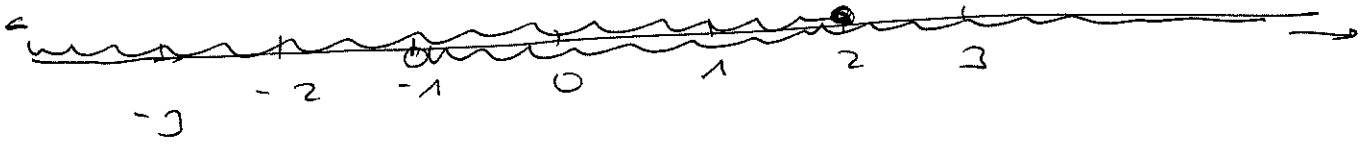
1) a) TEÓRICO b) TEÓRICO

c) $A \cap B = (-1, 2] = \{ x \in \mathbb{R} \mid -1 < x \leq 2 \}$



2) a) $(32 + \sqrt{5} + 4\sqrt{40}) - (10 + 1 - 2\sqrt{10}) + 10\sqrt{10} - 3\sqrt{10} =$

$= \overline{32} + \overline{\sqrt{5}} + \underline{8\sqrt{10}} - \overline{10} - \overline{1} + \underline{2\sqrt{10}} + \underline{10\sqrt{10}} - \underline{3\sqrt{10}} =$

$= \underline{\underline{26 + 17\sqrt{10}}}$

b) $\dfrac{\sqrt{3}}{2\sqrt{5} - \sqrt{3}} = \dfrac{\sqrt{3}(2\sqrt{5} + \sqrt{3})}{(2\sqrt{5} - \sqrt{3})(2\sqrt{5} + \sqrt{3})} = \dfrac{\sqrt{3}(2\sqrt{5} + \sqrt{3})}{20 - 3} = \dfrac{\sqrt{3}(2\sqrt{5} + \sqrt{3})}{17}$

$\dfrac{\sqrt{2}}{\sqrt{27} - \sqrt{3}} = \dfrac{\sqrt{2}}{3\sqrt{3} - \sqrt{3}} = \dfrac{\sqrt{2}}{2\sqrt{3}} = \dfrac{\sqrt{2}\sqrt{3}}{6} = \dfrac{\sqrt{6}}{6}$

$\dfrac{\sqrt{3}(2\sqrt{5} + \sqrt{3})}{17} - \dfrac{\sqrt{6}}{6} = \dfrac{6\sqrt{3}(2\sqrt{5} + \sqrt{3}) - 17\sqrt{6}}{102} =$

$= \dfrac{12\sqrt{15} + 18 - 17\sqrt{6}}{102} = \dfrac{18 + 12\sqrt{15} - 17\sqrt{6}}{102}$ //

3) a)

	1	m	-11	-12
$\sqrt{2}$		$\sqrt{2}$	$\sqrt{2}m + 2$	$2m - 9\sqrt{2}$
	1	$m + \sqrt{2}$	$\sqrt{2}m - 9$	$2m - 9\sqrt{2} - 12$

$2m - 9\sqrt{2} - 12 = 0$ / $\quad m = \dfrac{9\sqrt{2} + 12}{2}$ //

b)

	1	m	−11	−12
−1		−1	−m+1	m+10
	1	m−1	−m+10	|m−2|

$m-2=0 \Rightarrow \underline{\underline{m=2}}$

	1	2	−11	−12
−1		−1	−1	12
	1	1	−12	0

$x^2+x-12=0 \quad x=\frac{-1\pm\sqrt{1+48}}{2}=\frac{-1\pm7}{2}=\begin{cases}3\\-4\end{cases}$

$P(x)=(x+1)(x-3)(x+4)$

4) $P(x)=x(3x^5+x^4-34x^3-60x^2-40x+32)$

	3	1	−34	−60	−40	−32
−2		−6	10	48	24	32
	3	−5	−24	−12	−16	0
−2		−6	22	4	16	
	3	−11	−2	−8	0	
4		12	4	8		
	3	1	2	0		

$3x^2+x+2=0 \quad x=\frac{-1\pm\sqrt{1-24}}{6}=\nexists\,\mathbb{R}$

$\underline{\underline{P(x)=x(x+2)^2(x-4)(3x^2+x+2)}}$

5) $\frac{3(x+3)(x-3)(x^2+2)}{(x+3)(x^2+2)\,6x^2(x-3)}=$

	1	3	2	6
−3		−3	0	−6
	1	0	2	0

$=\frac{1}{2x}$ //

1) a) $\frac{40\sqrt{2}}{2\sqrt{2}-\sqrt{3}} - \frac{12}{\sqrt{6}} = \frac{10\sqrt{2}(2\sqrt{2}+\sqrt{3})}{(2\sqrt{2}-\sqrt{3})(2\sqrt{2}+\sqrt{3})} - \frac{12\sqrt{6}}{\sqrt{6}\sqrt{6}} =$

$= \frac{40+10\sqrt{6}}{8-3} - \frac{12\sqrt{6}}{6} = \frac{40+10\sqrt{6}}{5} - \frac{12\sqrt{6}}{6} =$

$= 8 + 2\sqrt{6} - 2\sqrt{6} = 8$ //

b)

	2	-3	-2	3
1		2	-1	-3
	2	-1	-3	0

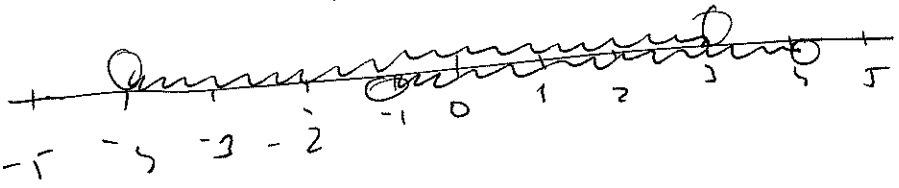
$2x^2 - x - 3 = 0$

$x = \frac{1 \pm \sqrt{1+24}}{4} = \frac{1 \pm 5}{4} = \begin{cases} 3/2 \\ -1 \end{cases}$

$x(4x^2-9) ; \quad 4x^2 - 9 = 0 ; \quad x^2 = \frac{9}{4} ; \quad x = \pm \frac{3}{2}$

$\frac{2(x-1)(x+1)(x-3/2)}{4x(x-3/2)(x+3/2)} = \frac{(x-1)(x+1)}{2x(x+3/2)}$

c)



$A \cup B = (-4, 4) \longrightarrow E(0, 4)$ //

$A \cap B = (-1, 3) \longrightarrow E(1, 2)$ //

2) a) i) TEÓRICO

ii) TEÓRICO

b) i)

	1	$-(5+\sqrt{3})$	$5\sqrt{3}+6$	m
$\sqrt{3}$		$\sqrt{3}$	$-5\sqrt{3}$	$6\sqrt{3}$
	1	-5	6	$m+6\sqrt{3}=0$

ii)

$m = -6\sqrt{3}$ //

$x^2 - 5x + 6 = 0 \quad \begin{matrix} \nearrow 2 \\ \searrow 3 \end{matrix}$

$P(x) = (x-\sqrt{3})(x-2)(x-3)$

3) a) $x^3 - 5x^2 = x^2 + 1 + 2x - x^2 - 11$

$x^3 - 5x^2 - 2x + 10 = 0$

	1	−5	−2	10
5		5	0	−10
	1	0	−2	0

Sol: $5, \sqrt{2}, -\sqrt{2}$

$x^2 - 2 = 0 \Rightarrow x = \pm\sqrt{2}$

b) $\sqrt{2x-1} = x + 2 - 4$; $\sqrt{2x-1} = x - 2$

$2x - 1 = (x-2)^2$; $2x - 1 = x^2 + 4 - 4x$

$0 = x^2 - 6x + 5$; $x = \frac{6 \pm \sqrt{36-20}}{2} = \frac{6 \pm 4}{2} = \begin{cases} 5 \text{ Sí} \\ 1 \text{ No} \end{cases}$

$x = 5 \rightarrow \sqrt{9} + 4 = 5 + 2$? → Sí

$x = 1 \rightarrow \sqrt{1} + 4 = 1 + 2$? → No

4) Alta → x

Mediana → y

Baja → z

$y = \frac{x+y+z}{4}$

$z = y + 10$

$x + 2y = 2z$

$$\begin{cases} -x + 3y - z = 0 \\ -y + z = 10 \\ x + 2y - 2z = 0 \end{cases}$$

$$\left(\begin{array}{ccc|c} -1 & 3 & -1 & 0 \\ 0 & -1 & 1 & 10 \\ 1 & +2 & -2 & 0 \end{array}\right) \sim \left(\begin{array}{ccc|c} -1 & 3 & -1 & 0 \\ 0 & -1 & 1 & 10 \\ 0 & 5 & -3 & 0 \end{array}\right)$$

$$\left(\begin{array}{ccc|c} -1 & 3 & -1 & 0 \\ 0 & -1 & 1 & 10 \\ 0 & 0 & +2 & 50 \end{array}\right)$$

$-x + 3y - z = 0$ → $x = 20$

$-y + z = 10$ → $y = 15$

$2z = 50$ → $z = 25$

(20, 15, 25)

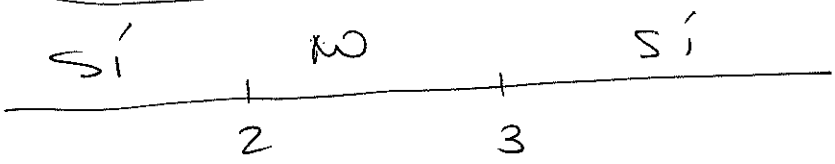
5) $\frac{2x-1}{x-2} - 5 < 0$ $\frac{2x-1}{x-2} - \frac{5(x-2)}{x-2} < 0$

$\frac{2x-1}{x-2} - \frac{5x-10}{x-2} < 0$ $\frac{2x-1-5x+10}{x-2} < 0$

$\frac{-3x+9}{x-2} < 0$

$-3x+9=0 \Rightarrow x=3$

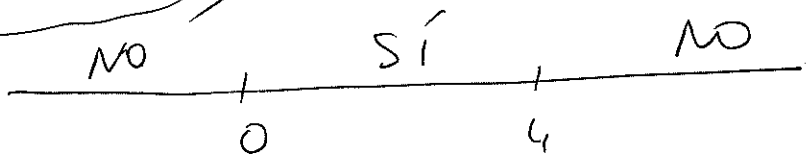
$x-2=0 \Rightarrow x=2$



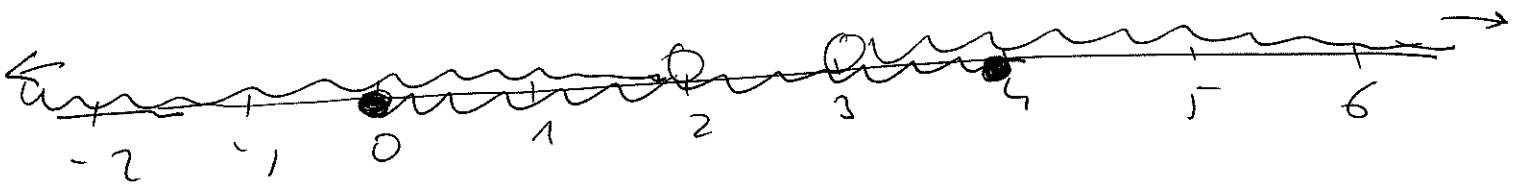
$S_1: (-\infty, 2) \cup (3, +\infty)$

$x^2 - 3x + x - 3 \leq 2x - 3$; $x^2 - 3x + x - 3 - 2x + 3 \leq 0$

$x^2 - 4x \leq 0$; $x^2 - 4x = 0$; $x(x-4)=0$ $x=0$ // $x=4$ //



$S_2: [0, 4]$



Sol sistema: $[0, 2) \cup (3, 4]$

1)

a) $\frac{3\sqrt{3}(\sqrt{5}+1)}{(\sqrt{5}-1)(\sqrt{5}+1)} - \frac{4\sqrt{5}(\sqrt{3}-1)}{(\sqrt{3}+1)(\sqrt{3}-1)} = \frac{3\sqrt{3}(\sqrt{5}+1)}{4} - \frac{4\sqrt{5}(\sqrt{3}-1)}{2} =$

