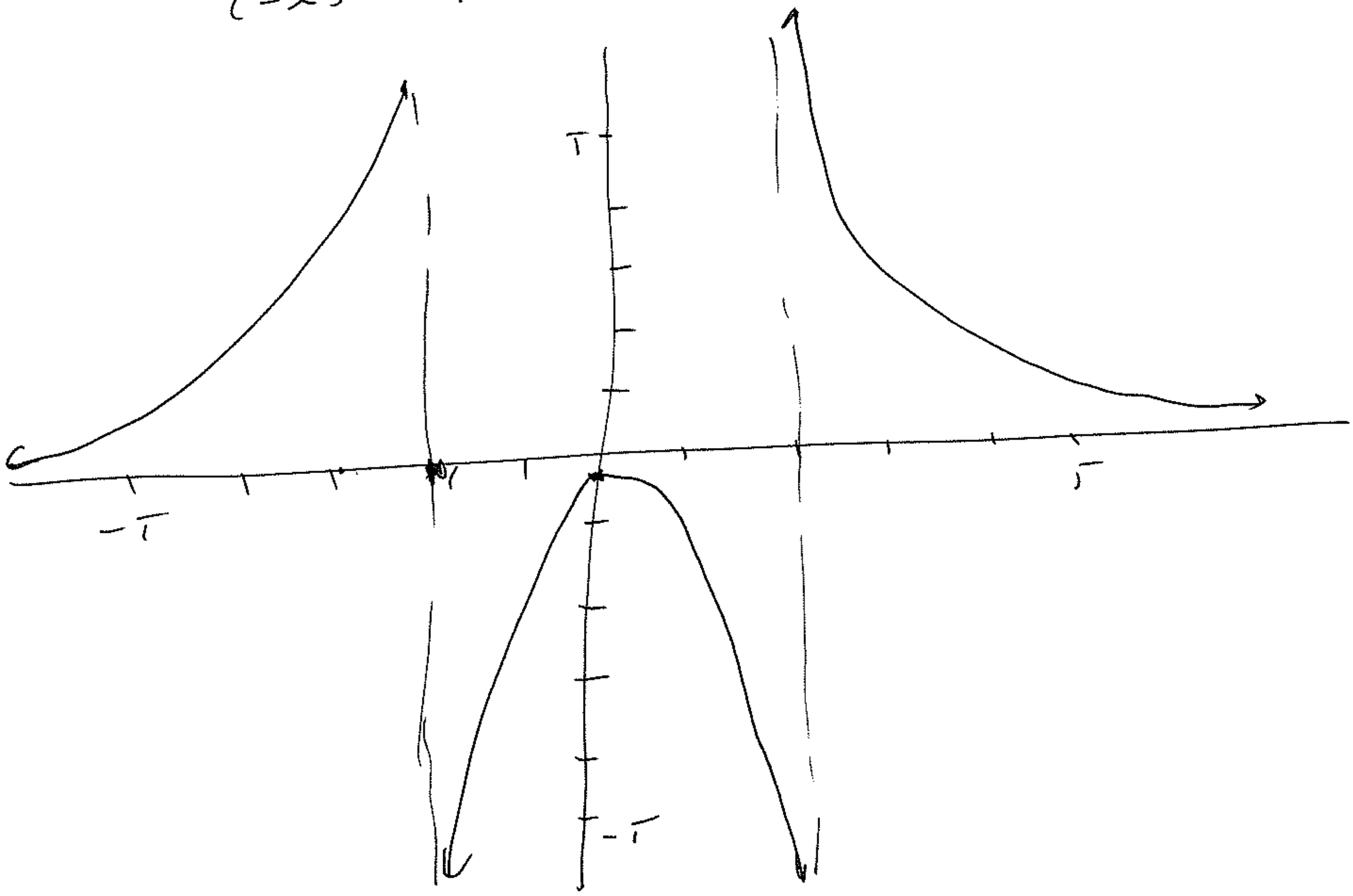
$f'>0$		$f'>0$		$f'<0$		$f'<0$
	-2		0		2	
Crec.		Crec		Decre		Decrec