$= \frac{3\sqrt{3}(\sqrt{5}+1) - 8\sqrt{5}(\sqrt{3}-1)}{4} = \frac{3\sqrt{15}+3\sqrt{3}-8\sqrt{15}+8\sqrt{5}}{4} = \frac{3\sqrt{3}+8\sqrt{5}-5\sqrt{15}}{4}$

b) $\frac{3(x+1)^2}{(x-2)(x-1)} \cdot \frac{(x-2)^2}{(x+2)(x-2)} = \frac{3(x+1)^2}{(x-1)(x+2)}$

2)

a)

	3	9	−3	m	−18
−2		−6	−6	18	$-2m-36$
	3	3	−9	$m+18$	$-2m-54$

$-2m-54=0$; $-2m=54$; $m=\frac{54}{-2}=-27$

	3	9	−3	−27	−18
−1		−3	−6	9	18
	3	6	−9	−18	0
−2		−6	0	18	
	3	0	−9	0	

$3x^2-9=0$; $3x^2=9$; $x^2=3$; $x=\pm\sqrt{3}$

$P(x) = 3(x+1)(x+2)(x+\sqrt{3})(x-\sqrt{3})$

$Q(x) = x(x^4+x^3-13x^2-25x-12) = x(x+1)^2(x+3)(x-4)$

b)

	1	1	−13	−25	−12
−1		−1	0	13	12
	1	0	−13	−12	0
−1		−1	1	12	
	1	−1	−12	0	

$x^2-x-12=0$; $x=\frac{1\pm\sqrt{1+48}}{2}=\frac{1\pm 7}{2}=\begin{cases}4\\-3\end{cases}$

3) a) $\sqrt{3x-5} = x-1$ · $3x-5 = (x-1)^2$

$3x-5 = x^2+1-2x$; $0 = x^2-5x+6$

$x = \frac{5 \pm \sqrt{25-24}}{2} = \frac{5 \pm 1}{2} = \begin{cases} 3 \to \text{Sí} \\ 2 \to \text{Sí} \end{cases}$

b) $(\sqrt{x^4+1-2x^2}) + (\sqrt{x^4+x^2+3x^2+3}) = \sqrt{x^4-x^2+8}$

$x^4+3x^2-4=0$; $\boxed{x^2=t}$

$t^2+3t-4=0$; $t = \frac{-3 \pm \sqrt{9+16}}{2} = \frac{-3 \pm 5}{2} = \begin{cases} 1 \\ -4 \end{cases}$

$x = \pm\sqrt{1} \begin{cases} 1 \\ -1 \end{cases}$ $x = \pm\sqrt{-4} \to \nexists$

4) Mayor $\to x$
Mediano $\to y$
Pequeño $\to z$

$x+y+z=50$
$2y = x+5$
$\frac{z}{2} = \frac{x}{5}$

$\left\{\begin{array}{l} x+y+z=50 \\ -x+2y=5 \\ -2x+5z=0 \end{array}\right.$

$\left(\begin{array}{ccc|c} 1 & 1 & 1 & 50 \\ -1 & 2 & 0 & 5 \\ -2 & 0 & 5 & 0 \end{array}\right) \sim \left(\begin{array}{ccc|c} 1 & 1 & 1 & 50 \\ 0 & 3 & 1 & 55 \\ 0 & 2 & 7 & 100 \end{array}\right) \sim \left(\begin{array}{ccc|c} 1 & 1 & 1 & 50 \\ 0 & 3 & 1 & 55 \\ 0 & 0 & 19 & 190 \end{array}\right)$

$x+y+z=50$
$3y+z=55$
$19z=190$

$z = \frac{190}{19} = 10$

$3y+10=55$; $3y=45 \Rightarrow y = \frac{45}{3} = 15$

$x+15+10=50$; $x = 50-15-10 = 25$

$(25, 15, 10)$

5) $\frac{3x-1}{2x+1} - 2 < 0$; $\frac{3x-1-4x-2}{2x+1} < 0$

$\frac{-x-3}{2x+1} < 0$

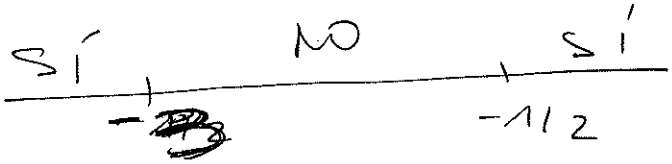
$x^2-3x \geq 0$

$-x-3=0 \Rightarrow x=-3$ ~~$x=-3$~~

$2x+1=0$

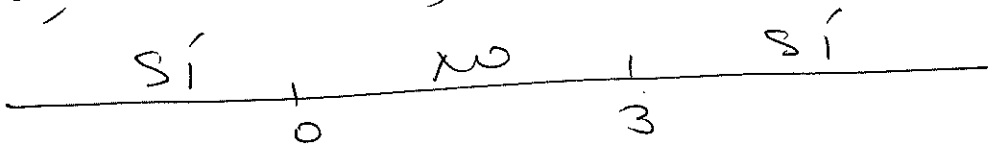
$-x-3=0 \Rightarrow x=-3$

$2x+1=0 \Rightarrow x=-1/2$

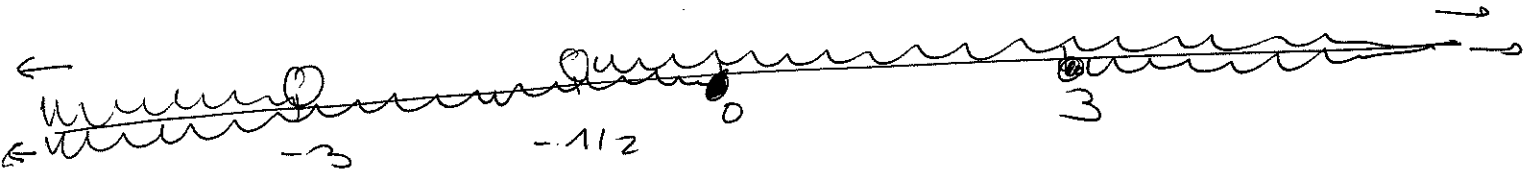


$S_1: (-\infty, -3) \cup (-1/2, +\infty)$

$x^2-3x=0; \quad x(x-3)=0 \rightarrow \begin{cases} x=0 \rightarrow x=0 \\ x-3=0 \rightarrow x=3 \end{cases}$



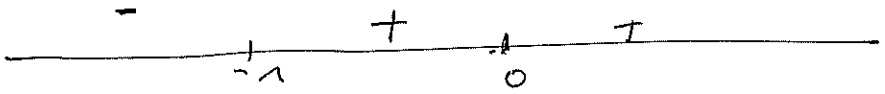
$S_2 = (-\infty, 0] \cup [3, +\infty)$



$S_{sistema} = S_1 \cap S_2 = (-\infty, -3) \cup (-1/2, 0] \cup [3, +\infty)$

1) a) $D_f = \mathbb{R}$

$D_g = \mathbb{R} - \{3\}$ $\quad D_h = [-1, +\infty) - \{0\}$



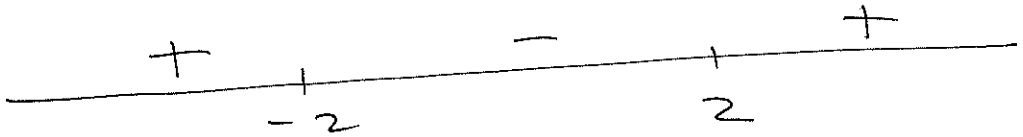
b) $2 = \frac{2x^2+8}{x^2+1}$; $2x^2+2 = 2x^2-8$ — NO PERTENECE

~~$2 = \frac{x+1}{x-3}$~~ ; ~~$2x-6 = x+1$~~ ; ~~$x = 7$~~ ; ~~PERTENECE~~

$2 = \sqrt{\frac{x+1}{x^2}}$; $4 = \frac{x+1}{x^2}$; $4x^2 - x - 1 = 0$; $x = \frac{1 \pm \sqrt{1+16}}{8}$ → SÍ PERT

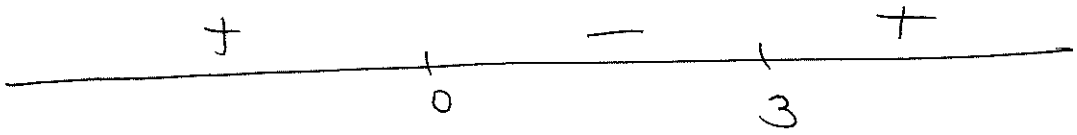
c) Corta en OX: $(-2,0)$ y $(2,0)$; AV: no tiene

$f(x)$:



Corta en OX: $(0,0)$; AV: $x = 3$

$g(x)$



d) $f(-x) = \frac{2(-x)^2-8}{(-x)^2+1} = \frac{2x^2-8}{x^2+1} = f(x)$ → PAR

$g(-x) = \frac{2(-x)}{(-x)-3} = \frac{-2x}{-x-3}$ → NO TIENE SIMETRÍAS

e) $\frac{2x^2-8}{x^2+1} = \frac{2x}{x-3}$; $(2x^2-8)(x-3) = 2x(x^2+1)$

$2x^3 - 6x^2 - 8x + 24 = 2x^3 + 2x$

$0 = 6x^2 + 10x - 24$; $0 = 3x^2 + 5x - 12$

$x = \frac{-5 \pm \sqrt{25+144}}{6} = \frac{-5 \pm 13}{6} = \begin{cases} \frac{8}{6} = \frac{4}{3} \\ \frac{-18}{6} = -3 \end{cases}$

$g(\frac{4}{3}) = \frac{8/3}{4/3-3} = \frac{8/3}{-5/3} = 8 \rightarrow (\frac{4}{3}, -8)$ //