$\downarrow$ $(0, -\frac{1}{4})$ Máx relat

v) Simetrías

$$f(-x) = \frac{(-x)^2+1}{(-x)^2-4} = \frac{x^2+1}{x^2-4} = f(x)$$

PAR ~~res~~ respecto al eje OY



x	y	f	xf	yf	xxf	yyf	xyf
0	0	3	0	0	0	0	0
0	1	2	0	2	0	2	0
1	0	3	3	0	3	0	0
1	1	2	2	2	2	2	2
3	1	4	12	4	36	4	12
3	2	1	3	2	9	4	6
4	4	5	20	20	80	80	80
		20	40	30	130	92	100

Med x	2		Cov	2
Med y	1,5		r	0,82514
Var x	2,5		Fiab	68,0851
Var y	2,35			
		Y (x)	0,8	-0,1
Dt x	1,58114			
		X(y)	0,85106	0,7234
Dt y	1,53297			

1)

a) r = 0'82514 - Relación directa no muy fuerte

b) x = 0'85106x + 0'7234. x(6) = 0'85106x 6 + 0'7234 = 5'82 Suspenso

No es muy fiable, la fiabilidad es del 68%

2)

+	1	1	1	2	2	3
1	2	2	2	3	3	4
1	2	2	2	3	3	4
1	2	2	2	3	3	4
2	3	3	3	4	4	5
2	3	3	3	4	4	5
3	4	4	4	5	5	6

a) i) E = {2, 3, 4, 5, 6}

$P(2) = \frac{9}{36}$; $P(3) = \frac{12}{36}$; $P(4) = \frac{10}{36}$; $P(5) = \frac{4}{36}$; $P(6) = \frac{1}{36}$