$g(-3) = \frac{-6}{-6} = 1 \rightarrow (-3, 1)$ //

2) $f(x) = \begin{cases} x^2+9 & \text{si } x < -2 \\ 6-x & \text{si } -2 \le x \le 3 \\ x^2-3x & \text{si } x > 3 \end{cases}$

x	-3	-2
δ	18	13

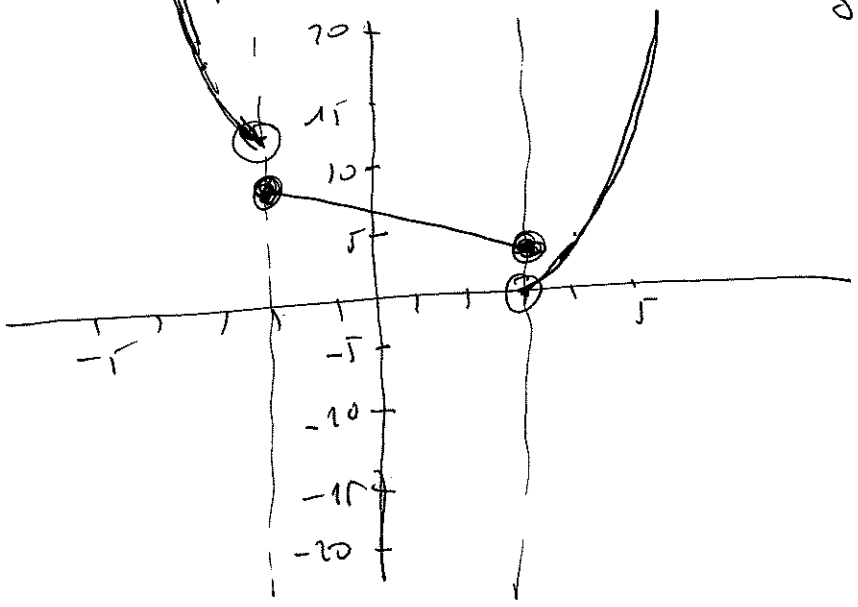
$V(0,9)$ mín

x	-2	3
δ	8	3

x	3	4
δ	0	4

$V(1'5, -2'25)$ mín

i)



ii) Discontinua en $x = -2$ y $x = 3$

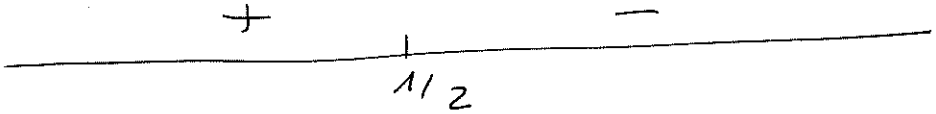
iii) Creciente: $(3, +\infty)$

Decreciente: $(-\infty, -2) \cup (-2, 3)$

iv) OX: No corta //

OY: $6 - x = 0 \Rightarrow x = 6 \rightarrow (0, 6)$ //

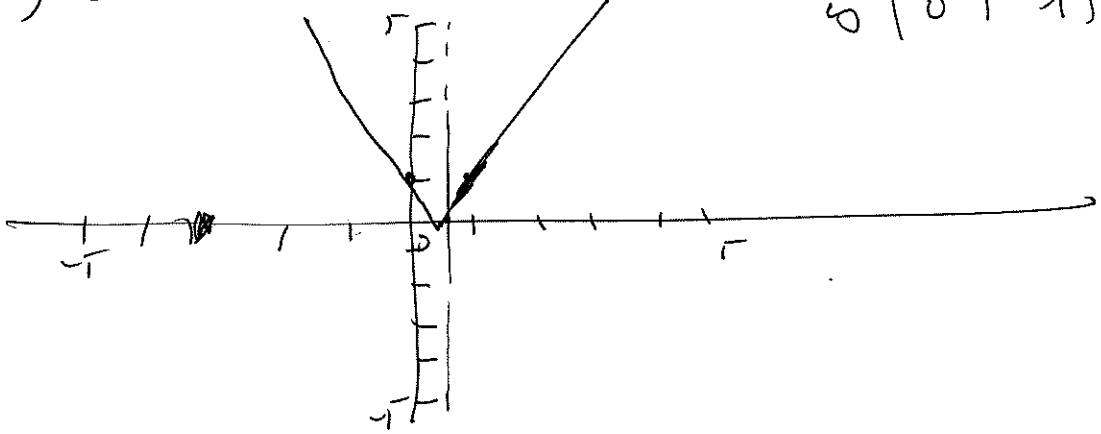
3) a) $-2x+1 = 0 \Rightarrow -2x = -1 ; \quad x = 1/2$



x	0	$1/2$
δ	1	0

x	$1/2$	1
δ	0	1

$f(x) = \begin{cases} -2x+1 & \text{si } x < 1/2 \\ 2x-1 & \text{si } x \ge 1/2 \end{cases}$

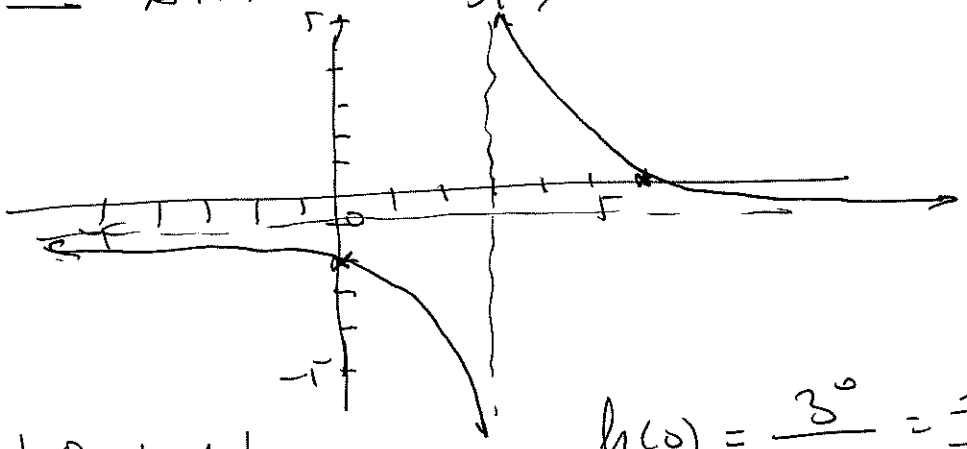


b) Corte eixos → OX: $y=0$ $\quad 0=\frac{-x+6}{x-3}$, $0=-x+6 \Rightarrow x=6$ $\quad (6,0)$

→ OY: $x=0$ $\quad y=\frac{-0+6}{0-3}=-2$ — $(0,-2)$

Assíntotas → AV: $x-3=0 \Rightarrow x=3$

— AH: $y=\frac{-1}{1}$; $y=-1$



c)

x	0	1
y	-1	3

$(0,-1)$ e $(1,3)$

$f(0)=\frac{3^0}{2.0-1}=\frac{1}{-1}=-1$

$f(1)=\frac{3^1}{2.1-1}=\frac{3}{1}=3$

$m=\frac{3-(-1)}{1-0}=\frac{4}{1}=4$; $\quad y=4x+n$

$-1=4.0+n$; $\quad n=-1$

$\boxed{y=4x-1}$

$y(0'6)=4.0'6-1=2'4-1=\underline{1'4}$

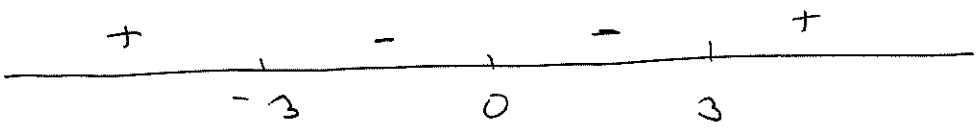
1) a) $x^2 = 0 \Rightarrow x = 0$; $D_f = \mathbb{R} - \{0\}$

$x + 1 = 0 \Rightarrow x = -1$; $D_g = \mathbb{R} - \{-1\}$

$x^2 - 4 = 0 \Rightarrow x = \pm 2$; $D_h = \mathbb{R} - \{-2, 2\}$

b) Cuts ox: $y = 0$; $x^2 - 9 = 0 \Rightarrow x = \pm 3$

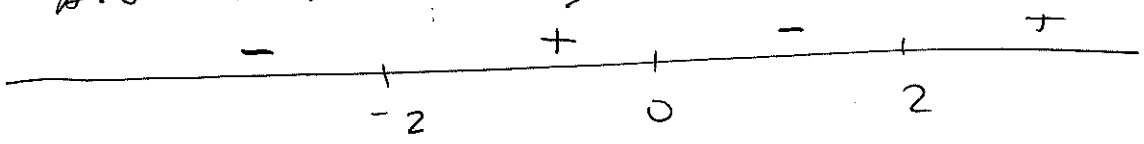
A. V: $x^2 = 0 \Rightarrow x = 0$



+: $(-\infty, -3) \cup (3, +\infty)$; -: $(-3, 0) \cup (0, 3)$

Cuts ox: $y = 0$; $x^2 = 0 \Rightarrow x = 0$

A. V: $x^2 - 4 = 0$; $x = \pm 2$



+: $(-2, 0) \cup (2, +\infty)$; -: $(-\infty, -2) \cup (0, 2)$

c) $\underline{f(x)}$ $0x$: $y = 0 \Rightarrow x^2 - 9 = 0 \Rightarrow x = \pm 3 \leftarrow \begin{matrix} (3,0) \\ (-3,0) \end{matrix}$

$0y$: $x = 0 \Rightarrow$ NO CORTA

$\underline{g(x)}$ $0x$: $y = 0 \Rightarrow \sqrt[3]{\frac{x-2}{x+1}} = 0 \Rightarrow \frac{x-2}{x+1} = 0 \Rightarrow x - 2 = 0 \Rightarrow x = 2$

$(2,0)$

$0y$: $x = 0 \Rightarrow \sqrt{\frac{0-2}{0+1}} = \sqrt[3]{-2}$ $(0, \sqrt[3]{-2})$

$\underline{h(x)}$ $0x$: $y = 0 \Rightarrow 0 = \frac{x^3}{x^2 - 4} \Rightarrow x^3 = 0 \Rightarrow x = 0 \rightarrow (0,0)$

$0y$: $x = 0 \Rightarrow \frac{0^3}{0^2 - 4} = 0 \rightarrow (0,0)$

d) $f(x)$ $\quad \frac{x^2-9}{x^2} = -1 \Rightarrow x^2-9 = -x^2$

$2x^2 - 9 = 0 \Rightarrow 2x^2 = 9 \; / \; x^2 = \frac{9}{2} \quad x = \pm\sqrt{\frac{9}{2}}$

$\left(\sqrt{\frac{9}{2}}, -1\right) \qquad \left(-\frac{\sqrt{9}}{2}, -1\right)$

$g(x)$ $\quad \sqrt[3]{\frac{x-2}{x+1}} = -1 \Rightarrow \frac{x-2}{x+1} = (-1)^3 = -1$

$x - 2 = -x - 1 \Rightarrow 2x = 1 \Rightarrow x = \frac{1}{2} \rightarrow \left(\frac{1}{2}, -1\right)$

2) i) $x^2 - 1$

x	0	1
y	-1	0

$x_v = \frac{0}{2} = 0 \qquad V(0,-1)$ mín

$y_v = 0^2 - 1 = -1$

$12 - 7x$

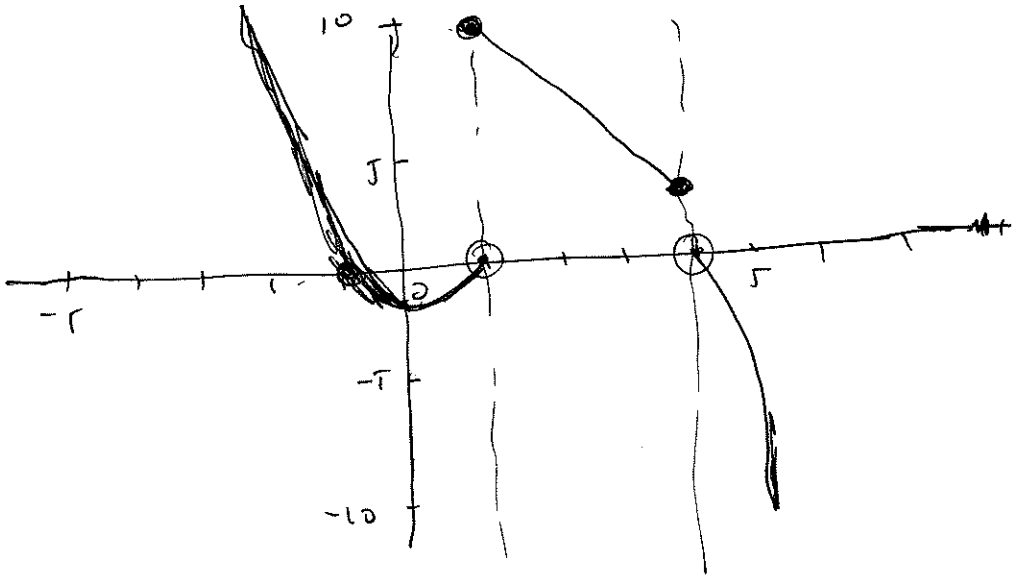
x	1	4
y	10	4

$-x^2 + 4x$

x	4	1
y	0	-9

$x_v = \frac{-4}{-2} = 2 \qquad V(2,4)$ máx

$y_v = -2^2 + 4 \cdot 2 = -4 + 8 = 4$



ii) Creciente: $(0,1)$; Decreciente: $(-\infty, 0) \cup (1,4) \cup (4, +\infty)$

iii) $0X: x^2 - 1 = 0 \Rightarrow x^2 = 1 \; / \; x = \pm 1 \rightarrow (-1,0)$ $(1,0)$

$0Y: 0^2 - 1 = -1 \Rightarrow (0,-1)$

iv) $f(1) = 12 - 7 = 10$ // $\qquad f(4) = 12 - 8 = 4$ //

3) a) $\frac{3^x}{3^2} + 2 \cdot 3^x \cdot 3 - 5 \cdot 3^x = 10$ $\boxed{3^x = t}$

$\frac{t}{9} + 6t - 15t = 10$; $t + 54t - 135t = 90$

$10t = 90 \Rightarrow t = \frac{90}{10} : 9 \Rightarrow 3^x = 9 \Rightarrow \underline{\underline{x = 2}}$

b) $(2^2)^x - 3 \cdot (2^2)^x + 5 \cdot 2^x = 3$; $\boxed{2^x = t}$

$t^2 - 3t^2 + 5t = 3 \cdot -2t^2 + 5t = 3$

$0 = 2t^2 - 5t + 3$; $t = \frac{5 \pm \sqrt{25 - 24}}{4} = \frac{5 \pm 1}{4} = \begin{cases} 3/2 \\ 1 \end{cases}$

$2^x = 1 \Rightarrow \underline{\underline{x = 0}}$

$2^x = \frac{3}{2} \Rightarrow \ln 2^x = \ln \frac{3}{2} \Rightarrow x \cdot \ln 2 = \ln \frac{3}{2}$

$x = \frac{\ln \frac{3}{2}}{\ln 2} = 0'58$

c)

x	1	2
y	2	3/4

$P(1, 2)$

$Q(2, 3/4)$

$f(1) = \frac{2^1 + 2}{3^1 - 1} = \frac{4}{2} = 2$

$f(2) = \frac{2^2 + 2}{3^2 - 1} = \frac{6}{8} = \frac{3}{4}$

$m = \frac{\frac{3}{4} - 2}{2 - 1} = \frac{3}{4} - 2 = \frac{-5}{4}$

$y - 2 = -\frac{5}{4}(x - 1)$ · $y - 2 = -\frac{5}{4}x + \frac{5}{4}$

$y = -\frac{5}{4}x + \frac{5}{4} + 2$; $\underline{\underline{y = -\frac{5}{4}x + \frac{13}{4}}}$

$y(1'3) = \frac{-5 \cdot 1'3}{4} + \frac{13}{4} = \frac{-6'5 + 13}{4} = \frac{6'5}{4} = \underline{\underline{1'625}}$

4) i) $(x^2+x+3)^{1/2} = 3$; $\sqrt{x^2+x+3} = 3$

$x^2+x+3 = 9$; $x^2+x-6 = 0$

$x = \frac{-1 \pm \sqrt{1+24}}{2} = \frac{-1 \pm 5}{2} = \begin{cases} 2 \\ -3 \end{cases}$

ii) $3^{3x-2} = 12$; $\log 3^{3x-2} = \log 12$

$(3x-2)\log 3 = \log 12$ $\quad (3x-2) = \frac{\log 12}{\log 3} = 2'26$

$3x = 4'26$; $x = \frac{4'26}{3} = 1'42$

iii) $x^2-7 = 16^{1/4} = \sqrt[4]{16} = 2$

$x^2 = 9 \Rightarrow x = \pm\sqrt{9} < \begin{matrix} 3 \\ -3 \end{matrix}$

b) $150 = 300 \cdot \left(1 + \frac{-4}{100}\right)^t$

$150 = 300 \cdot 0'96^t$; $\frac{150}{300} = 0'96^t$

$0'5 = 0'96^t$; $\log 0'5 = \log 0'96^t = t \cdot \log 0'96$

$t = \frac{\log 0'5}{\log 0'96} = 16'97$ días

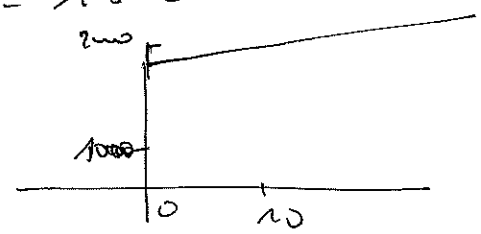
c) TEÓRICO

1) a) x = años trabajados

y = sueldo (en €)

$y = 1820 + \frac{45x}{3}$; $y = 1820 + 15x$

x	0	10
y	1820	1970



b) i) $D_f = \mathbb{R}$; $D_g = (-\infty, -4] \cup [3, +\infty)$

$x^2 + x - 12 = 0$ $\begin{cases} x = 3 \\ x = -4 \end{cases}$

ii) $12 = \sqrt{x^2 + x - 12}$; $144 = x^2 + x - 12$

$0 = x^2 + x - 156$; $x = \frac{-1 \pm \sqrt{1 + 624}}{2}$ — sí

iii) $f(-x) = \frac{2(-x) - 4}{(-x)^2 + 1} = \frac{-2x - 4}{x^2 + 1}$ = $\begin{cases} f(x) \to \text{NO} \\ -f(x) \to \text{NO} \end{cases}$

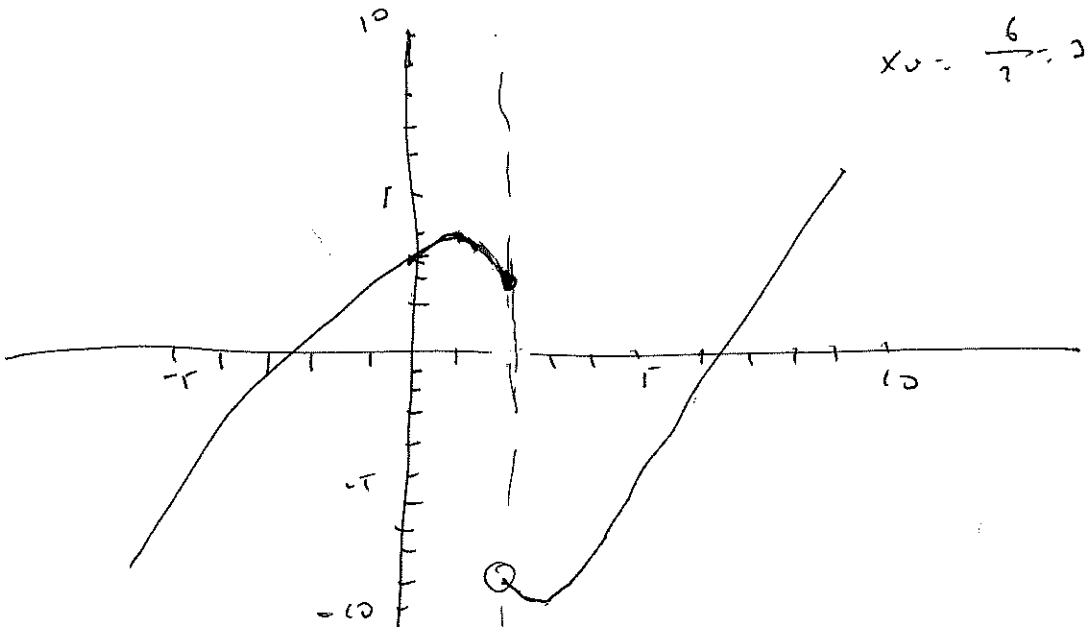
2) a) $f(x) = \begin{cases} -x^2 + 2x + 3 & \text{si } x \leq 2 \\ x^2 - 6x & \text{si } x > 2 \end{cases}$

x	0	2
y	3	3

$x_v = \frac{-2}{-1} = 1$; $y_v = 4$

x	2	3
y	-8	-9

$x_v = \frac{6}{2} = 3$; $y_v = -9$



b)

x	1	2
y	-1	r

$m = \frac{5-(-1)}{2-1} = \frac{6}{1} = 6$

$y = 6x + n$; $r = 6 \cdot 2 + n$; $r = 12 + n$; $n = -7$

$y = 6x - 7$ $\quad y(1'4) = 6 \cdot 1'4 - 7 = 8'4 - 7 = \underline{1'4}$

3) a) $4^x - 3 \cdot 2^x - 4 = 0$; $(2^2)^x - 3 \cdot 2^x - 4 = 0$

$2^x = t$ $\quad t^2 - 3t - 4 = 0$; $t = \frac{3 \pm \sqrt{9+16}}{2} = \frac{3 \pm 5}{2} \begin{cases} 4 \\ -1 \end{cases}$

$2^x = 4 \Rightarrow \underline{x = 2}$ $\quad 2^x = -1 \Rightarrow \underline{\nexists x \in \mathbb{R}}$

b) i) $x^{-4} = \frac{1}{81}$; $\frac{1}{x^4} = \frac{1}{81}$; $x^4 = 81$; $x = \pm\sqrt[4]{81} \begin{cases} 3 \\ -3 \end{cases}$

ii) $2^x = \frac{1}{\sqrt{2}}$; $\log 2^x = \log \frac{1}{\sqrt{2}}$; $x \log 2 = \log \frac{1}{\sqrt{2}}$