ii) $A = \{2, 3\}$ $P(A) = \frac{21}{36}$

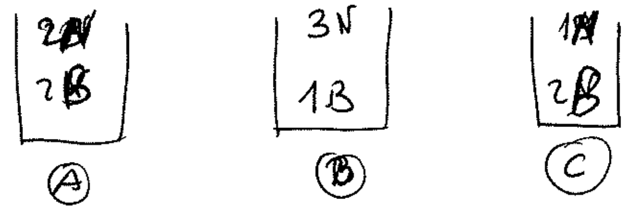
$B = \{3, 6\}$ $P(B) = \frac{13}{36}$

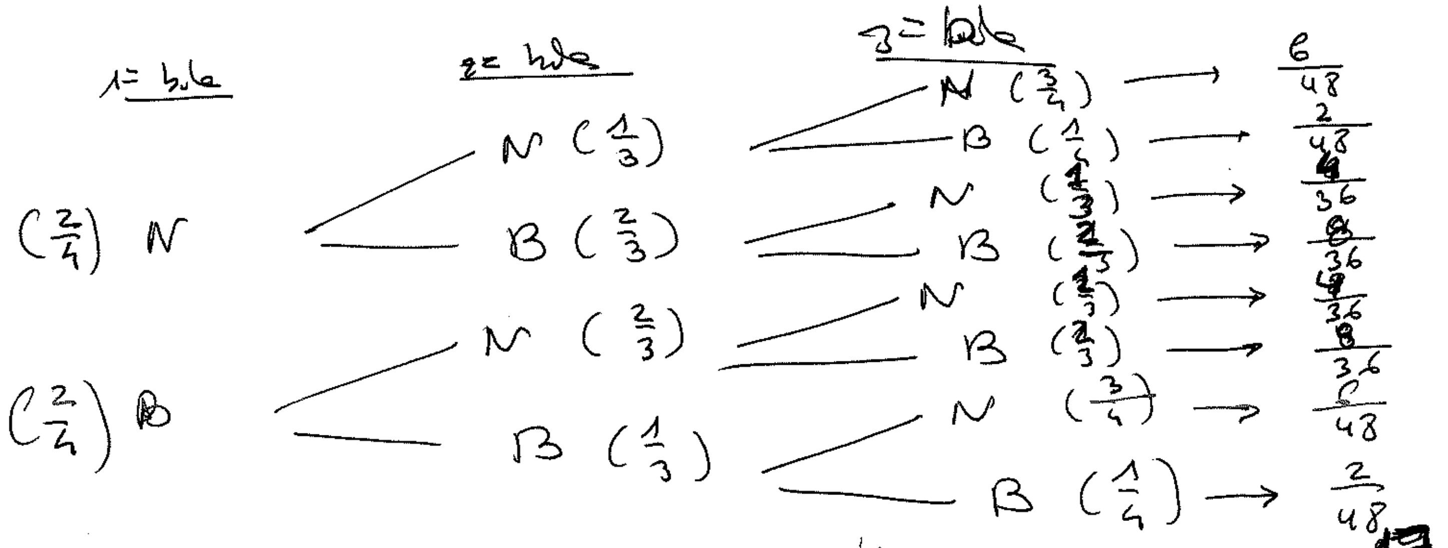
$A \cup B = \{2, 3, 6\}$ $P(A \cup B) = \frac{22}{36}$

$A \cap B = \{3\}$ $P(A \cap B) = \frac{12}{36}$

b) TEÓRICO

3)





a) P(última negra) $= \frac{6}{48} + \frac{4}{36} + \frac{4}{36} + \frac{6}{48} = \frac{8}{36} + \frac{12}{48} = \frac{2}{9} + \frac{1}{4} = \frac{17}{36}$ //

b) P(2 últimas blancas) $= \frac{8}{36} + \frac{2}{48} = \frac{2}{9} + \frac{1}{24} = \frac{16}{72} + \frac{3}{72} = \frac{19}{72}$ //

c) P(3 mismo color) $= \frac{6}{48} + \frac{2}{48} = \frac{8}{48} = \frac{1}{6}$ //

4) $X \longrightarrow B(10, 0'85)$ $\quad n = 10 \quad p = 0'85 \quad q = 0'15$

a) $P(X=8) + P(X=9) + P(X=10) = \underbrace{\binom{10}{8}(0'85)^8(0'15)^2 + \binom{10}{9}(0'85)^9(0'15)^1 + \binom{10}{10}(0'85)^{10}}_{0'82}$

b) 0E, 1E, 2E, 3E, 4E, 5E, 6E, 7E $= 1 - (8E + 9E + 10E) =$

$= 1 - [P(X=8) + P(X=9) + P(X=10)] = 1 - 0'82 = \underline{0'18}$

c) $5F + 6F + 7F = 5E + 4E + 3E =$

$= P(X=5) + P(X=4) + P(X=3) = \underbrace{\binom{10}{5}(0'85)^5(0'15)^5 + \binom{10}{4}(0'85)^4(0'15)^6 + \binom{10}{3}(0'85)^3(0'15)^7}_{0'0098}$

5) a) $P(X < 80) = P(Z < -1) = 1 - P(Z < 1) = 1 - 0'8413 = \underline{0'1587}$

b) $P(82 \leq X \leq 87) = P(-0'6 \leq Z \leq 0'4) = P(Z \leq 0'4) - P(Z \leq -0'6) =$

$= P(Z \leq 0'4) - [1 - P(Z \leq 0'6)] = 0'6554 - [1 - 0'7257] = \underline{0'3811}$