$x = \frac{\log \frac{1}{\sqrt{2}}}{\log 2} = \underline{-0'5}$

iii) $4^{3/2} = x$; $x = \sqrt{4^3} = \sqrt{64} = 8$

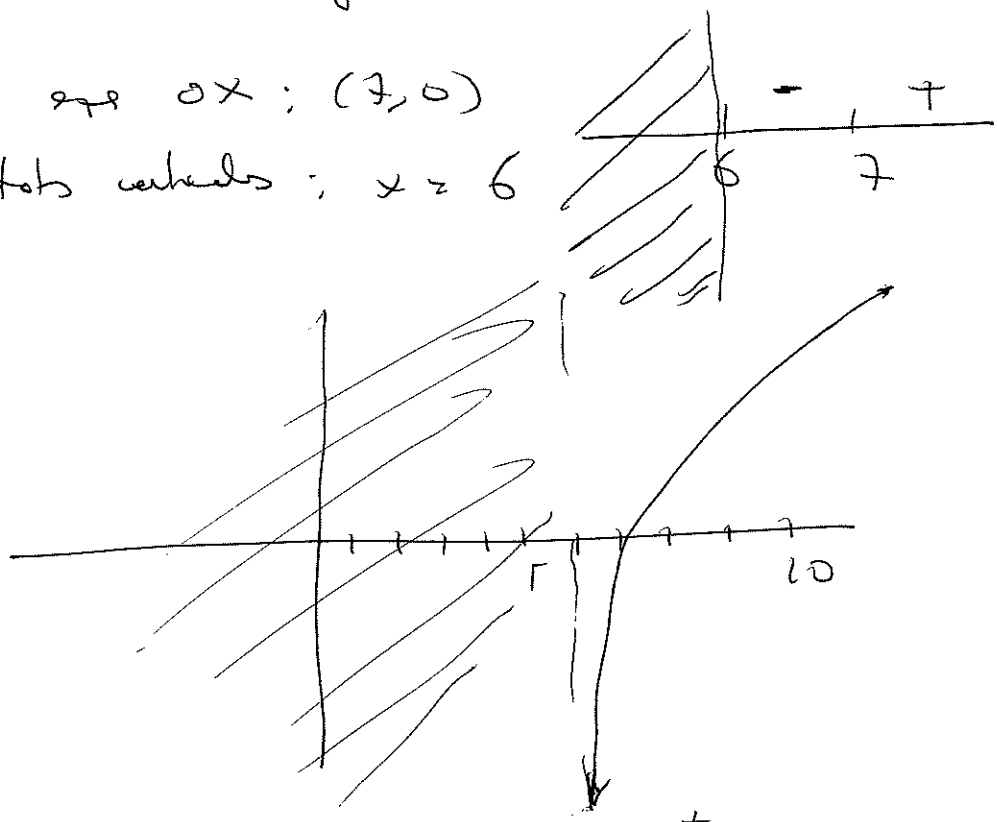
4) a) $f(x) = \log(x-6)$

i) Dominio: $x - 6 > 0 \Rightarrow x > 6$; $D_f = (6, +\infty)$

ii) Corte ejes $\begin{cases} OX:\ y=0 \Rightarrow 0=\log(x-6) \Rightarrow x-6=1 \\ \Rightarrow x=7 \rightarrow (7,0) \\ OY:\ x=0 \Rightarrow \log(0-6)=\log(-6) \rightarrow \nexists \text{ (No vale)} \end{cases}$

iii) Signo: Corte eje OX: $(7,0)$

Asíntota vertical: $x=6$



+: $(7,+\infty)$

-: $(6,7)$

b)

i) $9839 = 8000\left(1+\frac{3}{100}\right)^t = 8000 \cdot 1'03^t$

$\frac{9839}{8000} = 1'03^t \quad 1'229875 = 1'03^t$

$\log 1'229875 = \log 1'03^t = t \cdot \log 1'03$

$t = \frac{\log 1'229875}{\log 1'03} = \underline{\underline{7 \text{ años}}}$

ii) $24.000 = 8000\left(1+\frac{\%TV}{\frac{100}{x}}\right)^{15}$

$\frac{24.000}{8000} = (1+x)^{15} \quad 3=(1+x)^{15}$

$1+x = \sqrt[15]{3} \quad 1+x = 1'0759$

$x = 1'0759-1 = 0'0759$

$\%TV = 100 \cdot 0'0759 = \underline{\underline{7'59\%}}$

1)

i) $\lim_{x\to+\infty} \dfrac{(\sqrt{x^2-x+2}-\sqrt{x^2-3x-1})(\sqrt{x^2-x+2}+\sqrt{x^2-3x-1})}{(\sqrt{x^2-x+2}+\sqrt{x^2-3x-1})} =$

$= \lim_{x\to+\infty} \dfrac{(x^2-x+2)-(x^2-3x-1)}{\sqrt{x^2-x+2}+\sqrt{x^2-3x-1}} = \lim_{x\to+\infty} \dfrac{2x+3}{x+x} = \dfrac{2x}{2x} = 1$ //

ii) $\lim_{x\to 0} \dfrac{(\sqrt{4+x}-2)(\sqrt{4+x}+2)}{(x^2+2x)(\sqrt{4+x}+2)} =$

$= \lim_{x\to 0} \dfrac{(4+x)-4}{(x^2+2x)(\sqrt{4+x}+2)} = \lim_{x\to 0} \dfrac{x}{x(x+2)(\sqrt{4+x}+2)} =$

$= \lim_{x\to 0} \dfrac{1}{(x+2)(\sqrt{4+x}+2)} = \dfrac{1}{2\cdot(2+2)} = \dfrac{1}{8}$ //

iii) $\lim_{x\to -1} \dfrac{(x+1)(x^2-x+1)}{(x+1)(x-4)} =$

	1	0	0	1
-1		-1	1	-1
	1	-1	1	0

$x^2-x+1=0 \quad x=\dfrac{1\pm\sqrt{1-4}}{2} \notin \mathbb{R}$

$= \lim_{x\to -1} \dfrac{x^2-x+1}{x-4} = \dfrac{(-1)^2-(-1)+1}{(-1)-4} = \dfrac{-3}{5}$ //

iv) $\lim_{x\to-\infty} \left(\dfrac{x^3-3x^2+1}{2x^3-x^2+2}\right)^{\frac{x^4-x}{x^3-3x^2}} = \left(\dfrac{1}{2}\right)^{-\infty} = 2^{+\infty} = +\infty$

v) $\lim_{x\to 2} \left(\dfrac{x(x^2-4)-x^2(x-2)}{(x-2)(x^2-4)}\right) = \lim_{x\to 2} \left(\dfrac{x^3-4x-x^3+2x^2}{(x-2)(x^2-4)}\right)$

$= \lim_{x\to 2} \dfrac{2x^2-4x}{(x-2)(x^2-4)} = \lim_{x\to 2} \dfrac{2x(x-2)}{(x-2)(x+2)(x-2)} =$

$= \lim_{x\to 2} \dfrac{2x}{(x+2)(x-2)} = \infty$

2) FUNCIONES

$x = 2$ D.I.S.I

$\lim_{x \to 2^-} f(x) = +\infty$

$\lim_{x \to 2^+} f(x) = -\infty$

$f(2) = \nexists$

CONEXIONES

$x = 1$ CONTINUA

$\lim_{x \to 1^-} f(x) = 2$

$\lim_{x \to 1^+} f(x) = \frac{-2}{1-2} = 2$

$f(1) = 2$

$\lim_{x \to 1^-} \frac{x^2-1}{x-1} = \lim_{x \to 1^-} \frac{(x+1)(x-1)}{(x-1)} = 2$

3) a) $\lim_{x \to 3^-} f(x) = \frac{3+1}{3-1} = \frac{4}{2} = 2$

$\lim_{x \to 3^+} f(x) = \frac{3+3k}{9}$

$f(3) = \frac{3+1}{3-1} = \frac{4}{2} = 2$

$\frac{3+3k}{9} = 2 \quad 3+3k = 18 \quad 3k = 15 \quad k = 5$

b) FUNCIONES

$x = 1$ D.I.S.I

$\lim_{x \to 1^-} f(x) = -\infty$

$\lim_{x \to 1^+} f(x) = +\infty$

$f(1) = \nexists$

CONEXIONES

$x = 0$ D.I.S.I

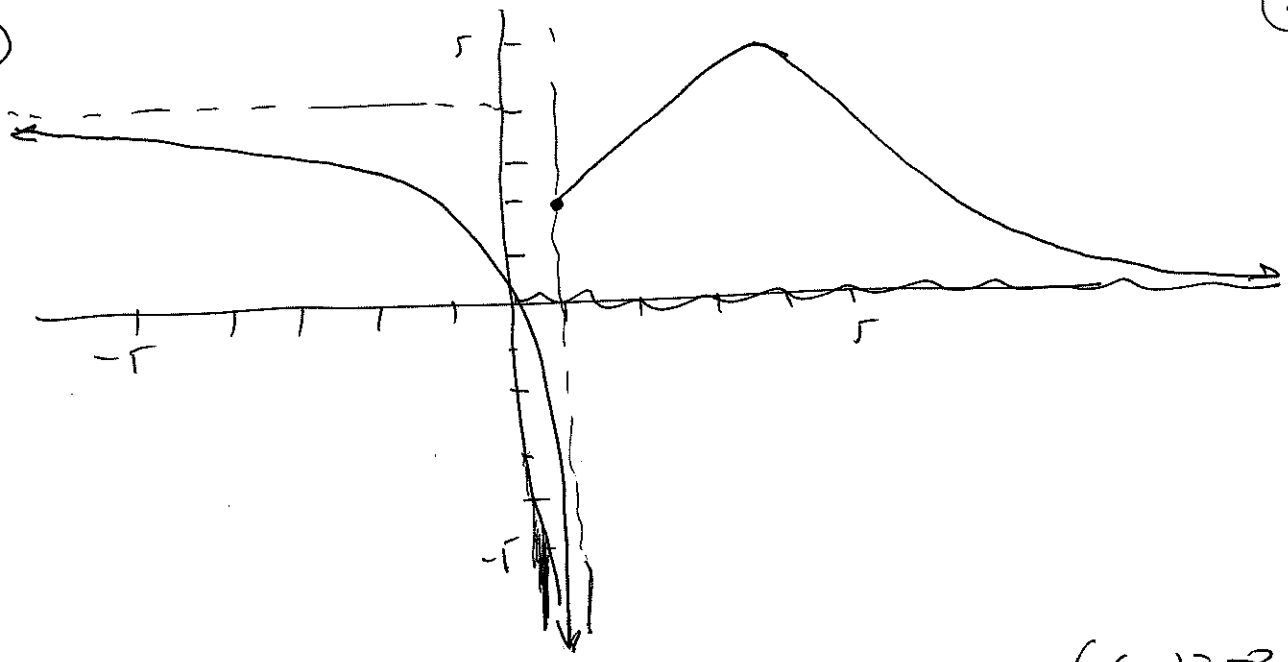
$x = 3$ CONTINUA

$\lim_{x \to 0^-} f(x) = -\infty$

$\lim_{x \to 0^+} f(x) = -1$

$f(0) = -1$

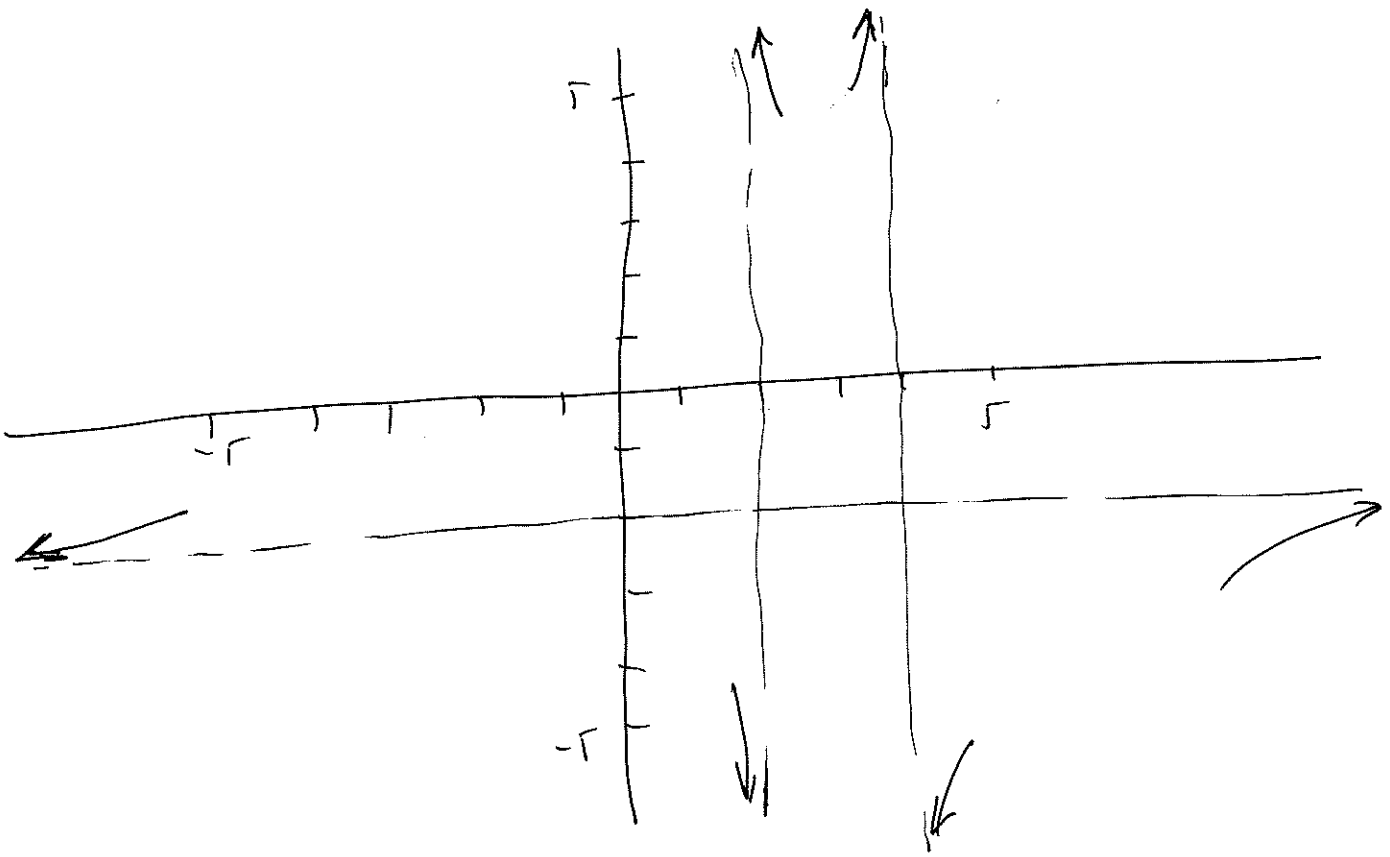
4) a)



b) A.H: $\lim_{x \to +\infty} f(x) = -2$; $\lim_{x \to -\infty} f(x) = -2$

$\boxed{y = -2}$ Dos lados.

AV: $x^2 - 6x + 8 = 0$ → $\boxed{x = 2}$; $\boxed{x = 4}$



1) a) i) $\lim_{x \to 1^-} f(x) = \frac{1+1}{1^2-4} = \frac{2}{-3} = -\frac{2}{3}$

$\exists \lim_{x \to 1^+} f(x) = \frac{a \cdot 1 + 2}{1+1} = \frac{a+2}{2}$

$f(1) = \frac{a \cdot 1 + 2}{1+1} = \frac{a+2}{2}$

$-\frac{2}{3} = \frac{a+2}{2} \Rightarrow -4 = 3a + 6; \quad 3a = -10; \quad a = \frac{-10}{3}$

ii)

FUNCIONES

$x^2 - 4 = 0 \rightarrow$ (x = −2 → D.I.S.I) → ~~x = 2~~

$x + 1 = 0 \rightarrow$ ~~x = −1~~

$x - 2 = 0 \rightarrow$ ~~x = 2~~

CONEXIONES

x = 1 → D.I.S.F

x = 4 → D.E

$\exists \lim_{x \to -2^-} f(x) = -\infty$	$\exists \lim_{x \to 1^-} f(x) = -\frac{2}{3}$	$\exists \lim_{x \to 4^-} f(x) = 2$
$\exists \lim_{x \to -2^+} f(x) = +\infty$	$\exists \lim_{x \to 1^+} f(x) = 2$	$\exists \lim_{x \to 4^+} f(x) = 2$
$f(-2) = \nexists$	$f(1) = 2$	$f(4) = \nexists$

2) i) $\lim_{x \to 3} \frac{(\sqrt{x^2-x+3} - 3)(\sqrt{x^2-x+3} + 3)}{(x^2 + 2x - 15)(\sqrt{x^2-x+3} + 3)} =$

$= \lim_{x \to 3} \frac{x^2 - x + 3 - 9}{(x^2+2x-15)(\sqrt{x^2-x+3} + 3)} = \lim_{x \to 3} \frac{x^2 - x - 6}{(x^2+2x-15)(\sqrt{x^2-x+3} + 3)} =$

$= \lim_{x \to 3} \frac{(x-3)(x+2)}{(x-3)(x+5)(\sqrt{x^2-x+3} + 3)} =$

$= \lim_{x \to 3} \frac{(x+2)}{(x+5)(\sqrt{x^2-x+3} + 3)} = \frac{5}{8 \cdot (3+3)} = \frac{5}{48}$ //

ii) $\lim_{x\to+\infty} \dfrac{\left(\sqrt{4x^4-3x^2+1}-\sqrt{4x^4-x^2-2}\right)\left(\sqrt{4x^4-3x^2+1}+\sqrt{4x^4-x^2-2}\right)}{\sqrt{4x^4-3x^2+1}+\sqrt{4x^4-x^2-2}} =$

$= \lim_{x\to+\infty} \dfrac{(4x^4-3x^2+1)-(4x^4-x^2-2)}{\sqrt{4x^4-3x^2+1}+\sqrt{4x^4-x^2-2}} =$

$= \lim_{x\to+\infty} \dfrac{4x^4-3x^2+1-4x^4+x^2+2}{\sqrt{4x^4-3x^2+1}+\sqrt{4x^4-x^2-2}} =$

$= \lim_{x\to+\infty} \dfrac{-2x^2+3}{2x^2+2x^2} = \lim_{x\to+\infty} \dfrac{-2x^2+3}{4x^2} = -\dfrac{2}{4} = -\dfrac{1}{2}$

iii) $\lim_{x\to+\infty} \left(\dfrac{(2x^2-x)(x-2)-(2x^2+1)(x+1)}{(x+1)(x-2)}\right) =$

$= \lim_{x\to+\infty} \dfrac{2x^3-4x^2-x^2+2x-2x^3-2x^2-x-1}{x^2-2x+x-2} =$

$= \lim_{x\to+\infty} \dfrac{-7x^2+x-1}{x^2-x-2} = -7$ //

3) a) $x=3$; $f(3)=3\cdot\sqrt{3+1}=3\cdot 2=6$

$P(3,6)$

$f'(x)=1\cdot\sqrt{x+1}+x\cdot\dfrac{1}{2\sqrt{x+1}}=\sqrt{x+1}+\dfrac{x}{2\sqrt{x+1}}$

$f'(3)=\sqrt{3+1}+\dfrac{3}{2\sqrt{3+1}}=2+\dfrac{3}{4}=\dfrac{11}{4}$

$$\boxed{y-6=\frac{11}{4}(x-3)}$$

b) i) $f(2) = -8$ $\quad f(x) = ax^3 - bx^2 - 12$

$f'(2) = 0$ $\quad f'(x) = 3ax^2 - 2bx$

$f(2) = a \cdot 2^3 - b \cdot 2^2 - 12 = 8a - 4b - 12 = -8$

$$\boxed{8a - 4b = 4}$$

$f'(2) = 3a \cdot 2^2 - 2 \cdot b \cdot 2 = \boxed{12a - 4b = 0}$

$$\left.\begin{array}{l} 8a - 4b = 4 \\ 12a - 4b = 0 \end{array}\right\} \quad \boxed{\begin{array}{l} a = -1 \\ b = -3 \end{array}}$$

$f(x) = -x^3 + 3x^2 - 12$

$f'(x) = -3x^2 + 6x = 0 \; ; \; -3x(x-2) = 0$

$x = 0 \; ; \quad x - 2 = 0 \rightarrow x = 2$

$E_1(0, -12)$ Min abs. $E_2(2, -8)$ Max relat

$f''(x) = -6x + 6 \rightarrow f''(0) = 6 > 0$

$f''(2) = -6 < 0$

$-6x + 6 = 0 \; ; \; -6x = -6 \; ; \; x = \frac{-6}{-6} = 1$

Inflex $(1, -10)$

Descr	Creix	Creix	Descr
	0	1	2
	∪	Inflex	∩

4) i) $D = \mathbb{R} - \{-2, 2\}$ $\quad x^2-4=0 \Rightarrow x = \begin{matrix}-2\\2\end{matrix}$

ii) C. tăi $\begin{cases} OX: \ y=0 \rightarrow \text{NU CONTA} \\ OY: \ x=0; \ y = \frac{4}{-1} = -4 \rightarrow (0,-4) \end{cases}$

iii) Asimptote:

Orizontale: $\lim_{x\to+\infty} \frac{4}{x^2-4} = 0$; $\lim_{x\to-\infty} \frac{4}{x^2-4} = 0$

$\boxed{y=0}$ Dă bine

Verticale: $x^2-4=0$; $\boxed{x=-2}$; $\boxed{x=2}$

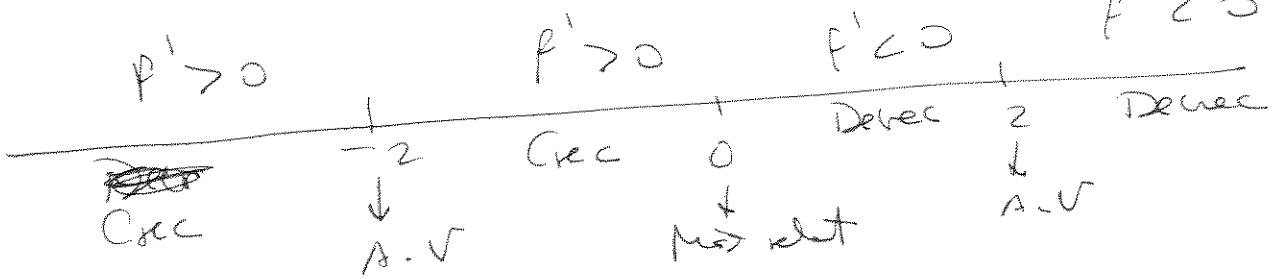
$\lim_{x\to-2^-} f(x) = +\infty$; $\lim_{x\to-2^+} f(x) = -\infty$

$\lim_{x\to 2^+} f(x) = +\infty$

$\lim_{x\to 2^-} f(x) = -\infty$;

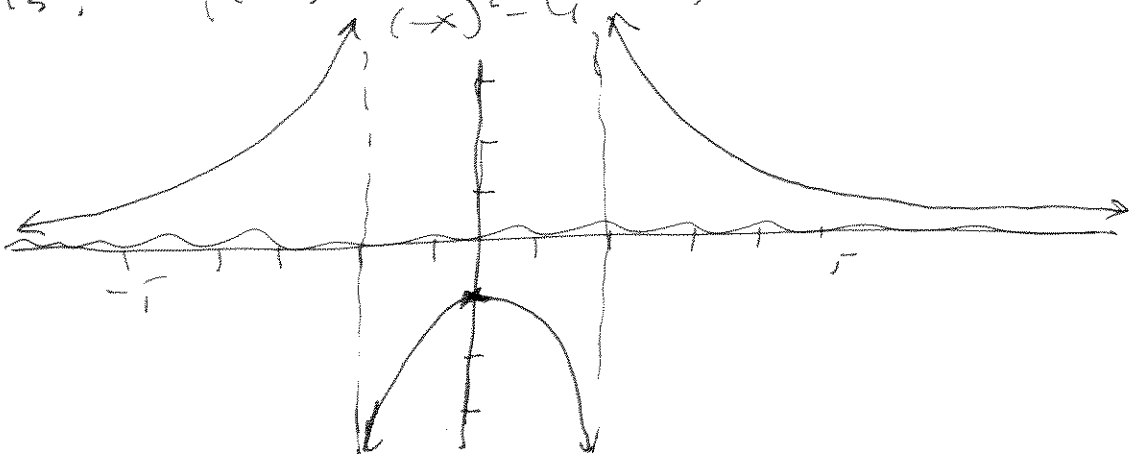
iv) Extreme elem:

$$f'(x) = \frac{0\cdot(x^2-4) - 2x\cdot 4}{(x^2-4)^2} = \frac{-8x}{(x^2-4)^2} = 0 \Rightarrow x = 0$$