6) $P(X > 100) = P(Z > 3) = 1 - P(Z < 3) = 1 - 0'9987 = 0'0013$

$0'0013 \times 40.000.000 =$ 52.000 personas

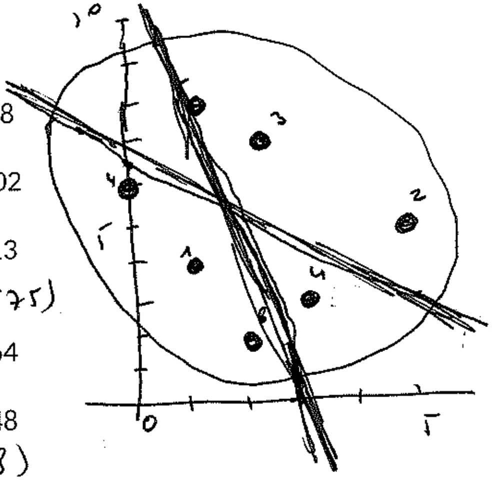
1)

a)

x	y	f	xf	yf	xxf	yyf	xyf
0	6	4	0	24	0	144	0
1	4	1	1	4	1	16	4
1	8	5	5	40	5	320	40
2	2	6	12	12	24	24	24
2	7	3	6	21	12	147	42
3	3	4	12	12	36	36	36
5	5	2	10	10	50	50	50
		25	46	123	128	737	196

Med x	1,84
Med y	4,92
Var x	1,7344
Var y	5,2736
Dt x	1,31697
Dt y	2,29643

Cov	-1,2128	
r	-0,40102	
Fiab	16,0813	
	(0, 6'2)	(5, 2'75)
Y (x)	-0,69926	6,20664
X(y)	-0,22998	2,97148
	(2'97, 0)	(1'1, 8)



b) $20 \times 0'16 = \underline{\underline{3'2}}$

2) a) i) $P(or, or) = \frac{10}{40} \cdot \frac{9}{39} = \frac{90}{1560}$

ii) $P(or, Cop) + P(Cop, or) = \frac{10}{40} \cdot \frac{10}{39} + \frac{10}{40} \cdot \frac{10}{39} = \frac{200}{1560}$