$E_1(0,-1)$ – Max elt.

v) Simetrie: $f(-x) = \frac{4}{(-x)^2-4} = \frac{4}{x^2-4} = f(x)$ – PAR



1) FUNCIONES

$x = 1 \rightarrow$ D.I.S.I

CONEXIONES

$x = -2 \rightarrow$ D.E

$x = 4 \rightarrow$ CONTINUA

$\lim_{x \to 1^-} f(x) = -\infty$

$\lim_{x \to 1^+} f(x) = +\infty$

$f(1) = \nexists$

$\lim_{x \to -2^-} f(x) = 0$

$\lim_{x \to -2^+} f(x) = 0$

$f(-2) = \nexists$

$\lim_{x \to 4^-} f(x) = 2$

$\lim_{x \to 4^+} f(x) = 2$

$f(4) = 2$

2) i) $\lim_{x \to 1} \dfrac{(\sqrt{2x-1} - x)(\sqrt{2x-1} + x)}{(x^2 + x - 2)(\sqrt{2x-1} + x)} =$

$= \lim_{x \to 1} \dfrac{(2x-1) - x^2}{(x^2 + x - 2)(\sqrt{2x-1} + x)} =$

$= \lim_{x \to 1} \dfrac{-x^2 + 2x - 1}{(x^2 + x - 2)(\sqrt{2x-1} + x)} = \lim_{x \to 1} \dfrac{-(x-1)^2}{(x-1)(x+2)(\sqrt{2x-1} + x)} =$

$= \lim_{x \to 1} \dfrac{-(x-1)}{(x+2)(\sqrt{2x-1} + x)} = \dfrac{-0}{3 \cdot (1+1)} = \dfrac{-0}{6} = 0 //$

ii) $\lim_{x \to 2} \frac{(x-2)(x^2+2x+4)}{x(x^2-3x+2)} =$

	1	0	0	−8
2		2	4	8
	1	2	4	0

$= \lim_{x \to 2} \frac{(x-2)(x^2+2x+4)}{x(x-2)(x-1)} =$

$= \lim_{x \to 2} \frac{x^2+2x+4}{x(x-1)} = \frac{4+4+4}{2 \cdot 1} = \frac{12}{2} = 6$ //

iii) $\lim_{x \to \infty} \left(\frac{2x^2(2x-3) - 4x^2(x-1)}{(x-1)(2x-3)} \right) =$

$= \lim_{x \to \infty} \left(\frac{4x^3 - 6x^2 - 4x^3 + 4x^2}{2x^2 - 3x - 2x + 3} \right) =$

$= \lim_{x \to \infty} \frac{-2x^2}{2x^2 - 5x + 3} = \frac{-2}{2} = -1$ //

3) a) $f'(x) = \frac{1}{2\sqrt{\frac{x+1}{x-2}}} \cdot \frac{1(x-2) - 1 \cdot (x+1)}{(x-2)^2} =$

$= \frac{1}{2\sqrt{\frac{x+1}{x-2}}} \cdot \frac{x - 2 - x - 1}{(x-2)^2} =$

$= \frac{\sqrt{x-2}}{2\sqrt{x+1}} \cdot \frac{-3}{(x-2)^2} = \frac{-3\sqrt{x-2}}{2\sqrt{x+1}\,(x-2)^2}$ //

$$g'(x) = 2\left(\frac{x^2+2}{x^2-1}\right)\cdot\frac{2x(x^2-1)-2x(x^2+2)}{(x^2-1)^2} =$$

$$= 2\left(\frac{x^2+2}{x^2-1}\right)\cdot\frac{2x^3-2x-2x^3-4x}{(x^2-1)^2} =$$

$$= 2\left(\frac{x^2+2}{x^2-1}\right)\frac{-6x}{(x^2-1)^2} = \frac{-12x(x^2+2)}{(x^2-1)^3} //$$

b) i) $f(2) = 7$; $f'(1) = 0$

$f'(x) = 3x^2 + a$

$f(2) = 2^3 + a\cdot 2 + b = 8 + 2a + b = 7 \cdot \quad 2a+b = -1$

$f'(1) = 3\cdot 1^2 + a = 3 + a = 0 \Rightarrow a = -3$ //

$2\cdot(-3) + b = -1 \cdot \quad -6 + b = -1 \cdot \quad b = 5$ //

ii) $f(x) = x^3 - 3x + 5$

$f'(x) = 3x^2 - 3 = 0 \Rightarrow 3x^2 = 3 \cdot x^2 = 1 \cdot x = \pm 1$

$f(1) = 1^3 - 3\cdot 1 + 5 = 3$

$f(-1) = (-1)^3 - 3(-1) + 5 = 7$

$E_1(1, 3)$ Mínimo local $\quad E_2(-1, 7)$ Máx relat

$f''(x) = 6x$

$f''(1) = 6 > 0$

$f''(-1) = -6 < 0$

$6x = 0 \Rightarrow x = 0$

$f(0) = 5$

$(0, 5)$ Inflexión

Crec + Derec + Derec + Crec

-1 0 1

4) i) $D = \mathbb{R}$

ii) Corte ejes $\begin{cases} OX: (0,0) \\ OY: (0,0) \end{cases}$ $2x=0 \Rightarrow x=0$

iii) Asintotas Horizontales

$\lim_{x\to+\infty} \frac{2x}{x^2+4} = 0$ $\lim_{x\to-\infty} \frac{2x}{x^2+4} = 0$

$y = 0$ Dos veces

Asintotas verticales

$x^2+4=0 \to \nexists$ sol no tiene

iv) Crec y Derec

$$f'(x) = \frac{2(x^2+4) - 2x\cdot 2x}{(x^2+4)^2} = \frac{2x^2+8-4x^2}{(x^2+4)^2} =$$

$$= \frac{-2x^2+8}{(x^2+4)^2} = 0$$

$-2x^2+8=0 \cdot \quad 2x^2=8; \quad x^2=4; \quad x=\pm 2$

$E_1\left(2, \frac{4}{8}\right)$ $E_2\left(-2, -\frac{4}{8}\right)$

$f'<0$ $f'>0$ $f'<0$

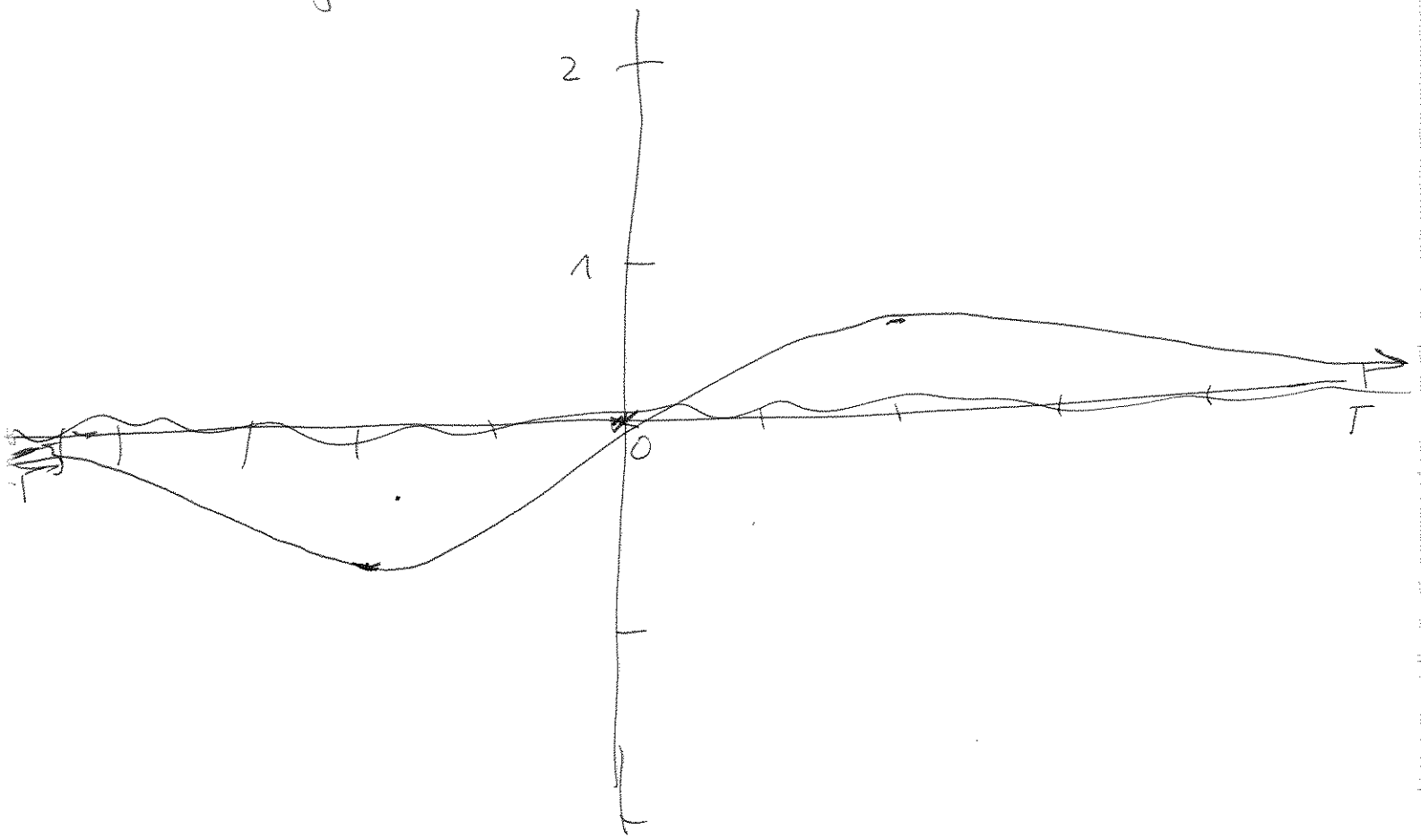
Derec | -2 | Crec | 2 | Derec

-2: mínimo 2: máximo

v) Simetría.

$$f(-x) = \frac{2(-x)}{(-x)^2+4} = \frac{-2x}{x^2+4} = \begin{cases} f(x) \to \text{NO} \\ -f(x) \to \text{SI} \end{cases}$$

Impar



1)

x	y	f	xf	yf	xxf	yyf	xyf
0	8	4	0	32	0	256	0
0	3	3	0	9	0	27	0
3	4	4	12	16	36	64	48
4	2	5	20	10	80	20	40
6	1	6	36	6	216	6	36
8	0	2	16	0	128	0	0
9	0	1	9	0	81	0	0
		25	93	73	541	373	124

Med x	3,72		Cov	-5,9024
Med y	2,92		r	-0,8357
Var x	7,8016		Fiab	69,8438
Var y	6,3936			
		Y (x)	-0,7566	5,73441
Dt x	2,79313			
		X(y)	-0,9232	6,41567
Dt y	2,52856			

a) Relación inversa y no muy fuerte

b) $X(5) = -0'92 \cdot 5 + 6'41 = 1'81$ No es muy fiable Fiabil=69'84%

2) a)