iii) $P(or, or) \times 4 = \frac{90}{1560} \times 4 = \frac{360}{1560}$

iv) ~~[illegible]~~

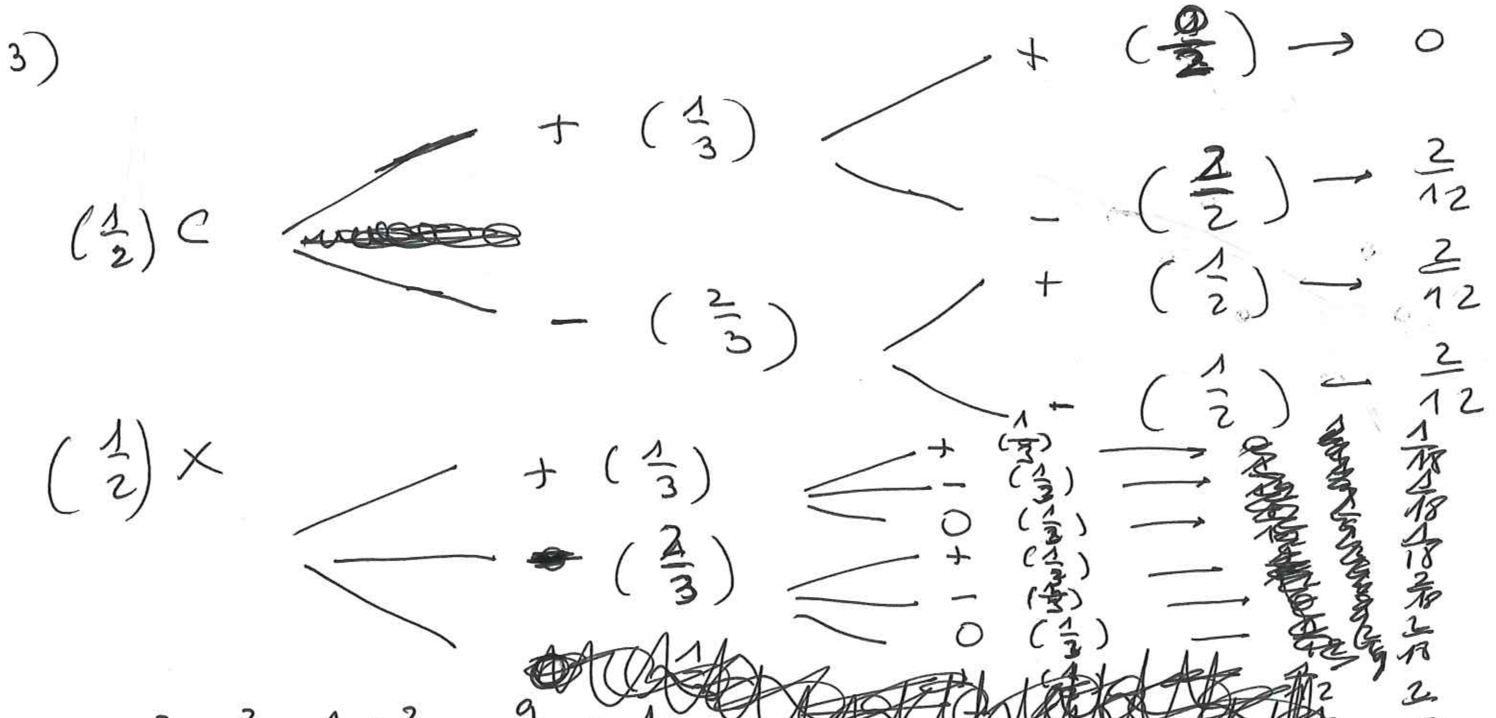
$P(Basto, No\ basto) + P(No\ basto, Basto) = \frac{10}{40} \cdot \frac{30}{39} + \frac{30}{39} \cdot \frac{10}{40} = \frac{600}{1560}$

b) i) TEÓRICO

ii) Como son incompatibles $A \cap B = \emptyset \Rightarrow P(A \cap B) = 0$

$P(A \cup B) = P(A) + P(B) - \underbrace{P(A \cap B)}_{0} = P(A) + P(B) = 0'4 + 0'3 = \underline{\underline{0'7}}$

3)



$\frac{2}{12}+\frac{2}{12}+\frac{1}{18}+\frac{2}{18} = \frac{9}{18} = \frac{1}{2}$

a) 3 + 3 + 1 + 2

b) $\frac{1}{18}+\frac{2}{18} = \frac{3}{18} = \frac{1}{6}$

c) $0 + \frac{2}{12} + \frac{1}{18} + \frac{2}{18} = \frac{6}{18} = \frac{1}{3}$ // ~~5/18~~ 3+2 7 4

4) $X \to B(6, \frac{1}{3})$

$\binom{6}{0}(\frac{1}{3})^0(\frac{2}{3})^6 = \underline{0'912}$

a) $1 - P(X=0) = 1 -$

b) 0F, 1F, 2F, 3F, 4F → 6E, 5E, 4E, 3E, 2E =

$= 1 - (0E + 1E) = 1 - \left[\binom{6}{0}(\frac{1}{3})^0(\frac{2}{3})^6 + \binom{6}{1}(\frac{1}{3})^1(\frac{2}{3})^5\right] =$

$= 1 - 0'351 = \underline{0'649}$

c) $P(X=3) = \binom{6}{3}(\frac{1}{3})^3(\frac{2}{3})^3 = \underline{0'219}$

5)

a) $P(X>10) = P(Z>-0'67) =$ $P(X<0'67) = \underline{0'7486}$

b) $P(X<5) = P(Z<-2'33) = 1 - P(Z<2'33) = \underline{0'0099}$

c) $P(X>15) = P(Z>1) = 1 - P(Z<1) = 0'1587$

$0'1587^3 = \underline{0'0039}$