1	2	3	4	5	6
2	4	6	8	10	12
3	6	9	12	15	18
4	8	12	16	20	24
5	10	15	20	25	30
6	12	18	24	30	36

b) TEÓRICO

c) $P(A) = \frac{9}{36}$ $P(B) = \frac{28}{36}$

$P(A \cup B) = \frac{29}{36}$

$P(A \cap B) = \frac{8}{36}$

E = {1, 2, 3, 4, 5, 6, 8, 9, 10, 12, 15, 16, 18, 20, 24, 25, 30, 36}

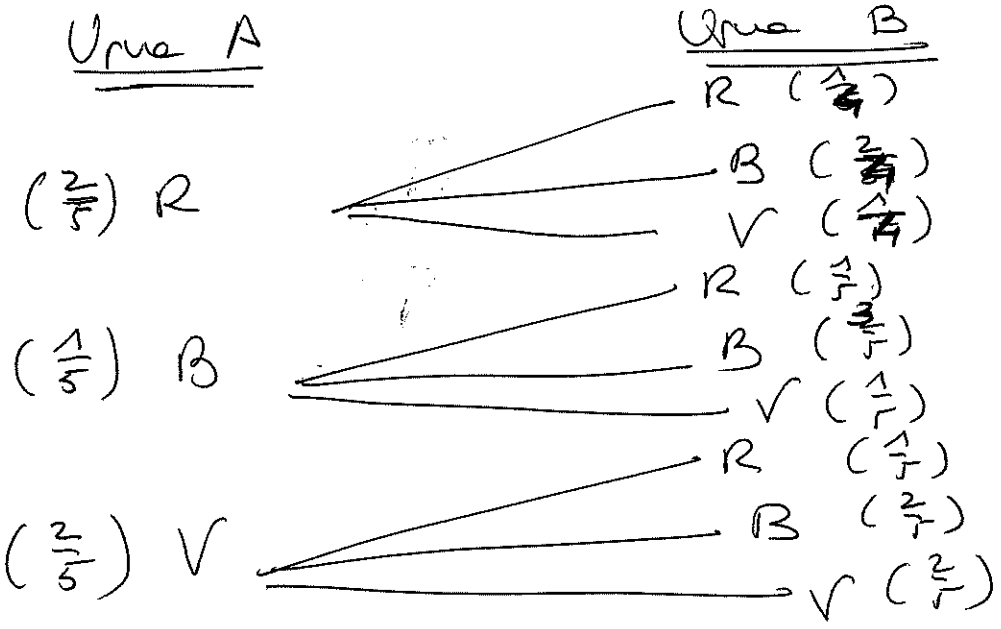
3)

2R
1B
2V

(A)

1R
2B
1V

(B)



$$P(R) = \frac{2}{5}\cdot\frac{1}{4} + \frac{1}{5}\cdot\frac{1}{5} + \frac{2}{5}\cdot\frac{1}{5} = \frac{2}{20} + \frac{1}{25} + \frac{2}{25} =$$

$$= \frac{10+4+8}{100} = \frac{22}{100} \rightarrow 22\% \;//$$

$$P(B) = \frac{2}{5}\cdot\frac{2}{4} + \frac{1}{5}\cdot\frac{3}{5} + \frac{2}{5}\cdot\frac{2}{5} = \frac{4}{20} + \frac{3}{25} + \frac{4}{25} =$$

$$= \frac{20+12+16}{100} = \frac{48}{100} \rightarrow 48\% \;//$$

$$P(V) = \frac{2}{5}\cdot\frac{1}{4} + \frac{1}{5}\cdot\frac{1}{5} + \frac{2}{5}\cdot\frac{2}{5} = \frac{2}{20} + \frac{1}{25} + \frac{4}{25} =$$

$$= \frac{10+4+16}{100} = \frac{30}{100} \rightarrow 30\% \;//$$

4) $n = 15$; $p = 0'9$; $q = 0'1$

$X \longrightarrow B(15;\ 0'9)$

a) $P(X=13) + P(X=14) + P(X=15) =$

$= \binom{15}{13}(0'9)^{13}(0'1)^2 + \binom{15}{14}(0'9)^{14}(0'1)^1 + \binom{15}{15}(0'9)^{15}(0'1)^0 =$

$= 0'266 + 0'343 + 0'205 = \underline{0'814} \rightarrow 81'4\%$

b) $P(X=7) + P(X=8) = \binom{15}{7}(0'9)^7(0'1)^8 + \binom{15}{8}(0'9)^8(0'1)^7 =$

$= 0'000038 + 0'0002 = \underline{0'000238} \rightarrow 0'038\%$

c) $1 - [P(X=13) + P(X=14) + P(X=15)] =$

$= 1 - 0'814 = \underline{0'188} \rightarrow 18'8\%$

5) $X \longrightarrow N(2'5;\ 0'6)$ $\qquad Z = \dfrac{X - 2'5}{0'6}$

a) $P(X \geq 3) = P(Z \geq 0'83) = 1 - P(Z \leq 0'83) =$

$= 1 - 0'7967 = \underline{0'2033} \rightarrow \underline{20'33\%}$

b) $P(X \leq 2'1) = P(Z \leq -0'66) = 1 - P(Z \leq 0'66) =$

$= 1 - 0'7454 = \underline{0'2546} \rightarrow \underline{25'46\%}$

c) $P(2'3 \leq X \leq 2'8) = P(-0'33 \leq Z \leq 0'5) =$

$= P(Z \leq 0'5) - P(Z \leq -0'33) = P(Z \leq 0'5) - [1 - P(Z \leq 0'33)] =$

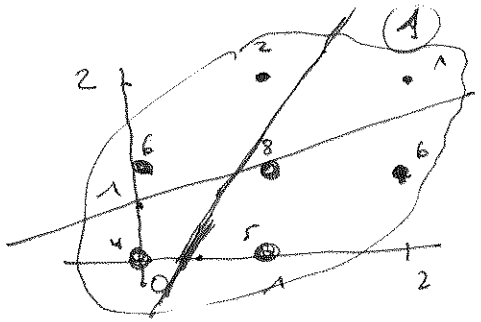
$= 0'6915 - [1 - 0'6293] = \underline{0'3208}$; $0'3208 \times 1000 =$ 1604 personas

1)

a)

x	y	f	xf	yf	xxf	yyf	xyf
0	0	4	0	0	0	0	0
1	0	5	5	0	5	0	0
0	1	6	0	6	0	6	0
1	1	8	8	8	8	8	8
2	1	6	12	6	24	6	12
1	2	2	2	4	2	8	4
2	2	1	2	2	4	4	4
		32	29	26	43	32	28

Med x	0,90625		Cov	0,13867
Med y	0,8125		r	0,3291
Var x	0,52246		Fiab	10,8304
Var y	0,33984			
Dt x	0,72281	Y (x)	0,26542	0,57196
Dt y	0,58296	X(y)	0,40805	0,57471



x	0	2
0	0'57	1'09

x	0'4	1'44
0	0	2

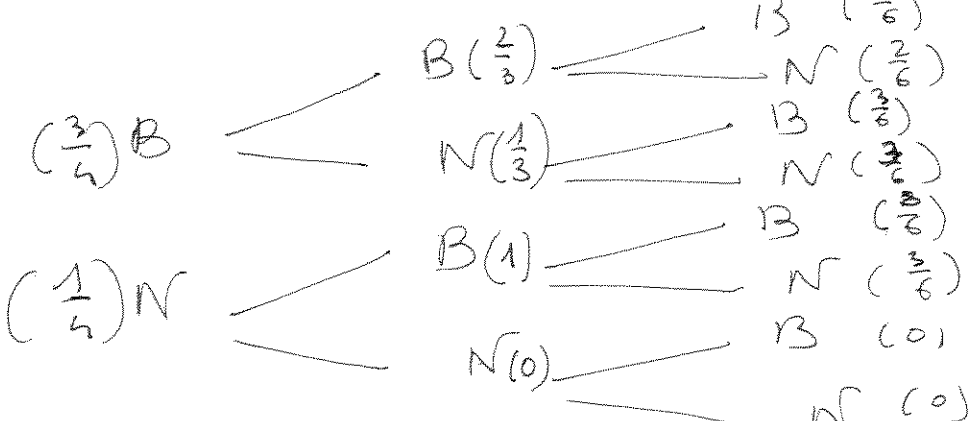
b) $x = 3 \Rightarrow y(3) = 0'26 \times 3 + 0'57 = 1'35$ kilos → no es fiable. Fiab = 10'83 ;

2) $E = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15\}$

$P(A) = 9/15$ $P(B) = 3/15$ $P(C) = 8/15$

$P(A \cup B) = 10/15$; $P(A \cup C) = 11/15$; $P(A \cap B) = \frac{2}{15}$; $P(B \cap C) = \frac{1}{15}$

3): $\boxed{3B\ 1N}$ (A) $\boxed{2B\ 2N}$ (B)



a) $P(B) = \frac{3}{4}\cdot\frac{2}{3}\cdot\frac{4}{6} + \frac{3}{4}\cdot\frac{1}{3}\cdot\frac{3}{6} + \frac{1}{4}\cdot 1\cdot\frac{3}{6} =$

$= \frac{24}{72} + \frac{9}{72} + \frac{3}{24} = \frac{24+9+9}{72} = \frac{42}{72} = \frac{7}{12}$

b) $P(N) = 1 - \frac{7}{12} = \frac{5}{12}$

c) $P(\text{Mismo color}) = \frac{3}{4}\cdot\frac{2}{3} + \frac{1}{4}\cdot 0 = \frac{6}{12} = \frac{1}{2}$ //

4) $n = 10$; $p = 0'2$; $q = 0'8$. $X \to B(10, 0'2)$

a) $P(X=9) + P(X=10) = \binom{10}{9}(0'2)^9(0'8)^1 + \binom{10}{10}(0'2)^{10}(0'8)^0$

$= 4'096\times10^{-6} + 1'02\times10^{-7} = 0'0000041$

b) $P(X=2) + \cdots + P(X=10) = 1 - [P(X=0) + P(X=1)]$

$= 1 - \left[\binom{10}{0}(0'2)^0(0'8)^{10} + \binom{10}{1}(0'2)^1(0'8)^9\right] =$

$= 1 - [0'10 + 0'26] = 1 - 0'36 = \underline{0'64} \to \underline{64\%}$.

c) $P(X\geq 4) + P(X=5) + P(X=6) =$

$= \binom{10}{4}(0'2)^4(0'8)^6 + \binom{10}{5}(0'2)^5(0'8)^5 + \binom{10}{6}(0'2)^6(0'8)^4$

$= 0'08 + 0'02 + 0'005 = \underline{0'105} \to \underline{10'5\%}$.

5) $X \rightarrow N(200, 25)$ $\quad z = \frac{x-200}{25}$ (2)

a) $P(170 \leq x \leq 210) = P(-1'2 \leq z \leq 0'4) =$

$= P(z \leq 0'4) - P(z \leq -1'2) =$

$= 0'6554 - [1 - P(z \leq 1'2)] = 0'6554 - (1 - 0'8849) =$

$= 0'6554 - 0'1151 = \underline{0'5403} \rightarrow \underline{54'03\%}$

b) $P(x \leq 180) = P(z \leq -0'8) = 1 - P(z \leq 0'8) =$

$= 1 - 0'7881 = \underline{0'2119} \rightarrow \underline{21'19\%}$

c) $P(x \geq 250) = P(z \geq 2) = 1 - P(z \leq 2) =$

$= 1 - 0'9772 = 0'0228$

$0'0228 \times 1500 \simeq \underline{34 \text{ pers}}$